Physical Meteorology 1

Lecture 9

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معادله حالت برای بخار آب

Ideal Gas Law for Dry Air

$$p\alpha_d = R_d T$$

p = pressure of dry air
α_d = specific volume of dry air
R_d = gas constant for dry air

Ideal Gas Law for Water Vapor

$$e\alpha_v = R_v T$$

e = vapor pressure of water vapor
α_v = specific volume of water vapor
R_v = gas constant for water vapor

Gas Constant of Water Vapor

$$R_v = \frac{R^*}{M_W}$$

$$R_v = \frac{8314\ J\ K^{-1}\ kmol^{-1}}{18\ kg\ kmol^{-1}}$$

$$R_v = 461\ J\ K^{-1}\ kg^{-1}$$

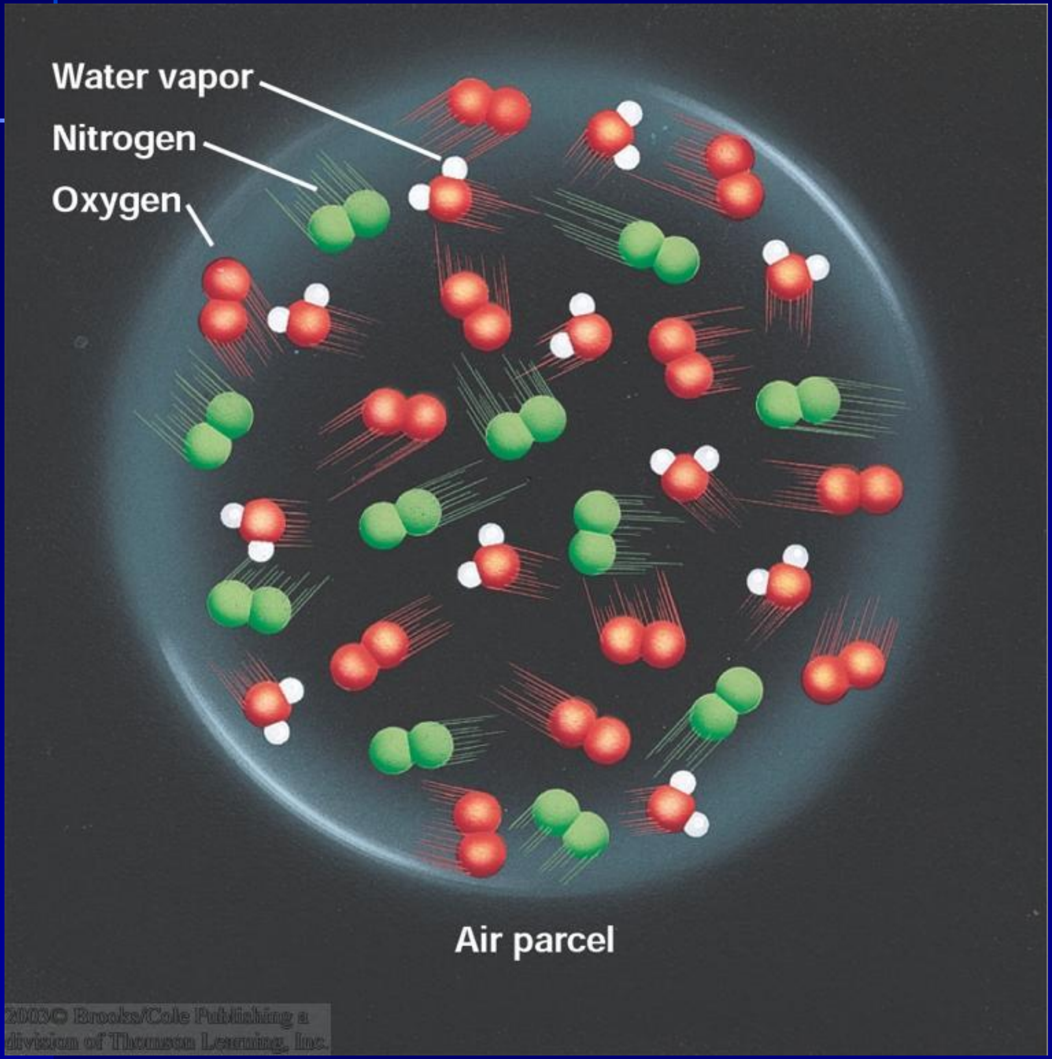
Molecular Weight (*M_w*)

Hydrogen = 1kg kmol^{-1}

Oxygen = 16 kg kmol^{-1}

Water = 18 kg kmol^{-1}

Describing Atmospheric Moisture



Atmospheric water vapor has been defined several different ways.

These terms include absolute humidity, specific humidity, mixing ratio, vapor pressure, and relative humidity.

Moisture Variables متغیرهای رطوبت

برای تعیین آثار تبدیلات انرژی در جو ضروری است پارامترهای مشخصی را که بر هوای مرطوب اعمال می شوند بررسی کنیم.

