Meteorology

Lecture 4

Sahraei

Physics Department

Razi University

http://www.razi.ac.ir/sahraei

The Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{V}) = 0 \qquad \frac{1}{\rho}\frac{d\rho}{dt} + \nabla.\vec{V} = 0$$

Boussinesq approximation

$$\rho = \rho_0 + \rho'(x, y, z, t)$$

$$\rho_0 \gg \rho'$$

$$\frac{\partial}{\partial t}(\rho_0 + \rho') + \nabla.((\rho_0 + \rho')\vec{V}) = 0$$

$$\frac{\partial \rho'}{\partial t} + \nabla.(\rho_0 \vec{V}) + \nabla.(\rho' \vec{V}) = 0$$

$$\rho'\frac{U}{L} \qquad \rho_0\frac{U}{L} \qquad \rho'\frac{U}{L}$$

$$but \quad \rho' \ll \rho_0 \qquad \nabla.(\rho_0\vec{V}) = 0 \qquad \nabla.\vec{V} = 0$$

According to this Boussinesq approximation, the continuity equation reduces to an expression that the flow is incompressible. That is, fluid chunks change their shape, but not their volume.

Thus, here, conservation of mass becomes conservation of volume.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial \rho}{\partial t} = -u\,\frac{\partial \rho}{\partial x} - v\,\frac{\partial \rho}{\partial y} - w\,\frac{\partial \rho}{\partial z} - \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$\frac{\partial \rho}{\partial t} = -u\frac{\partial \rho}{\partial x} - v\frac{\partial \rho}{\partial y} - w\frac{\partial \rho}{\partial z}$$

$$\rho = \rho_0 + \bar{\rho}(z) + \rho'(x, y, z, t)$$

$$\rho_0 \gg \bar{\rho} \gg \rho'$$

This requires that the vertical variation of density is small relative to the mean, and that the horizontal and temporal variations are small relative to the vertical.

This is a good approximation for the ocean where we have

$$\begin{aligned} \rho_o &\cong 1000\,\mathrm{kg/m^3} \\ \bar{\rho} &\cong 10\,\mathrm{kg/m^3} \\ \rho' &\cong 0.1\,\mathrm{kg/m^3} \end{aligned}$$

But not so good for the atmosphere over the bottom 15 km where

$$\begin{aligned} \rho_o &\cong 0.5\,\mathrm{kg/m^3} \\ \bar{\rho} &\cong 0.5\,\mathrm{kg/m^3} \\ \rho' &\cong 0.005\,\mathrm{kg/m^3} \end{aligned}$$

Atmosphere: the analastic approximation

Boussinesq is okay for the ocean, but not so good for the atmosphere. In particular, the difference between ρ_0 *and* $\bar{\rho}(z)$ are large

$$\rho = \bar{\rho}(z) + \rho'(x, y, z, t), \qquad \bar{\rho} \gg \rho' \qquad \rho = \bar{\rho}(z)$$

Substitution yields (recalling that ¯ρ does not depend on t):

$$\frac{\partial \rho'}{\partial t} + \nabla.(\bar{\rho}\vec{V}) + \nabla.(\rho'\vec{V}) = 0$$

Scale analysis shows us that:

$$\rho'\frac{U}{L} \qquad \bar{\rho}\frac{U}{L} \qquad \rho'\frac{U}{L}$$

$$\sin ce \quad \bar{\rho} \gg \rho' \quad we\ have \quad \nabla.(\bar{\rho}\vec{V}) = 0 \qquad \frac{\partial}{\partial x}\bar{\rho}u + \frac{\partial}{\partial y}\bar{\rho}v + \frac{\partial}{\partial z}\bar{\rho}w = 0$$

