Atmospheric Physics

Lecture 4

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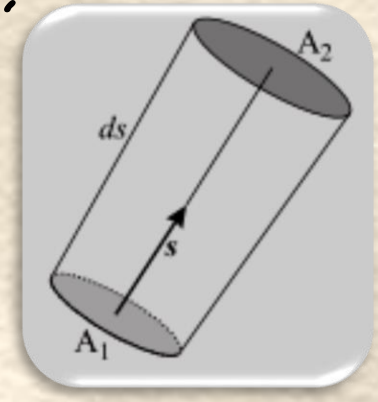
Extinction and emission

Consider a beam radiation of unit cross-sectional area, moving in a small range of solid angles $\Delta\Omega$ about the direction ***s;***



If the photons experience absorption or scattering in a small distance *ds* along the beam, due to the presence of a radiatively active gas,

then the spectral radiance *L* will be reduced

Lambert's law

$$mass \rightarrow \rho_a ds \qquad \rho_a \rightarrow density\ of\ RAG$$

$$dL_\nu = -k_\nu(s)\rho_a(s)L_\nu(s)ds$$

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k_ν the extinction coefficient $k_\nu = a_\nu + s_\nu$

$k_\nu \rho_a J_\nu ds$ $J_\nu(s)$ source function

$$\frac{dL_\nu}{ds} = -k_\nu \rho_a (L_\nu - J_\nu)$$

the radiative-transfer equation
(also called Schwarzschild's equation

$$\chi_\nu(s) = \int_{s_0}^{s} k_\nu(s')\rho_a(s')ds'$$

the optical path

$$\frac{dL_\nu}{d\chi_\nu} + L_\nu = J_\nu \qquad L_\nu e^{\chi_\nu} = \int J_\nu e^{\chi'_\nu} d\chi'_\nu + constant$$

$$L_\nu(s) = \int_0^{\chi_\nu} J_\nu(\chi')e^{-(\chi_\nu - \chi')}d\chi' + L_{\nu 0}e^{-\chi_\nu}$$

Note that, in the absence of emission (J_ν=0), the spectral radiance falls exponentially, decreasing by a factor of e over a distance corresponding to unit optical path

$$L_\nu(s) = L_{\nu 0}e^{-\chi_\nu}$$

This exponential decay is known as Beer's law

$\chi_\nu \gg 1$ A region is said to be optically thick

$\chi_\nu \ll 1$ A region is said to be optically thin

A photon is likely to be absorbed or scattered within an optically thick region, but is likely to traverse an optically thin region without absorption or scattering

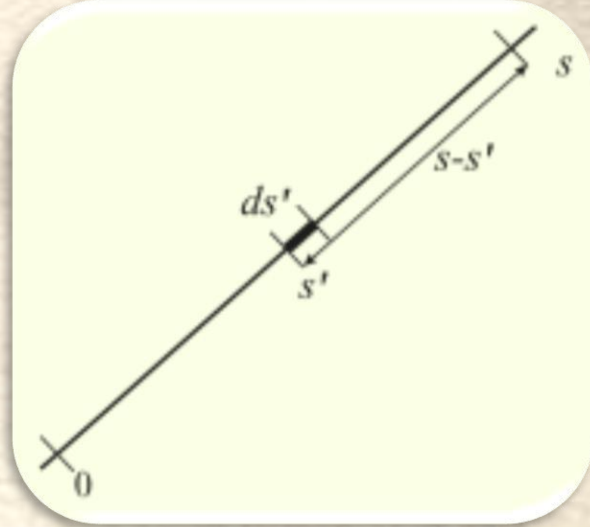
As a simple example, suppose that the extinction coefficient k_ν and the density ρ_a of the radiatively active gas are both constant and take s_0=0, then from

$$\chi_\nu(s) = \int_{s_0}^{s} k_\nu(s')\rho_a(s')ds'$$ the optical path

The optical path is proportional to the distance s $\chi_\nu = k_\nu \rho_a s$ so

$$L_\nu(s) = k_\nu \rho_a \int_0^s J_\nu(s') e^{-k_\nu \rho_a (s-s')} ds' + L_{\nu 0} e^{-k_\nu \rho_a s}$$

$$e^{-k_\nu \rho_a (s-s')}$$



Under local thermodynamic equilibrium conditions in the absence of scattering, the source function equals the black-body spectral radiance, i.e. the Planck function. This can be shown by using Kirchhoff's law, which holds under LTE as follows

$$mass \rightarrow \rho_a ds$$

$$R.E. \rightarrow k_\nu \rho_a ds\ J_\nu$$

$$R.A. \rightarrow k_\nu \rho_a ds\ L_\nu$$

$$\frac{dL_\nu}{ds} = -k_\nu \rho_a (L_\nu - J_\nu)$$

Hence the spectral emittance, the ratio of the emitted radiance to the radiance emitted by a black body, is

$$\varepsilon_\nu = k_\nu \rho_a ds\ J_\nu \ / \ B_\nu$$

The spectral absorptance, the fraction of incident radiance that is absorbed is

$$\alpha_\nu = k_\nu \rho_a ds\, L_\nu \,/\, L_\nu = k_\nu \rho_a ds$$

neglecting scattering.

However, Kirchhoff's law states that $\varepsilon_\nu = \alpha_\nu$

$$J_\nu = B_\nu$$

The diffuse approximation

In radiative calculations we can often assume that the properties of the atmosphere and the radiation depend only on the vertical coordinate z.

This is the **plane-parallel atmosphere** assumption

$$\frac{dL_\nu}{d\chi_\nu} + L_\nu = J_\nu$$

(with J_ν = B_ν assuming LTE conditions)

$$\frac{dF_\nu^\downarrow}{d\chi_\nu^*} + F_\nu^\downarrow = \pi B_\nu$$

for the downward spectral irradiance $F_\nu^\downarrow$ along a *vertical path, where*

$$\chi_\nu^* \approx 1.66\chi_\nu$$

The quantity χ_ν is the optical path measured downwards from the top of the atmosphere

The quantity χ_ν^* is a scaled optical depth

The factor π arises from the calculation of the black-body irradiance, as in equation

$$F_\nu(\hat{r},\hat{n}) = \int_{2\pi} L_\nu \hat{n}.\hat{s}\, d\Omega(s) = 2\pi B_\nu \int_0^{\pi/2} \cos\phi \sin\phi d\phi = \pi B_\nu(T)$$

A similar equation, but with a change in sign of χ_ν^* holds for the upward spectral irradiance