Atmospheric Physics

Lecture 3

J. Sahraei

Physics Department,

Razi University

http://www.razi.ac.ir/sahraei

The Planck function

Black-body radiation

Planck's law states that the spectral energy density of black-body radiation at absolute temperature *T is given by*

$$u_\nu(T) = \frac{8\pi h\nu^3}{c^3(e^{h\nu/k_B T} - 1)}$$

where k_B is Boltzmann's constant

Since the photons carrying this energy are moving isotropically, the energy density associated with the group of photons moving within a small solid angle $\Delta\Omega$

$$u_\nu \Delta\Omega / (4\pi)$$

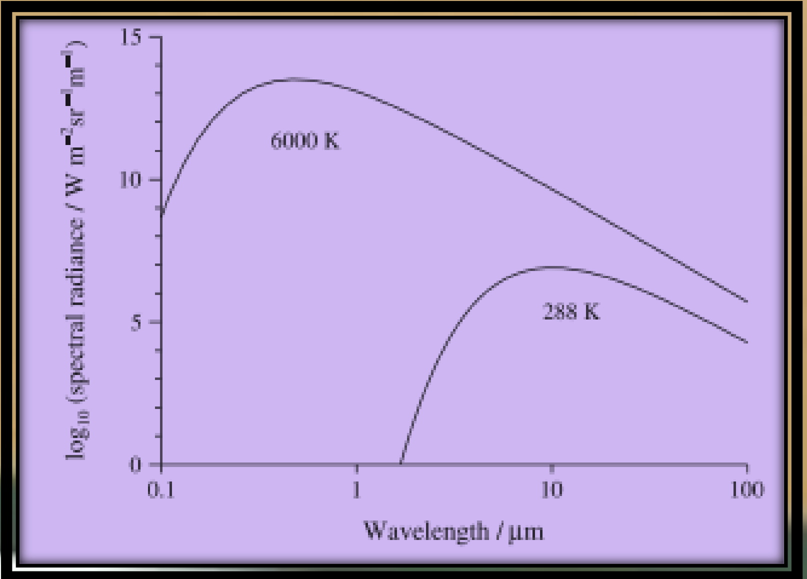
The power per unit area, per unit solid angle, per unit frequency interval (the spectral radiance) for black-body radiation at temperature *T is*

$$B_\nu(T) = \frac{2h\nu^3}{c^2(e^{h\nu/k_BT} - 1)}$$

The Planck function

The black-body spectral radiance can also be written in terms of the power per unit area, per unit solid angle, per unit wavelength interval,

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5(e^{hc/\lambda k_BT} - 1)}$$



If B_λ is integrated over all wavelengths, we obtain the black-body radiance

$$\int_0^\infty B_\lambda(T)d\lambda = \frac{\sigma}{\pi}T^4$$

where σ is the Stefan–Boltzmann constant

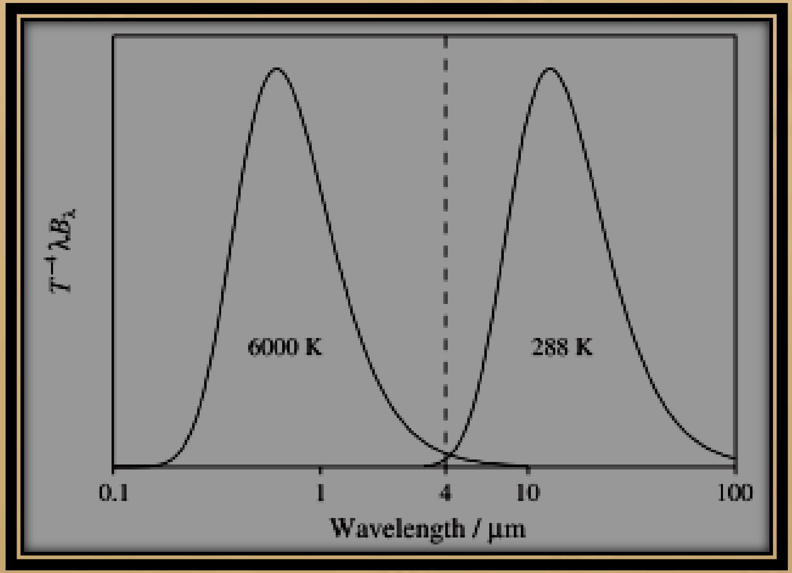
In terms of an integral over $\ln(\lambda)$, this gives

$$T^{-4}\int_{-\infty}^{\infty} \lambda B_\lambda(T)\, d(\ln\lambda) = \frac{\sigma}{\pi}$$

