Atmospheric Physics

Lecture 8

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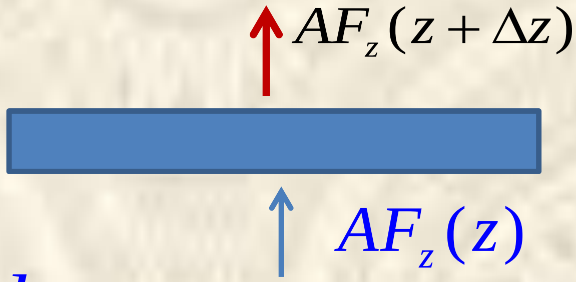
Heating rates

Basic ideas



$A\Delta z$

$$A\left[F_z(z) - F_z(z+\Delta z)\right] \approx -(A\Delta z)\,dF_z / dz$$

$$-dF_z / dz$$

$$Q = -\frac{1}{\rho(z)}\frac{dF_z}{dz} \qquad Q / c_p$$

$$F_z (= F^{\uparrow} - F_{\downarrow})$$

Short-wave heating

ρQ_ν^{sw} $\rho_a z$ $$\chi_\nu(z) = \int_z^\infty k_\nu(z')\rho_a(z')dz'$$

$$L_\nu(s) = \int_0^{\chi_\nu} J_\nu(\chi')e^{-(\chi_\nu-\chi')}\,d\chi' + L_{\nu 0}e^{-\chi_\nu}.$$

$$F_\nu^\downarrow(z) = F_{\nu\infty}^\downarrow e^{-\chi_\nu(z)}$$ $F_{\nu\infty}^\downarrow$ $e^{-\chi_\nu(z)}$

$\tau_\nu(z,\infty)$ $F_\nu^\uparrow$ $$F_{z\nu}(z) = -F_{\nu\infty}^\downarrow e^{-\chi_\nu(z)}$$

$$\rho Q_\nu^{sw} = \frac{d}{dz}(F_{\nu\infty}^\downarrow e^{-\chi_\nu(z)}) = F_{\nu\infty}^\downarrow(-\frac{d\chi_\nu}{dz})e^{-\chi_\nu(z)}$$

$$= F_{\nu\infty}^\downarrow k_\nu(z)\rho_a(z)e^{-\chi_\nu(z)}$$

$$\rho_a(z) = \rho_a(0)e^{-z/H_a}$$

$$\chi_\nu(z) = H_a k_\nu \rho_a(0)e^{-z/H_a} = \chi_\nu(0)e^{-z/H_a}$$

this shows how the optical depth increases as the solar radiation penetrates downwards, i.e. as z decreases.

Substitution into $F_{z\nu}(z) = -F_{\nu\infty}^{\downarrow} e^{-\chi_\nu(z)}$

then gives the vertical irradiance $F_{z\nu} = -F_{\nu\infty}^{\downarrow} e^{-\chi_\nu(0)e^{-z/H_a}}$

and differentiation gives the monochromatic volume heating rate,

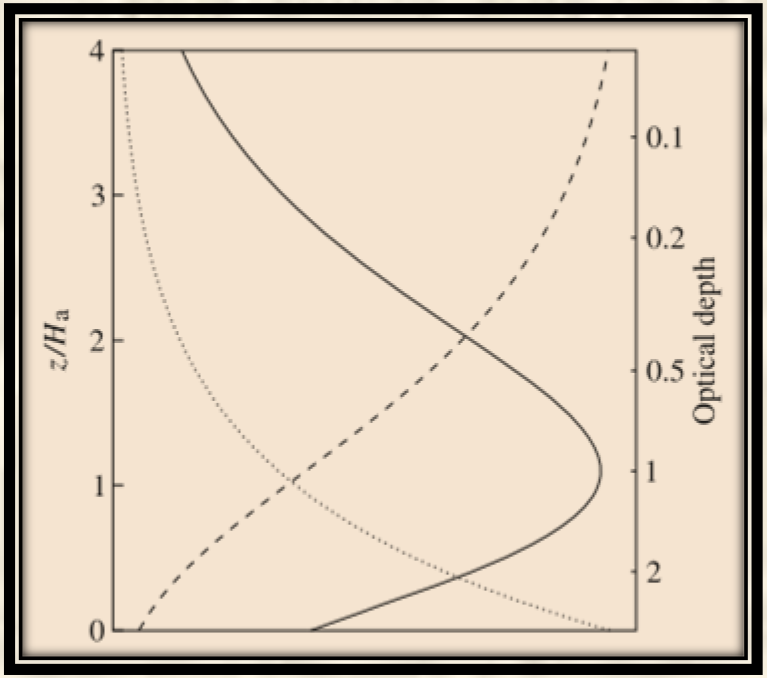
$$\rho Q_\nu^{sw}(z) = F_{\nu\infty}^{\downarrow} k_\nu \rho_a(0)e^{-z/H_a - \chi_\nu(0)e^{-z/H_a}}$$

$$\rho Q_{\nu}^{sw}$$

$$F_{z\nu}(z)$$

$$\rho_a(z)$$

$$\rho_a \propto e^{-z/H_a}$$



Long-wave heating and cooling

The upward thermal irradiance at frequency v and height *z is*

$$F_\nu^{\uparrow}(z) = \pi \int_0^z B_\nu(z') \frac{\partial \tau_\nu^*(z', z)}{\partial z'} dz' + \pi B_\nu(0) \tau_\nu^*(0, z)$$

$\tau_\nu^*(z', z)$ The spectral transmittance,

$B_\nu(0)$ The Planck function $J_\nu = B_\nu$

Similarly, the downward irradiance is

$$F_\nu^{\downarrow}(z) = -\pi \int_z^\infty B_\nu(z') \frac{\partial \tau_\nu^*(z', z)}{\partial z'} dz'$$