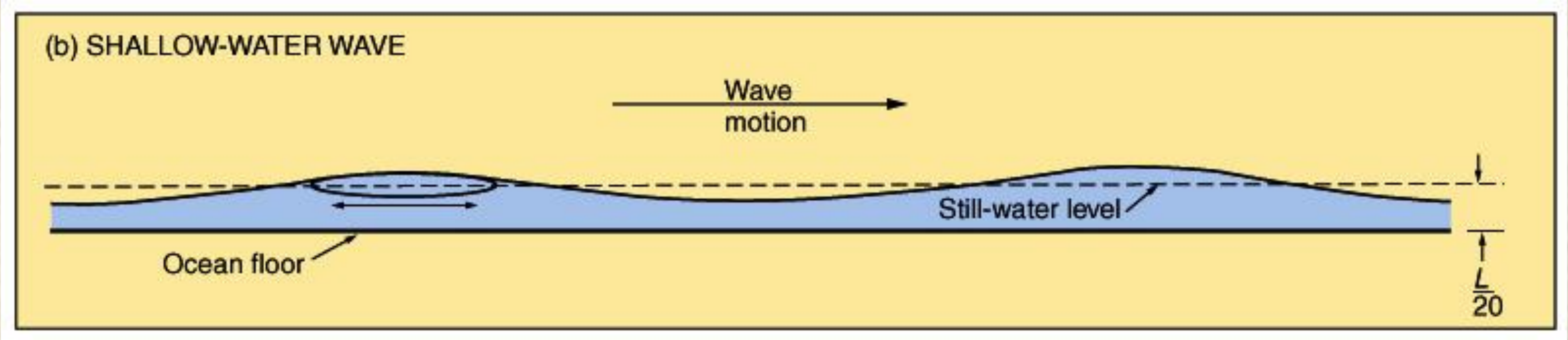
$$\bar{\rho}\nabla.\vec{V} + \vec{V}.\nabla\bar{\rho} = 0$$

This is called the analastic approximation; volume is not conserved, but conservation of vertical part of mass occurs.

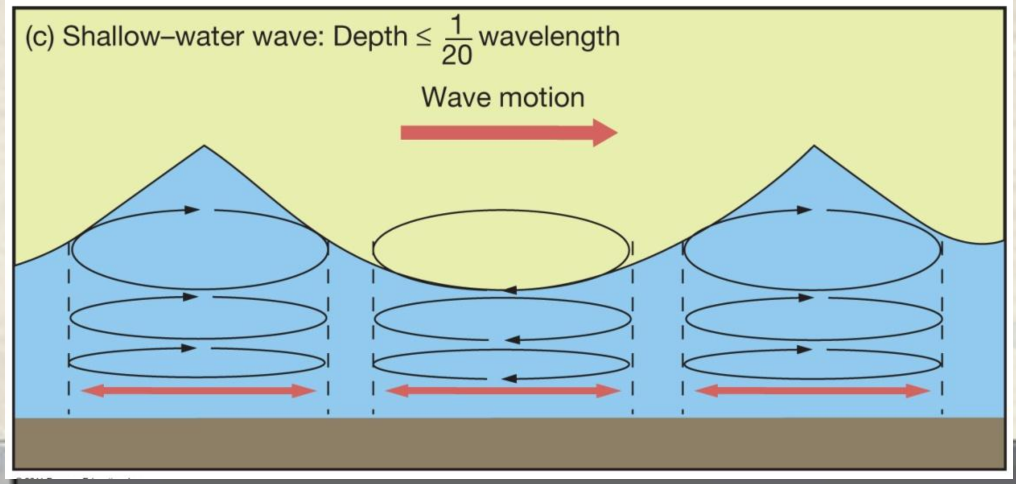
Shallow-water wave

Water depth is less than 1/20 wavelength

Wave speed (celerity) is proportional to depth of water



Orbital motion is flattened



Shallow-Water Wave

The example of pure wave motion concerns the horizontally propagating oscillations known as shallow water waves.

1- Incompressible and homogeneous flow, where ρ_0 is a constant density => no sound waves (simplifies equations)

2- The flow is assumed to be inviscid

3-The water is so shallow that the flow velocity, $V(x, y)$, is constant with depth.

4- Shallow water, Require $\lambda_x \gg h$. Otherwise too deep for hydrostatic assumption.

$$\frac{\partial p}{\partial z} = \rho_0 g$$ hydrostatic approximation

Shallow Water Equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial x} - fv = 0$$

$$\frac{d\rho}{dt} = 0$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial y} + fu = 0$$

$$\rho = \rho_0$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{d\rho}{dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

$$\frac{\partial p}{\partial z} = -\rho_0 g$$

$$\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial p}{\partial x}\right) = 0$$

$$\frac{\partial p}{\partial z} = -\rho_0 g$$

$$\int_{z(P_S)}^{z(P_T)} \frac{\partial p}{\partial z} dz = -\rho_0 g \int_{z(P_S)}^{z(P_T)} dz ,$$

$$P_S - P_T = g \rho_0 h ,$$

If $P_T = 0$ or $P_T \ll P_S$

$$\frac{P_S}{\rho_0} = gh$$

$$\frac{1}{\rho_0} \frac{\partial P_S}{\partial x} = g \frac{\partial h}{\partial x} .$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\int_0^z \frac{\partial w}{\partial z} dz = w_z - w_s = -\int_0^z \left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) dz .$$

$$w_h - w_S = -\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) h$$

$$w_h = \frac{dh}{dt}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+g\frac{\partial h}{\partial x}-fv=0$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+g\frac{\partial h}{\partial y}+fu=0$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}+h\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0$$

