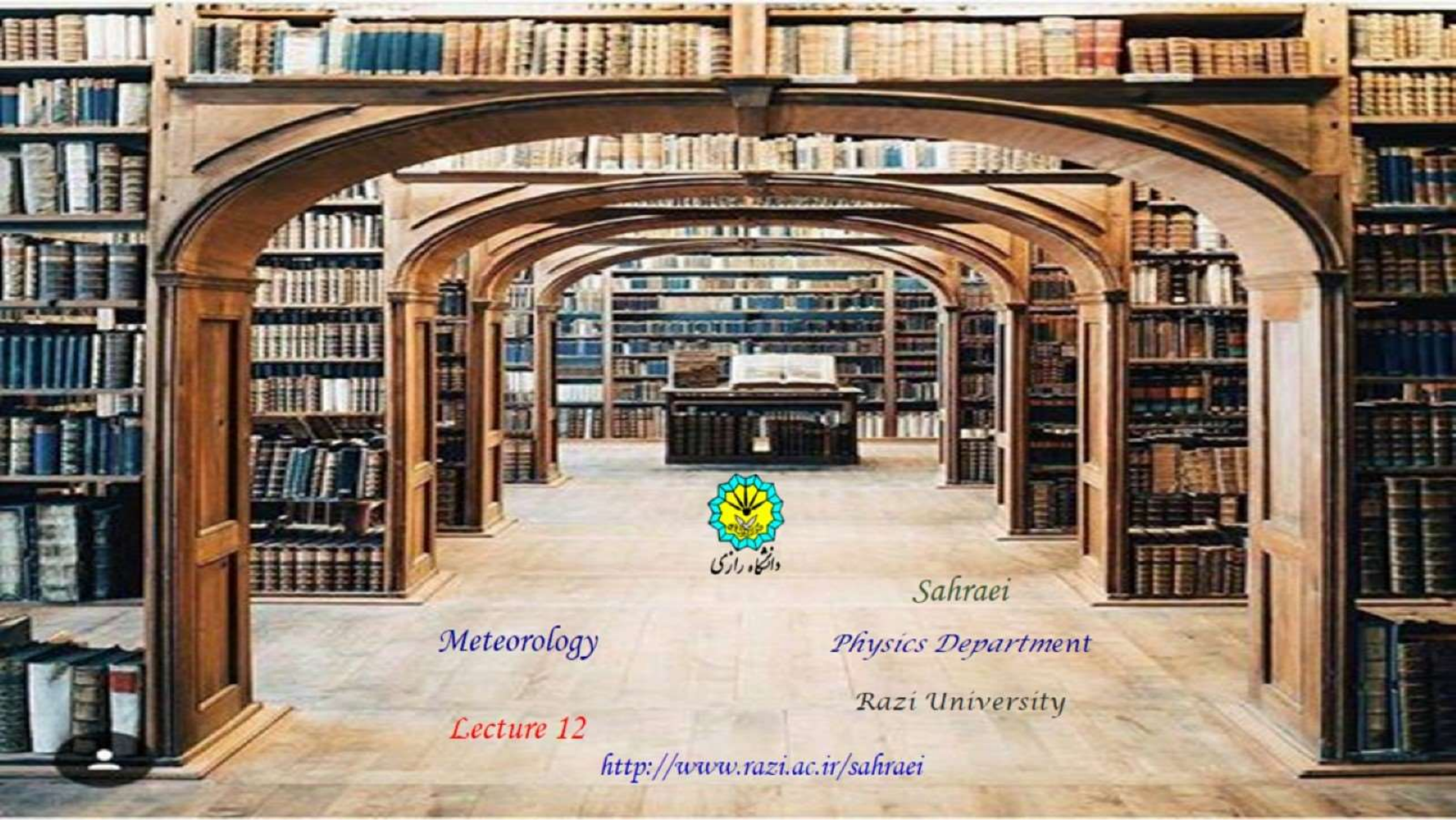

دانشگاه رازی

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Lecture 12

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