The net upward long-wave spectral irradiance

$$F_{z\nu}(z) = F_\nu^{\uparrow}(z) - F_\nu^{\downarrow}(z)$$

Q_ν^{lw} can be calculated using $$Q = -\frac{1}{\rho(z)}\frac{dF_z}{dz}$$

These heating and cooling terms can also be obtained directly

The derivation of the radiative-transfer

$$\frac{dL_\nu}{ds} = -k_\nu \rho_a (L_\nu - J_\nu).$$

shows that the spectral power emitted in a vertical direction from this slab is

$$k_\nu \rho_a J_\nu A \Delta z$$

where *Jv* is the source function, equal to *Bv* under LTE

The fraction of this power that escapes to space is given by the transmittance

$$\tau_\nu(z,\infty) = \exp(-\int_z^\infty k_\nu \rho_a dz')$$

Noting that
$$\frac{\partial \tau_\nu(z,\infty)}{\partial z} = k_\nu(z)\rho_a(z)\tau_\nu(z,\infty)$$

we find that the power escaping to space from the slab in a purely vertical beam is

$$B_\nu(z)\frac{\partial \tau_\nu(z,\infty)}{\partial z} A\Delta x$$

Now, integrating over all slanting paths as above and replacing τ_ν by , τ_ν^* we obtain a contribution to the heating rate per unit mass

$$Q_\nu^{cts}(z) = \frac{\pi B_\nu(z)}{\rho(z)} \frac{\partial \tau_\nu^*(z,\infty)}{\partial z}$$

A useful simplification for some purposes is the coolingto-space approximation, in which the loss of photon energy to space dominates the other contributions; therefore, under this approximation,

$$Q_{\nu}^{lw} \approx Q_{\nu}^{cts}$$

All gases that absorb and emit at frequency ν must in principle be included in τ_{ν}^{*}

Then $Q_{\nu}^{lw}(z)$ *must be integrated over all relevant frequencies to*

obtain the total long-wave cooling $Q^{lw}(z)$

Net radiative heating rates

$$Q^{sw} / c_p$$

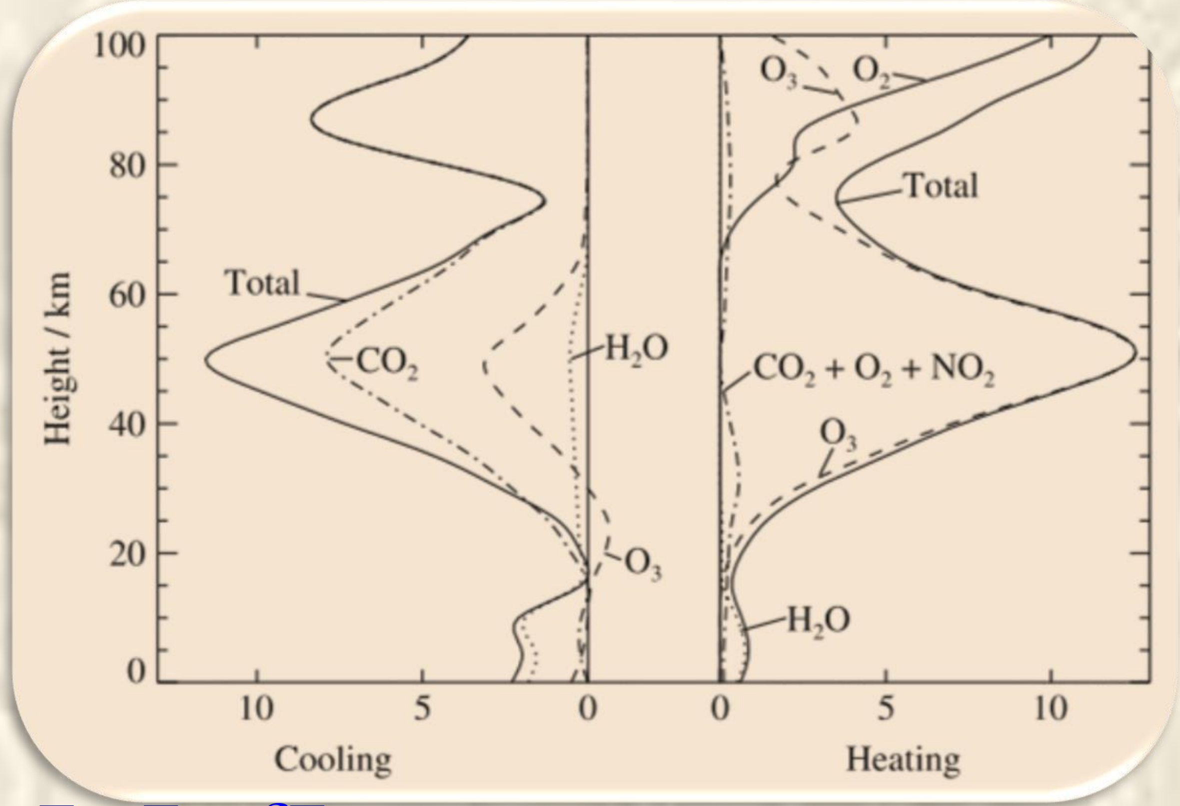
$$-Q^{lw} / c_p$$

$$Q = Q^{sw} + Q^{lw}$$

$$Q = 0$$

$$T_r(r)$$

$$Q(T_r(r)) = 0$$



$$T = T_r + \delta T$$

$$Q(T_r + \delta T) \approx Q(T_r) + \delta T \frac{\partial Q}{\partial T}\Big|_{T=T_r} = \delta T \frac{\partial Q}{\partial T}\Big|_{T=T_r}$$