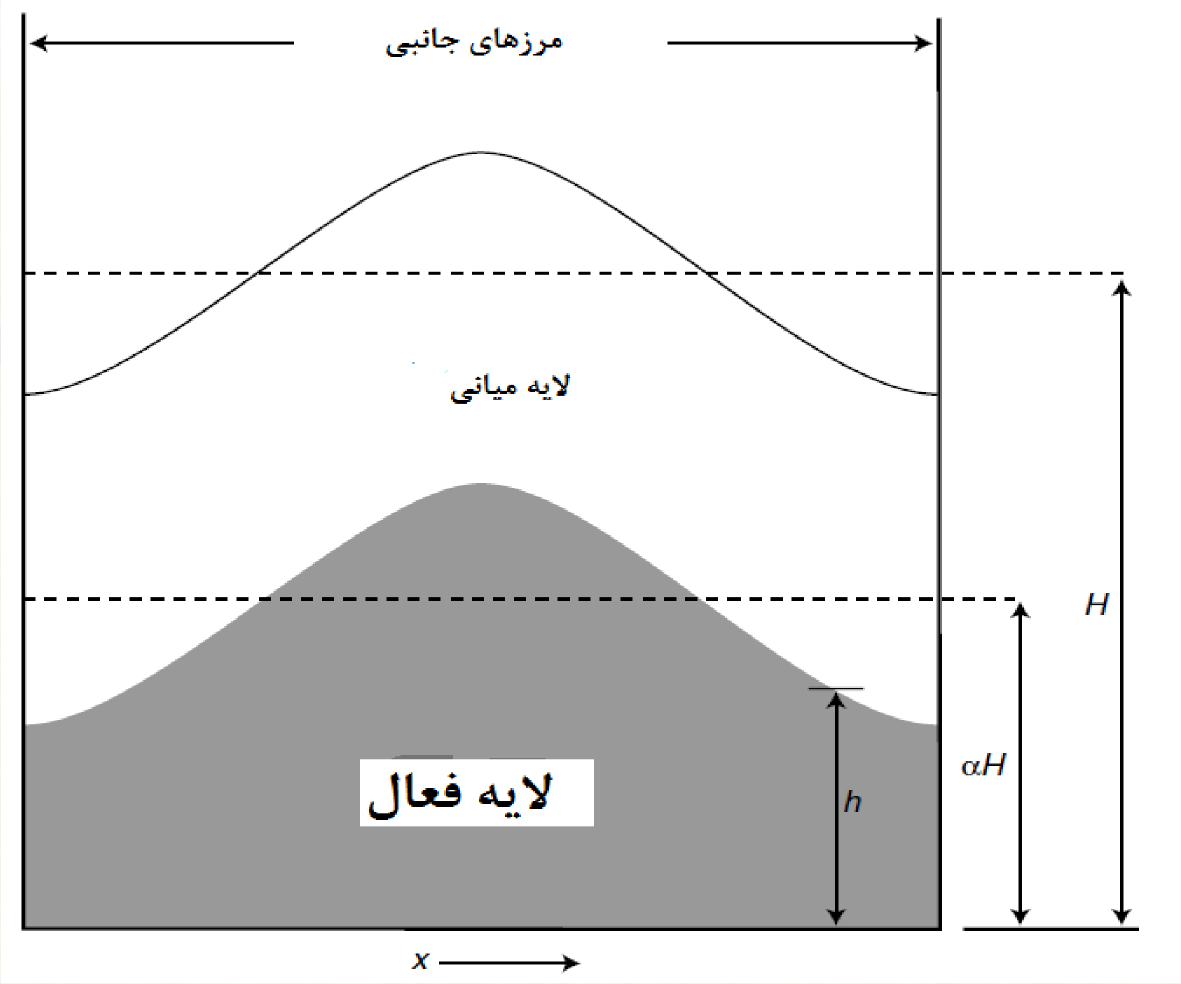
$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial h}{\partial x}-fv=0$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+g\frac{\partial h}{\partial y}+fu=0$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}+h(\frac{\partial u}{\partial x})=0$$

$$\frac{\partial h}{\partial y}=-\frac{f}{g}\bar{U}$$

$$H=\frac{RT_0}{g},$$


مرزهای جانبی
لایه میانی
لایه فعال
H
αH
h
x