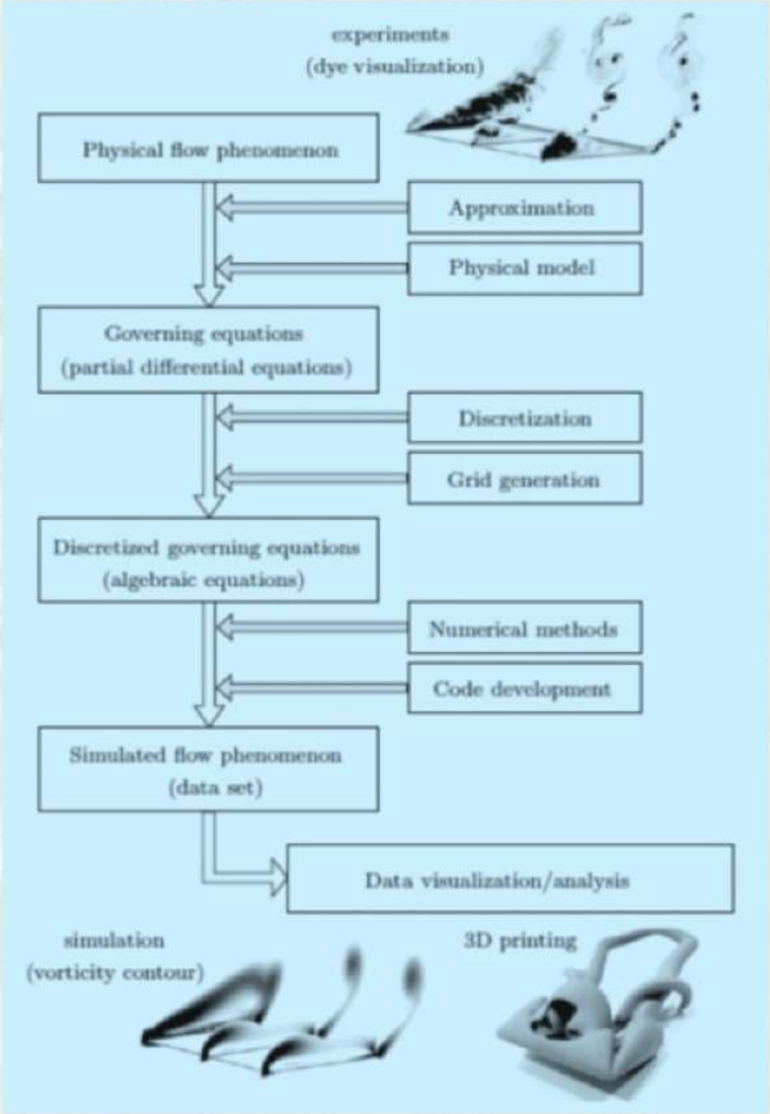
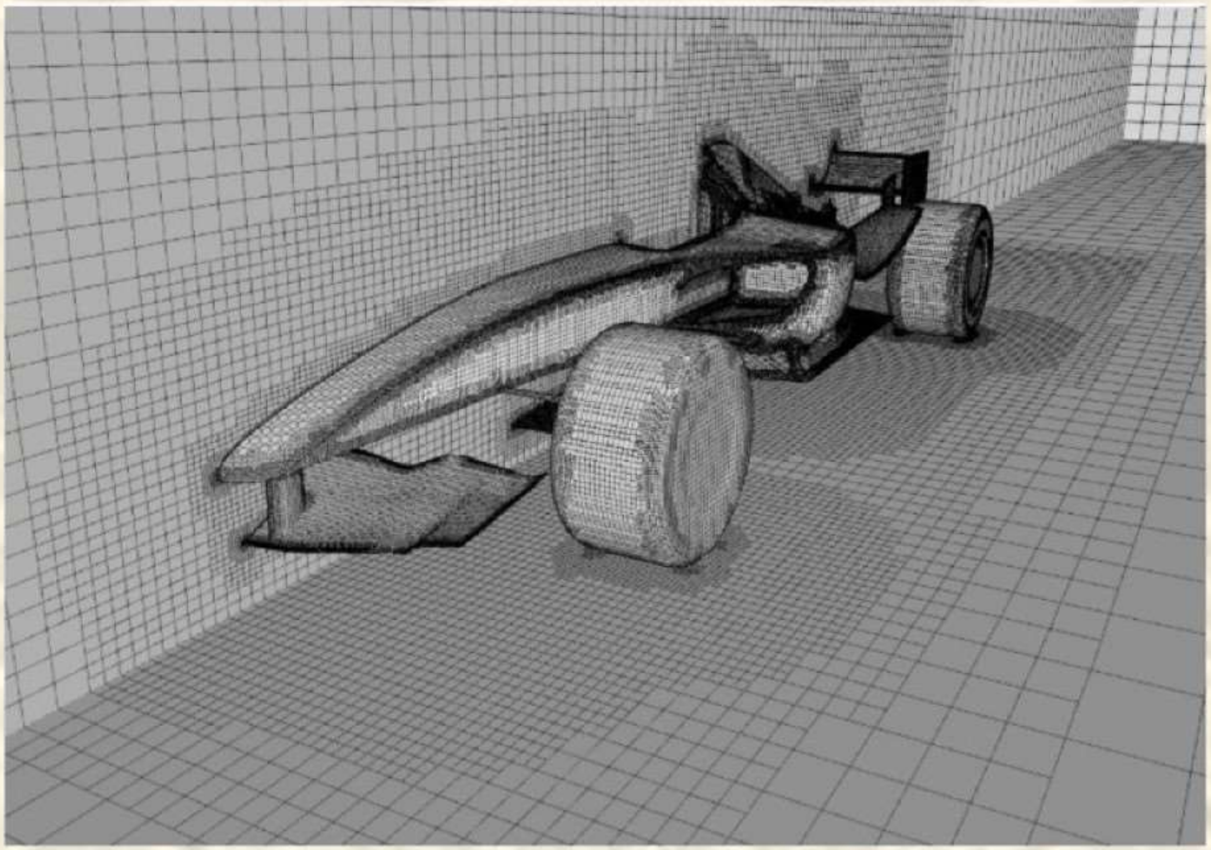
General process of simulating fluid flows.