$$Q(T_r) = 0$$

$$= -c_p \frac{\delta T}{\tau_r}$$

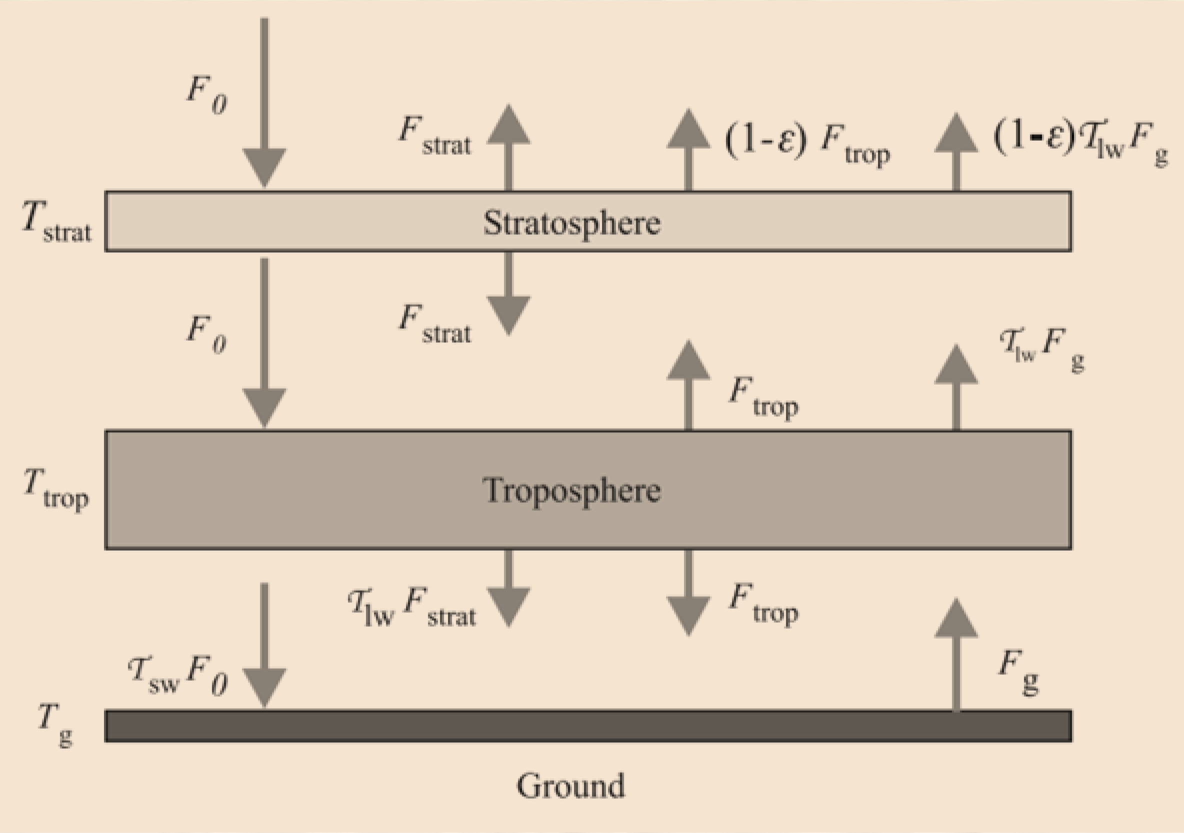
$$\tau_r = c_p (\partial Q / \partial T\big|_{T=T_r})^{-1}$$

The greenhouse effect revisited

Two-layer atmosphere in radiative equilibrium, including an optically thin stratosphere

T_{trop} τ_{sw} τ_{lw}

$$T_c \equiv \left(\frac{F_0}{\sigma}\right)^{1/4} \approx 255K$$



$$F_0 = F_{strat} + (1+\varepsilon)(F_{trop} + \tau_{lw}F_g)$$

$$F_{strat} = \sigma\varepsilon T_{strat}^4 \quad , \quad F_{trop} = \sigma(1-\tau_{lw})T_{trop}^4 \quad , \quad F_g = \sigma T_g^4$$

$$F_0 + F_{strat} = F_{trop} + \tau_{lw} F_g$$

$$2F_{strat} = \varepsilon(F_{trop} + \tau_{lw} F_g)$$

$$F_{trop} + \tau_{lw} F_g$$

$$F_0 + F_{strat} = (1-\varepsilon)(F_0 + F_{strat})$$

$$\sigma\varepsilon T_{strat}^4 = F_{strat} = \frac{\varepsilon F_0}{2-\varepsilon}$$

$$\varepsilon \ll 1 \qquad \sigma T_{strat}^4 \approx \frac{F_0}{2} \qquad T_{strat} \approx \frac{T_c}{2^{1/4}} = 214K$$

$$F_{trop} + \tau_{lw} F_g$$

$$F_{trop} = \frac{2F_0}{2-\varepsilon} - \tau_{lw} F_g$$

$$\tau_{sw} F_0 + F_{lw} F_{strat} + F_{trop} = F_g$$

Continuously stratified atmosphere in radiative equilibrium

$$-\frac{dF^{\uparrow}}{d\chi^*} + F^{\uparrow} = \pi B(T)$$

$$\pi B(T) = \sigma T^4$$

$$\frac{dF^{\downarrow}}{d\chi^*} + F^{\downarrow} = \pi B(T)$$

$$Q^{sw} = 0 \qquad Q^{lw} = 0$$

$$F_z = F^{\uparrow} - F^{\downarrow} = \text{constant} \qquad F^{\downarrow}(0) = 0 \qquad F_z = F^{\uparrow}(0)$$

$$F_z = F^{\uparrow} - F^{\downarrow} = F_0$$

$$-\frac{d}{d\chi^*}(F^{\uparrow} - F^{\downarrow}) + F^{\uparrow} - F^{\downarrow} = 2\pi B(T)$$

$$\pi B(T) = \frac{1}{2}(F^{\uparrow} + F^{\downarrow})$$

$$\frac{d}{d\chi^*}(F^{\uparrow} + F^{\downarrow}) = F^{\uparrow} - F^{\downarrow} = F_0$$