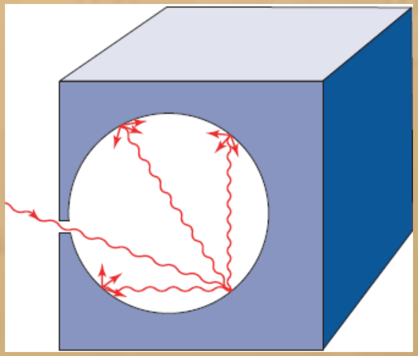
This suggests plotting $T^{-4}\lambda B_\lambda$ against $\ln\lambda$:

the area under the resulting curve is then independent of T.

Note that, with this normalisation, there is little overlap between the black-body spectral radiances at 6000



A black body is defined as a body that completely absorbs all radiation falling on it.



The concept of a black body is an idealisation

A real body will emit less radiation than this

The spectral emittance ε_ν of a body is the ratio of the spectral radiance from that body to the spectral radiance from a black body;

therefore $\varepsilon_\nu \leq 1$

It follows that a black body emits the maximum possible amount of energy in each frequency interval, at a given temperarture

We can also define the spectral absorptance α_ν

as the fraction of energy per unit frequency interval falling on a body that is absorbed

Kirchhoff's law states that $\varepsilon_\nu = \alpha_\nu$

that is, at a given temperature and frequency the spectral emittance of a body equals its spectral absorptance

Local thermodynamic equilibrium

$$B_\nu(T) = \frac{2h\nu^3}{c^2(e^{h\nu/k_BT} - 1)}$$

The Planck function

h = 6.6262x10^{-34} joule sec
k =1.3806x10^{-23} joule deg^{-1}
c =2.99793x10^{+8} m/s
T = object temperature in Kelvins

$$n_1 = g_1 e^{-E_1/k_BT}$$

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} e^{-(E_1 - E_2)/(k_BT)}$$

The radiative-transfer equation

Radiometric quantities

Several different, but related, quantities are used in the description and measurement of radiation. The most important are as follows.

The spectral radiance (or monochromatic radiance) $L_v(r, s)$

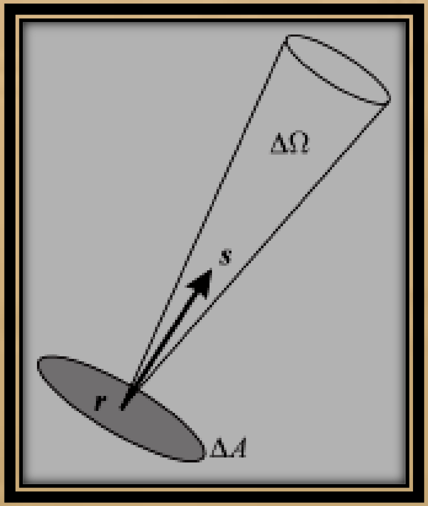
is the power per unit area, per unit solid angle, per unit frequency interval in the neighbourhood of the frequency v, at a point r, in the direction of the unit vector s.