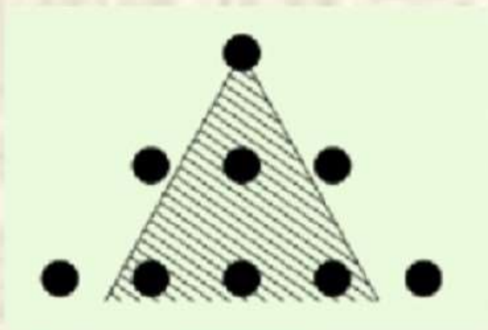




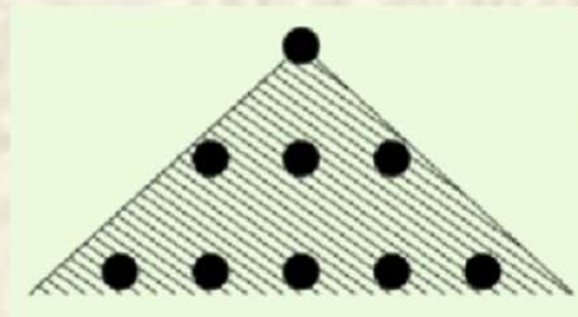
CFL Stability Condition

For $U\Delta t \leq \Delta x$, the numerical domain of dependence is larger than the physical domain of dependence (shaded region), and the system is stable.

For $U\Delta t > \Delta x$ we have the opposite situation, and the system is unstable.



$U\Delta t \leq \Delta x$



$U\Delta t > \Delta x$

What Courant Friedrichs and Lewy pointed out was that a great deal can be learned by considering the domains of dependence of a partial differential equation and of its discrete approximation.

As suggested in Figure a consider an initial-value problem for a partial differential equation and let (x ,t) be some point with $t > 0$.

Despite the picture the spatial grid need not be regular or one-dimensional.

The mathematical domain of dependence of u(x,t), denoted by X(x, t) is the set of all points in space where the initial data at t=0 may have some ffect on the solution u(x,t).

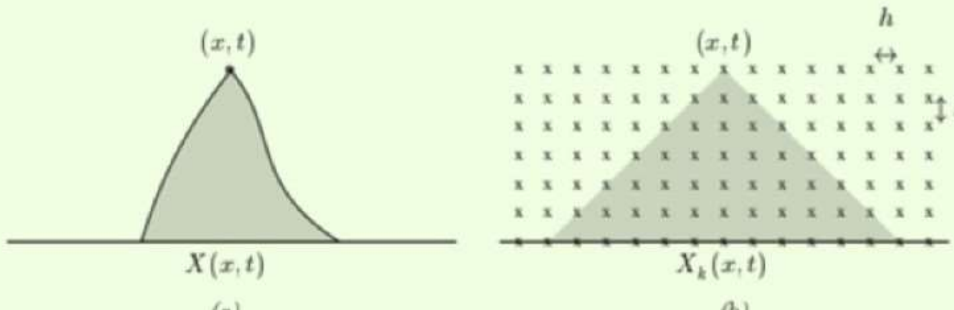


Figure 4.3.1. Mathematical and numerical domains of dependence.

as in Figure a are the characteristic curves for the partial differential equation and these are straight lines in simple examples but usually more general curves in problems containing variable coefficients or nonlinearity.

A numerical approximation also has a domain of dependence and this is suggested in Figure b.

Why is the CFL Condition Important

This is a question that always worries engineers more than mathematicians or applied physicists. I will say why later once I coves the following point:

1. How can I find the right value of the CFL condition why would I care if its 5 or 10 or a 100000. Once you run your numerical simulation you will require to validate the data with experimental. That is when you need play around with the different input parameters to assure that you can get the closest set of data that mimics the real life condition. Having a result of the researcher having confidence of the selected input values.

2. The CFL condition is a value that can assure that you are solving the differential equations (using approximation methods) with the right input parameters.

Grid anisotropy is the directional dependence of the accuracy of your numerical solution if you do not use enough points per wavelength.

Friction term is always positive and acts to dissipate kinetic energy. This term denotes the *dissipation rate of kinetic energy*. The existence of the viscous diffusion term in the momentum equation does not alter momentum conservation. However, there is a non-conservative dissipative term in the kinetic energy equation.

Any numerical solution has to be checked if it converges to the correct solution (as we have seen there are different options when using FD and not all do converge!)

The number of grid points per wavelength is a central concept to all numerical solutions to wave like problems. This desired frequency for a simulation imposes the necessary space increment.

The Courant criterion, the smallest grid increment and the largest velocity determine the (global or local) time step of the simulation

Examples on the exercise sheet.