$$F^{\uparrow} + F^{\downarrow} = F_0 \chi^* + \text{constant}$$

$$F^{\uparrow} + F^{\downarrow} = F_0 (1 + \chi^*)$$

$$F^{\uparrow} = \frac{1}{2} F_0 (2 + \chi^*)$$

$$F^{\downarrow} = \frac{1}{2} F_0 \chi^*$$

$$\pi B(T) = \sigma T^4 = \frac{1}{2} F_0 (1 + \chi^*)$$

$$F_0 (1 + \chi_g^*) / 2$$

$$\pi B(T_g) = \sigma T_g^4$$

$$\sigma T_g^4 = F_0(1+\frac{1}{2}\chi_g^*) = \sigma T_c^4(1+\frac{1}{2}\chi_g^*)$$

$$T_c \approx 255K \qquad \chi_g^* > 0$$

$$T_g > T_c$$

$$\rho_c(z) = \rho_a(0)e^{-z/H_a}$$

$$\chi^*(z) = \chi_g^* e^{-z/H_a}$$

$$F^{\uparrow}(z) = \frac{1}{2}F_0(2+\chi_g^* e^{-z/H_a}) \qquad F^{\downarrow}(z) = \frac{1}{2}F_0\chi_g^* e^{-z/H_a}$$

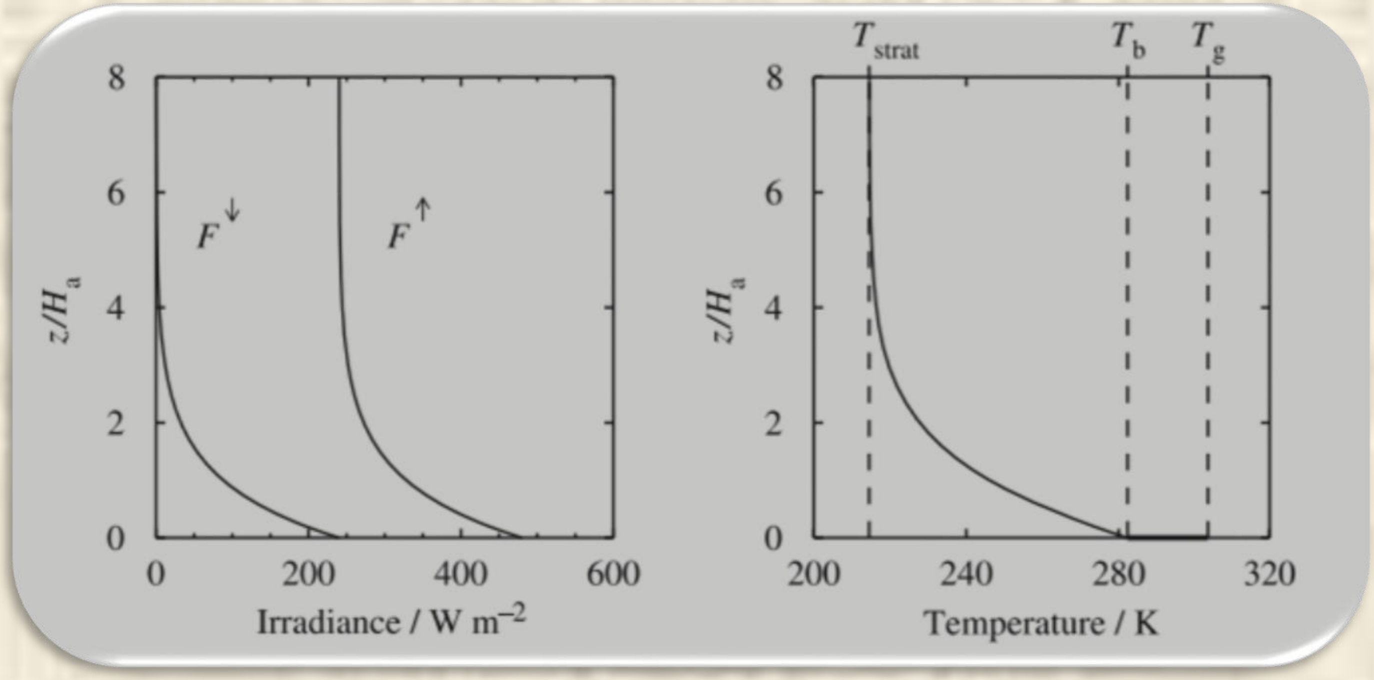
$$T(z) = \left[\frac{F}{2\sigma_0}(1+\chi_g^* e^{-z/H_a})\right]^{1/4}$$

z / H_a $\chi_g^* = 2$ $F_0 = 240\ W/m^{-2}$

$$T \to \left(\frac{F}{2\sigma}\right)^{1/4} \text{ as } z \to \infty$$

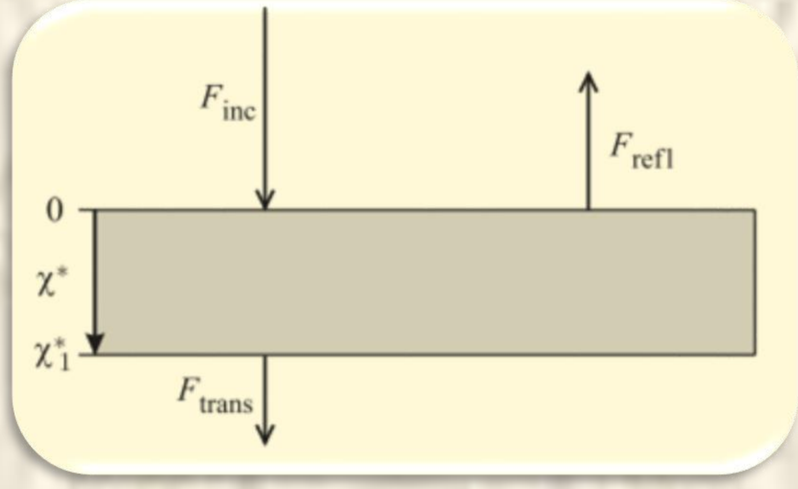
$$T_{strat} = 2^{-1/4} T_c$$

$$T(z) \to T_b \equiv T_c \left(\frac{1+\chi_g^*}{2}\right)^{1/4} \text{ as } z \downarrow 0$$