It is measured in W m^{-2} $stradian^{-1}$ Hz^{-1}

The spectral radiance can be visualised in terms of the photons emerging from a small area ΔA with unit normal s, centred at a point r

$$L_\nu \, \Delta A \, \Delta\Omega \, \Delta\nu$$

Is energy transferred by these photons, per unit time, from 'below ' the area A to 'above'. (Here 'below' means in the direction -s **and 'above' means in the direction s.)**



We have already encountered a special case of the spectral radiance, for isotropic black body radiation in an isothermal cavity, when

$$L_\nu = B_\nu(T),$$

The radiance L(r,s) is the power per unit area, per unit solid angle at a point r in the direction of the unit vector s; in other words it is the integral of L_ν over frequency

$$L(r,s) = \int_0^\infty L_\nu(r,s)\, d\nu$$

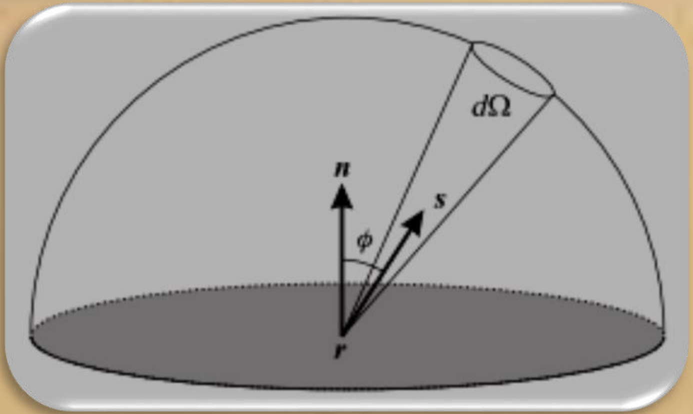
Its units are in W m^{-2} $stradian^{-1}$

The spectral irradiance (or monochromatic irradiance) $F_\nu(r, n)$:

is the power per unit area, per unit frequency interval in the neighbourhood of the frequency ν, at a point r through a surface of normal n;

$$F_\nu(\hat{r},\hat{n}) = \int_{2\pi} L_\nu(\hat{r},\hat{s})\, \hat{n}.\hat{s}\; d\Omega(s)$$

its units are W m^{-2} Hz^{-1}



The irradiance (or flux density) F(r, n) is the power per unit area at a point r through a surface of normal n, i.e. the integral of F_ν over frequency, and also the integral of the radiance L over a hemisphere:

$$F(\hat{r},\hat{n}) = \int_0^\infty F_\nu(\hat{r},\hat{n})d\nu = \int_{2\pi} L(\hat{r},\hat{s})\,\hat{n}.\hat{s}\,d\Omega(s)$$

its units are W m^{-2}

$$F^\downarrow = F(\hat{r},-\hat{n}) \qquad F_z = F^\uparrow - F^\downarrow \qquad L_\nu = B_\nu(T)$$

Equation $$F_\nu(\hat{r},\hat{n}) = \int_{2\pi} L_\nu(\hat{r},\hat{s})\, \hat{n}.\hat{s}\, d\Omega(s)$$

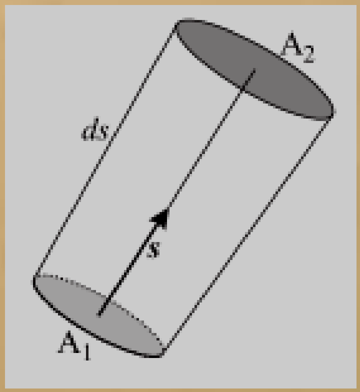
for the spectral irradiance becomes

$$F_\nu(\hat{r},\hat{n}) = \int_{2\pi} L_\nu \hat{n}.\hat{s}\, d\Omega(s) = 2\pi B_\nu \int_0^{\pi/2} \cos\phi \sin\phi d\phi = \pi B_\nu(T)$$

$$d\Omega = 2\pi \sin\phi d\phi$$

since there is axisymmetry around the normal. Integrating over all v we obtain the Stefan-Boltzmann law for the irradiance