$$T_g \equiv T_c \left(\frac{2+\chi_g^*}{2}\right)^{1/4}$$

A simple model of scattering


F_{inc}
F_{refl}
0
χ^*
χ_1^*
F_{trans}

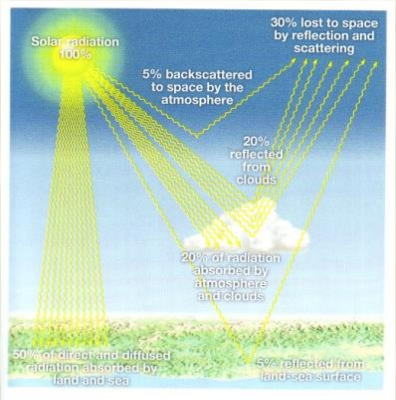
Shortwave Radiation

S_o = 1368 w m^{-2} is the **solar constant** for Earth

Insolation

$$R_0 = S_0 \left(\frac{d_m}{d} \right)^2 \cos \gamma$$

$$I_0 = \int_{t_1}^{t_2} R_0(t)\, dt$$



Stefan-Boltzmann Law

This law expresses the rate of radiation emission per unit area

$$R = \sigma T^4 \qquad \sigma = 5.67 \times 10^{-8}\, W / m^2 K^4$$

Compare the difference between the radiation emission from the sun and the Earth.

The sun with an average temperature of 6000 K emits 73,483,200 W/m^2

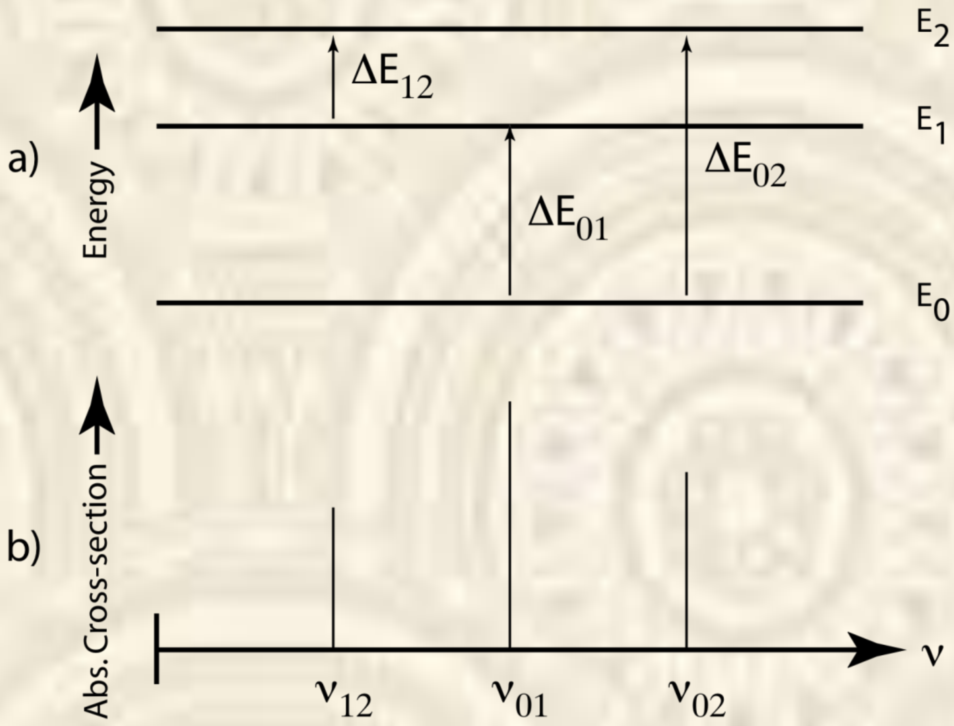
By contrast, Earth with an average temperature of 300 K emits 459 W/m^2

The sun has a temperature 20 times higher than Earth and thus emits about 160,000 times more radiation

This makes sense,

$$20^4 = 160{,}000$$

Absorption spectra of molecules



Hypothetical molecule with three allowed energy levels

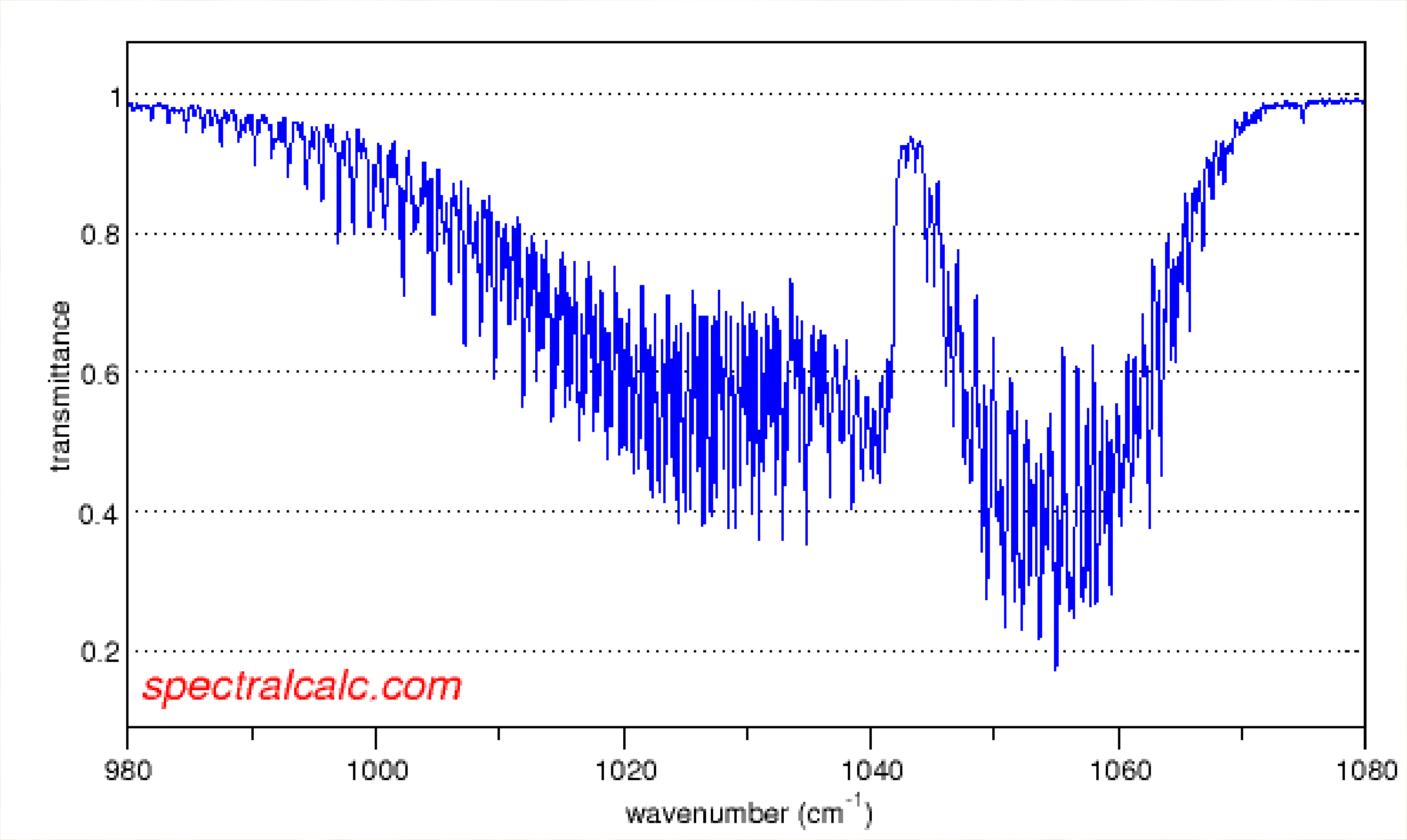
Note relationship to emission!

$$\nu_{ij} = \Delta E_{ij}/h$$

a) allowed transitions
b) positions of the absorption lines in the spectrum of the molecule

Line positions are determined by the energy changes of allowed transitions
Line strengths are determined by the fraction of molecules that are in a particular initial state required for a transition
Multiple degenerate transitions with the same energy may combine

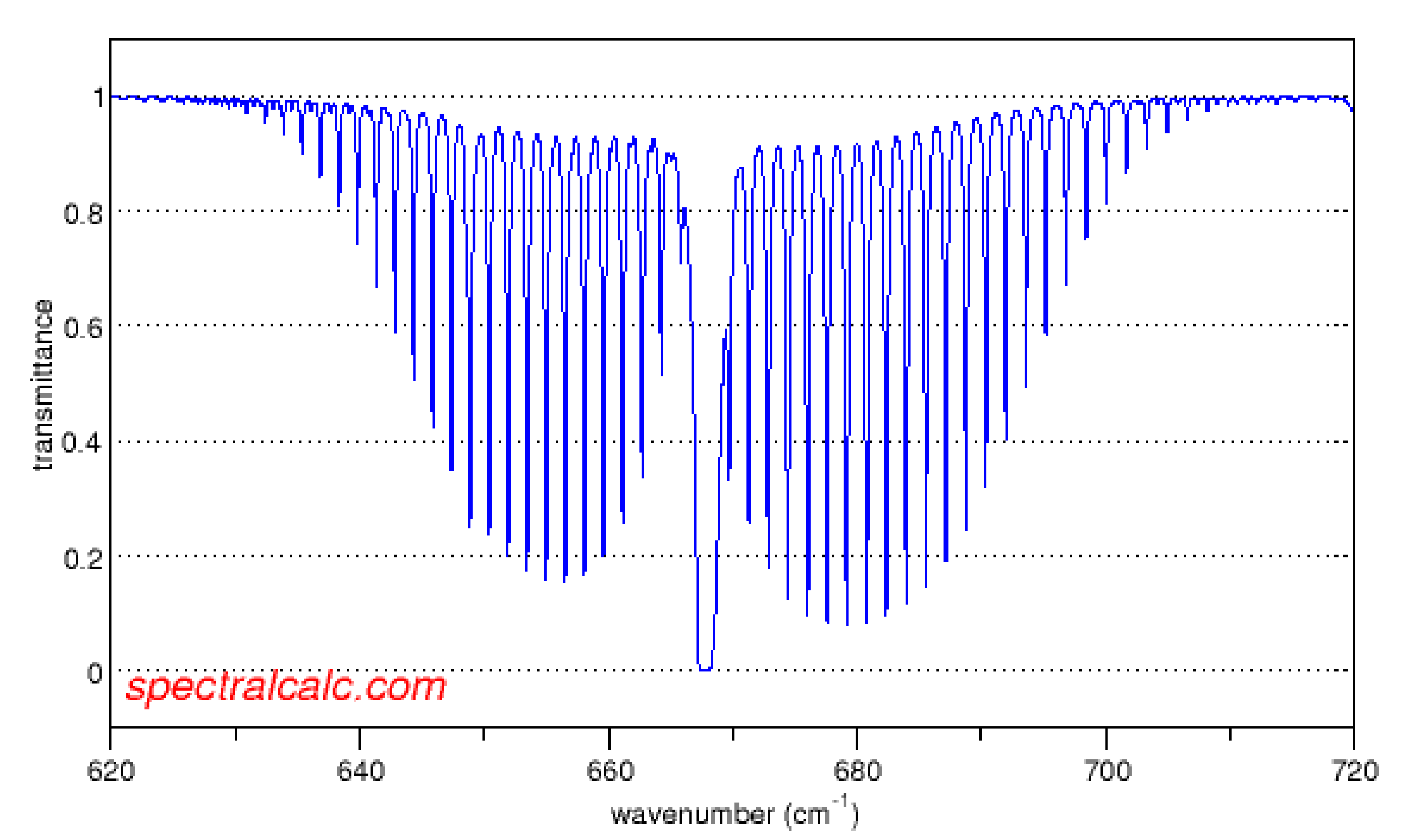
Transmittance spectrum for ozone (O_3)