$$F(\hat{r},\hat{n}) = \pi \int_0^\infty B_\nu(T) d\nu = \sigma T^4$$


A_2
ds
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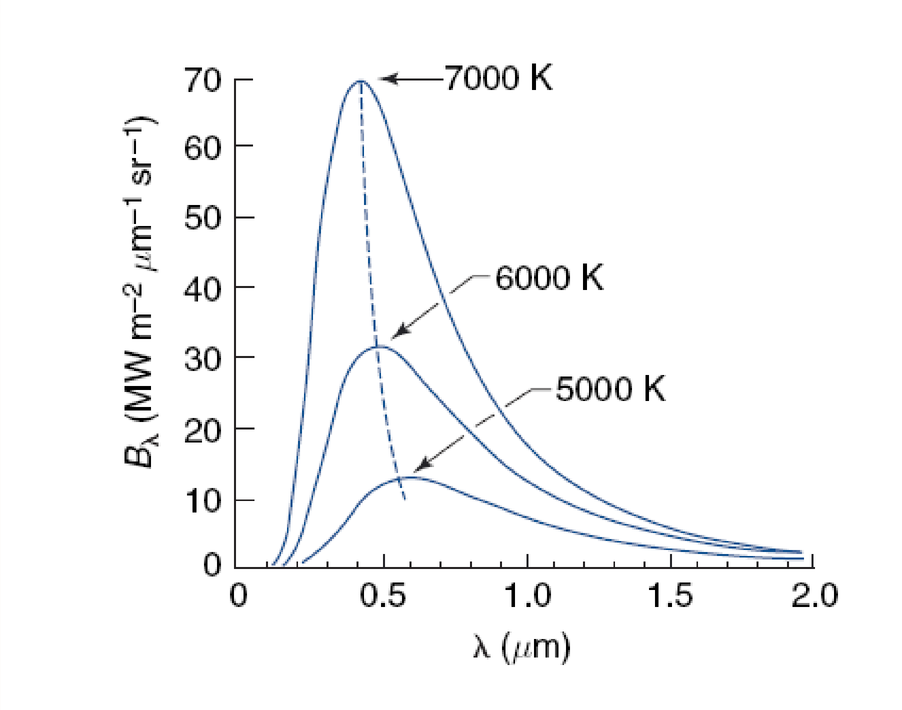
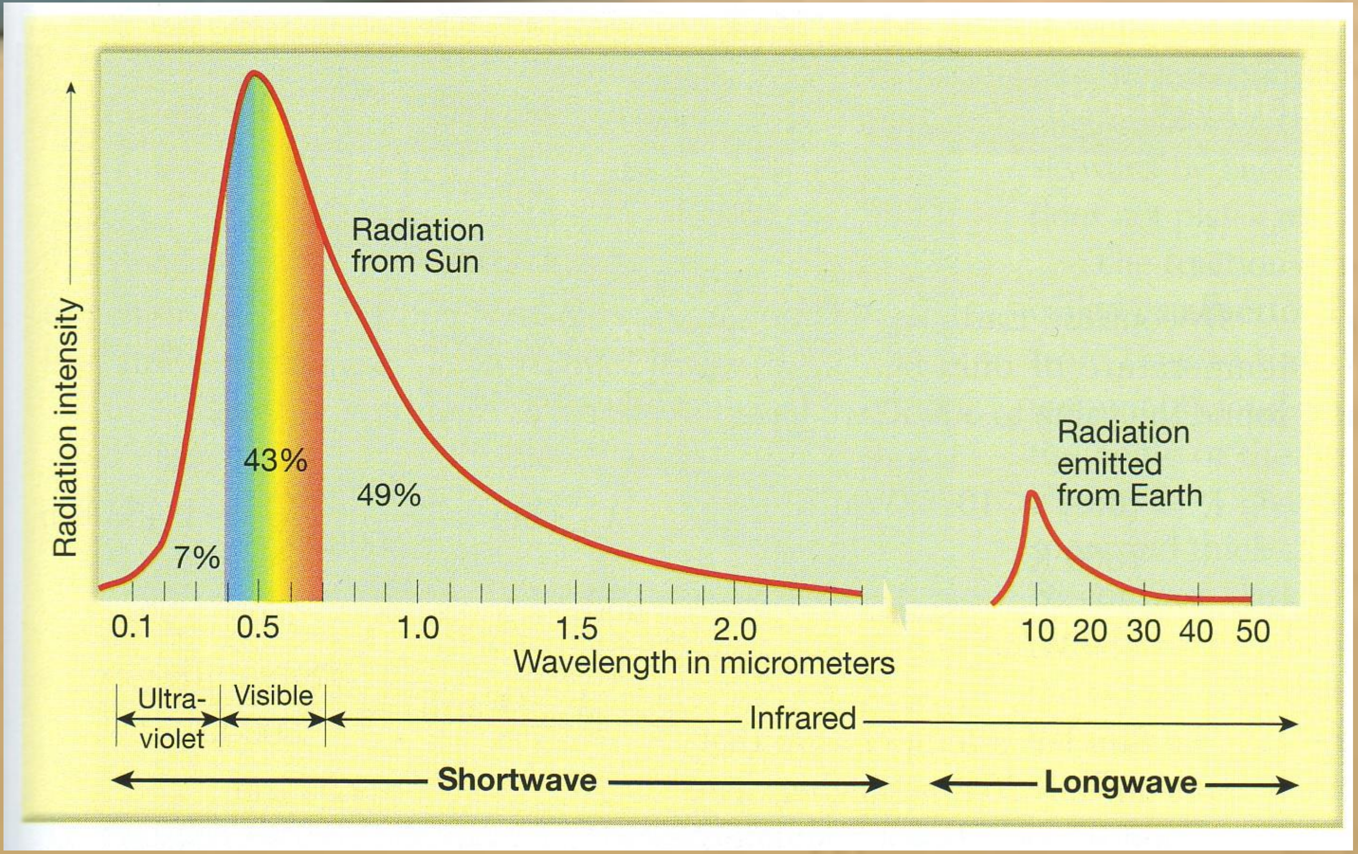
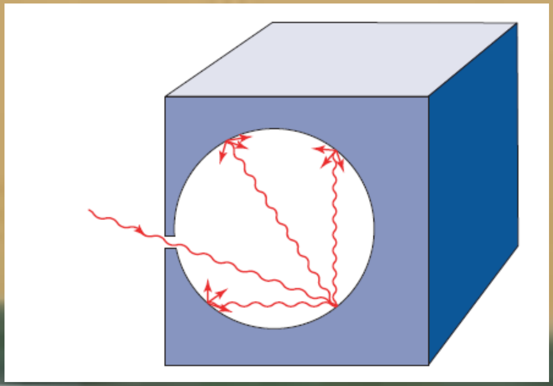