http://www.spectralcalc.com/calc/spectralcalc.php

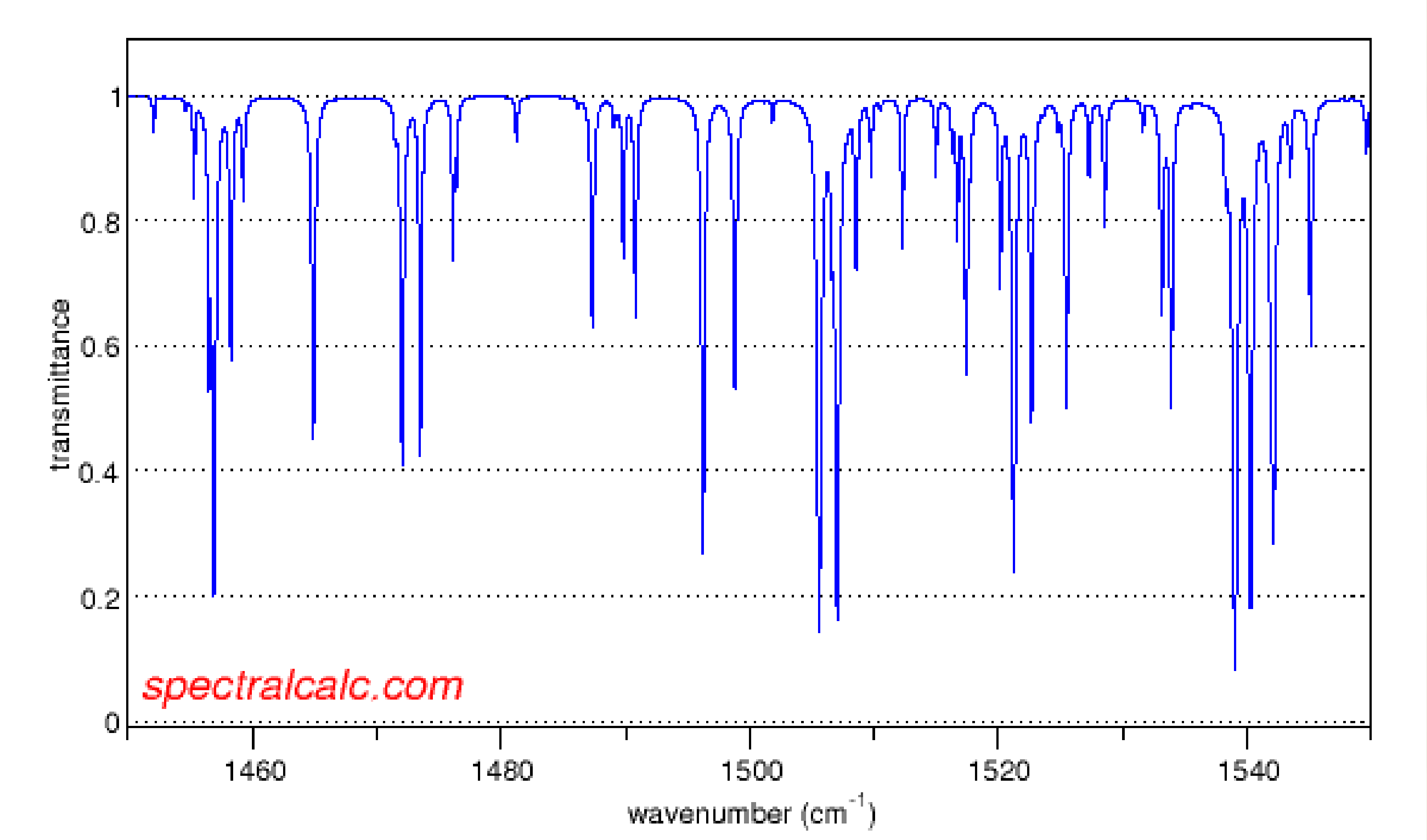
Transmittance spectrum for CO_2

http://www.spectralcalc.com/calc/spectralcalc.php



Transmittance spectrum for H_2O

http://www.spectralcalc.com/calc/spectralcalc.php

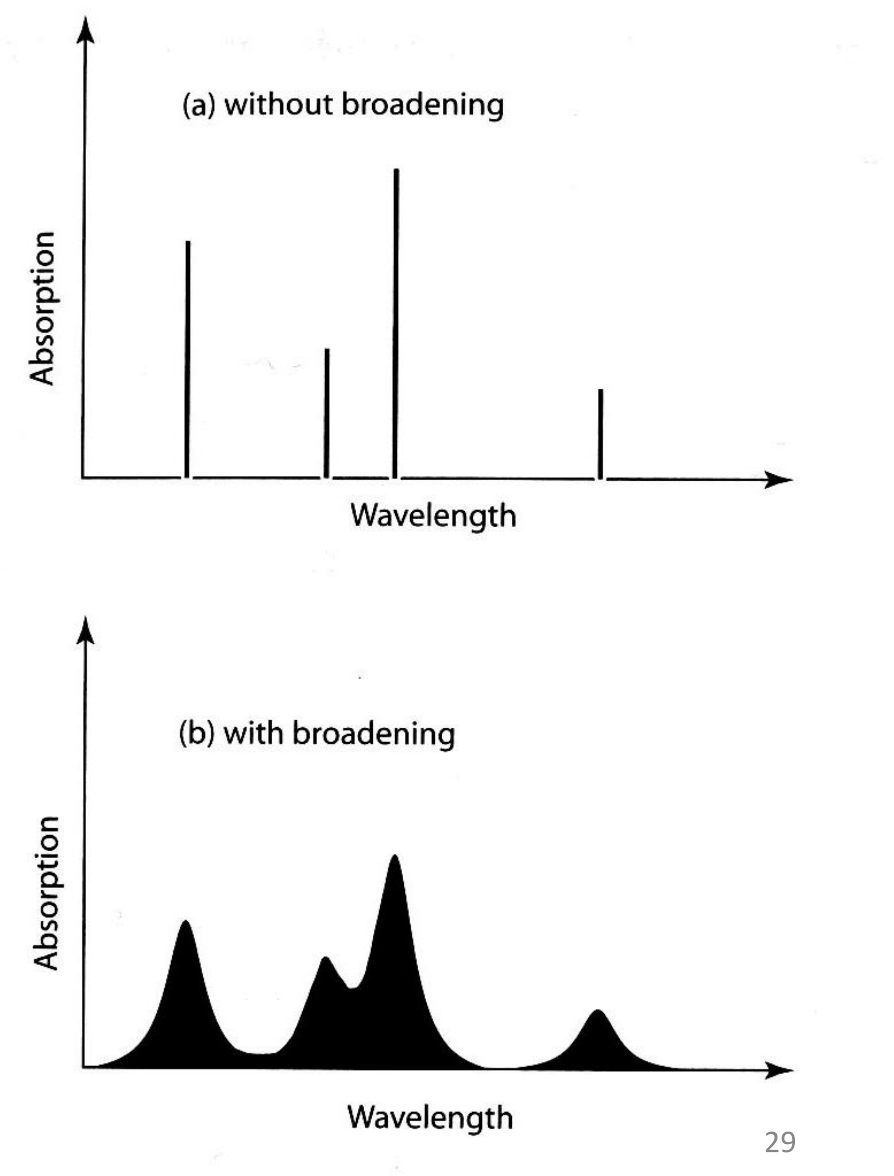


Absorption line shapes

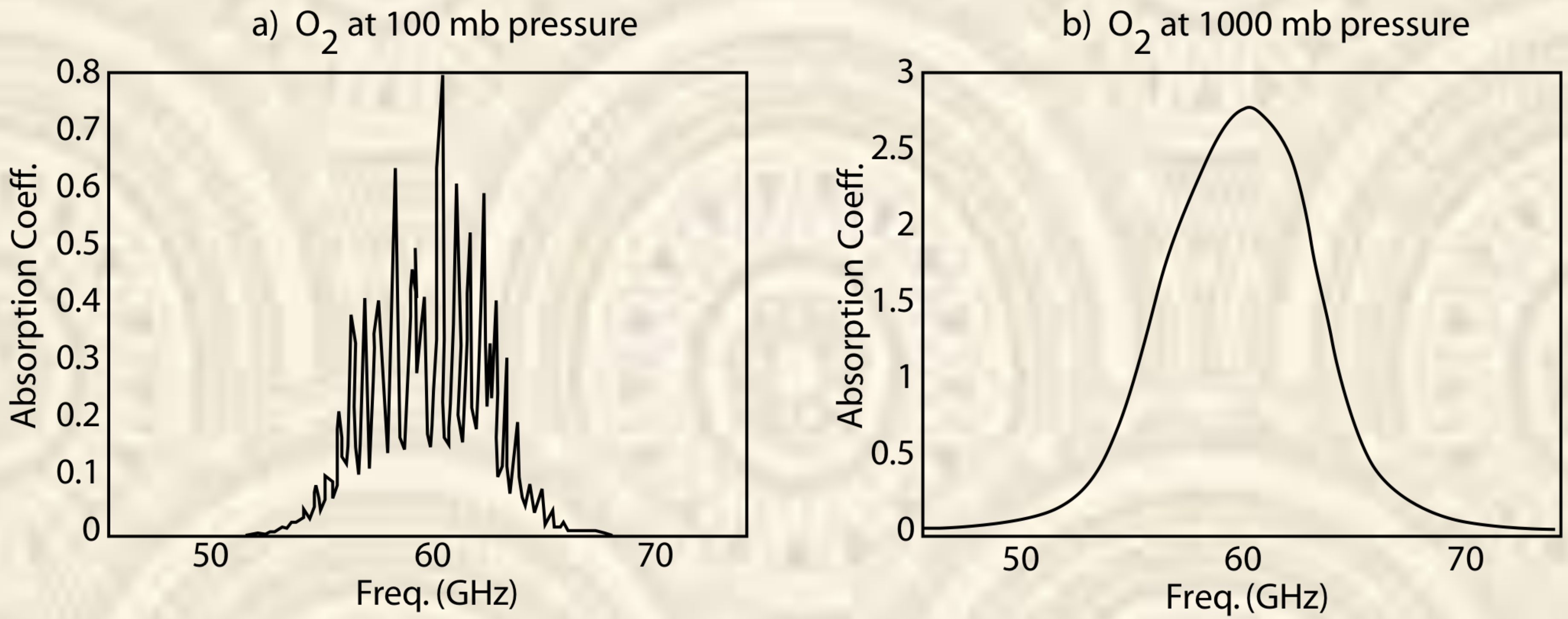
• **Doppler broadening:** random translational motions of individual molecules in any gas leads to Doppler shift of absorption and emission wavelengths (important in upper atmosphere)

• **Pressure broadening:** collisions between molecules randomly disrupt natural transitions between energy states, so that absorption and emission occur at wavelengths that deviate from the natural line position (important in troposphere and lower stratosphere)

• Line broadening closes gaps between closely spaced absorption lines, so that the atmosphere becomes opaque over a continuous wavelength range.



Pressure broadening



- Absorption coefficient of O_2 in the microwave band near 60 GHz at two different pressures. Pressure broadening at 1000 mb obliterates the absorption line structure.