Blackbody radiation curves at three different temperatures (K; see legend in upper right corner).


Radiation intensity
Radiation from Sun
43%
49%
7%
Radiation emitted from Earth
0.1
0.5
1.0
1.5
2.0
10 20 30 40 50
Wavelength in micrometers
Ultra- violet
Visible
Infrared
Shortwave
Longwave



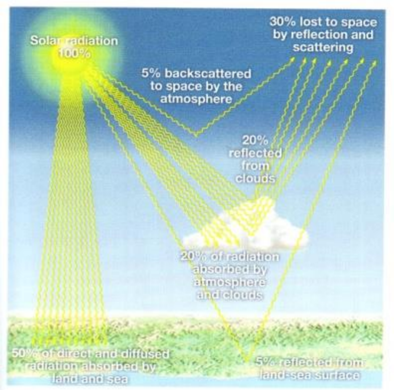
Shortwave Radiation

S_o = 1368 w m^{-2} is the **solar constant** for Earth

Insolation

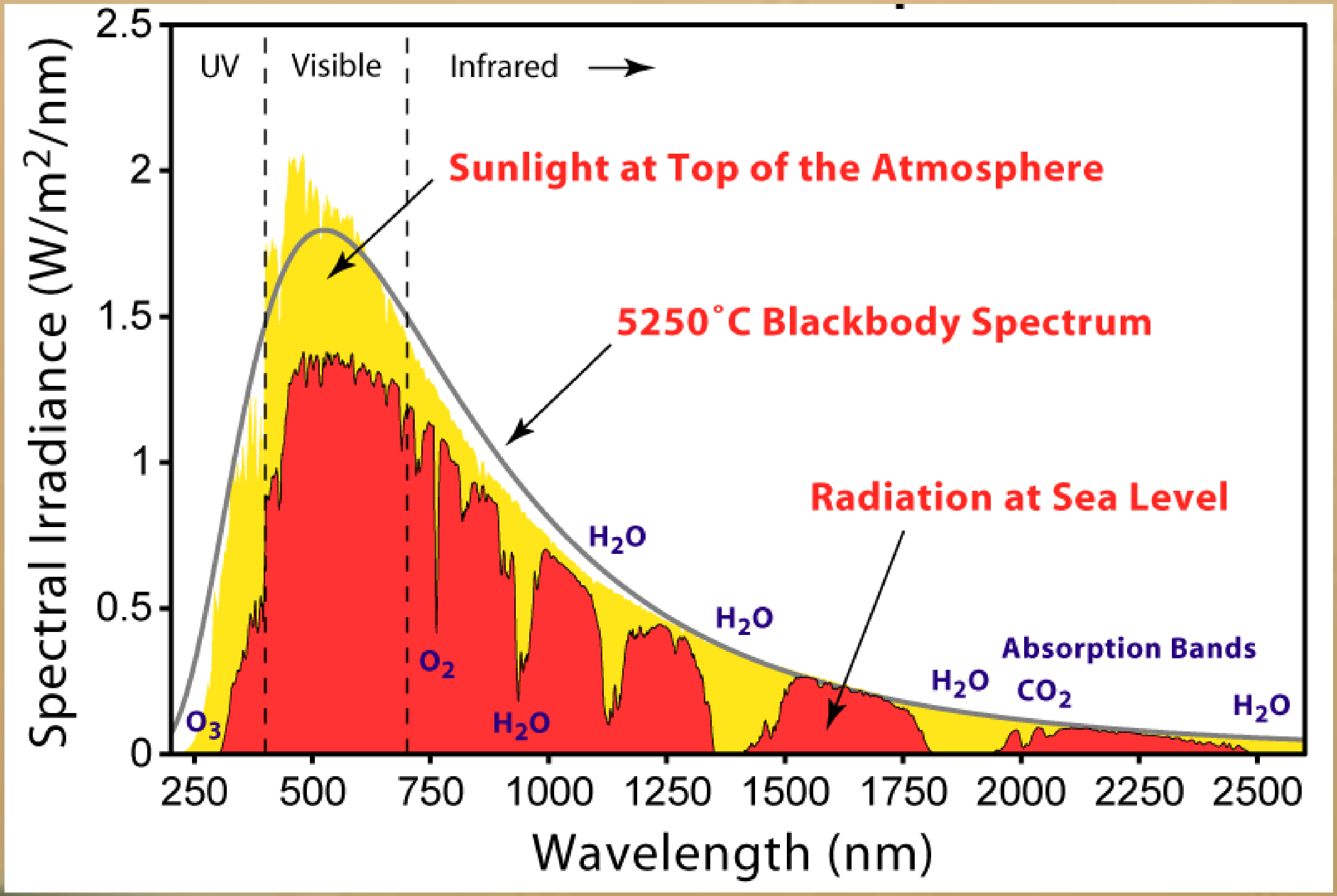
$$R_0 = S_0\left(\frac{d_m}{d}\right)^2 \cos\gamma$$

$$I_0 = \int_{t_1}^{t_2} R_0(t)\,dt$$



Radiation in the Atmosphere

Deviations from blackbody due to absorption by the solar atmosphere, absorption and scattering by the earth's atmosphere (below).



Stefan-Boltzmann Law

This law expresses the rate of radiation emission per unit area

$$R = \sigma T^4 \qquad \sigma = 5.67 \times 10^{-8}\ W / m^2 K^4$$

Compare the difference between the radiation emission from the sun and the Earth.

The sun with an average temperature of 6000 K emits 73,483,200 W/m^2

By contrast, Earth with an average temperature of 300 K emits 459 W/m^2

The sun has a temperature 20 times higher than Earth and thus emits about 160,000 times more radiation

This makes sense, $20^4 = 160{,}000$