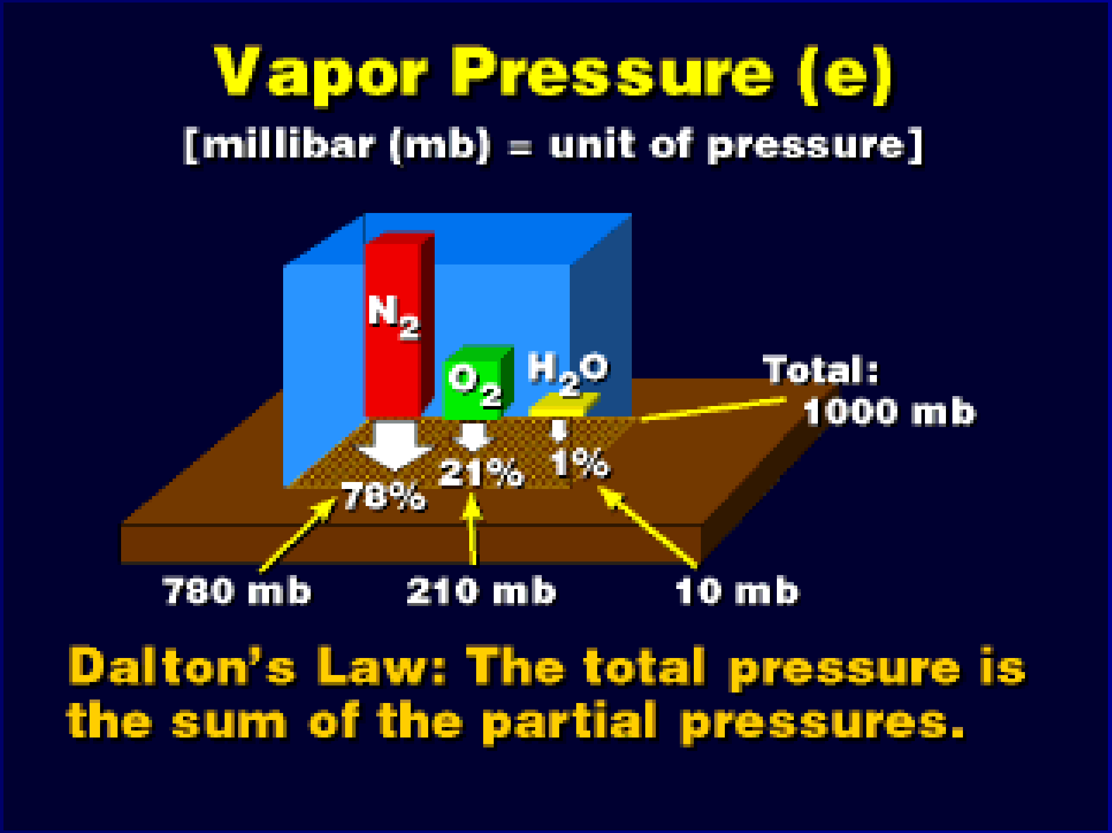
There are numerous ways to quantify the amount of water vapor in the air.

Vapour Presure (e): *part of the atmospheric pressure due to water vapor.*

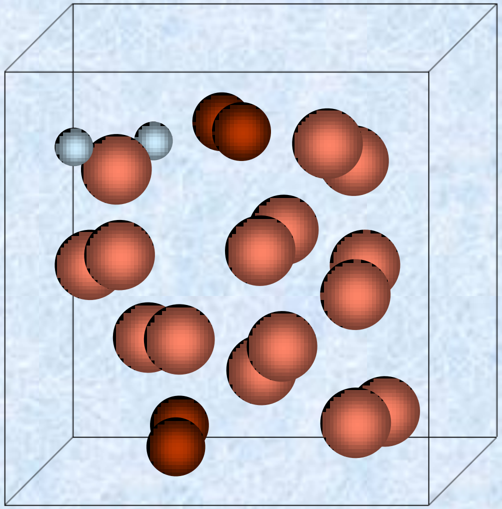
This is the fundamental way to measure the amount of vapor.

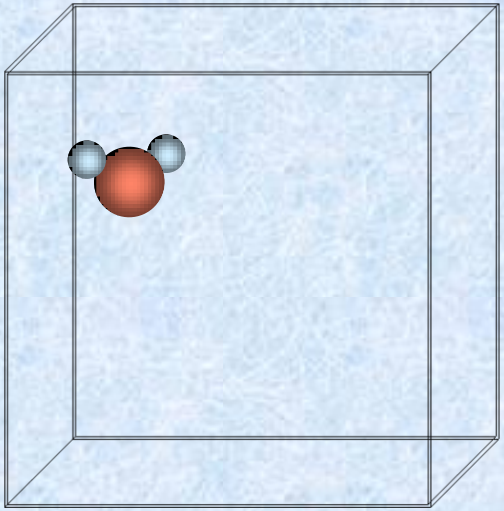
- *Remember Dalton's Law?*
  - *Law of Partial Pressures*
  - *Let's look at the contribution of water*

$$p = p_1 + p_2 + p_3 + \ldots$$



Partial pressure that water vapor exerts





Total Pressure

$$p = p_{O_2} + p_{N_2} + p_{H_2O}$$

Water Vapor Pressure

$$e = p_{H_2O}$$

Saturation vapour pressure فشار بخار اشباع

هنگامی که فضای بلافاصله بالای سطح اشباع شود فشار بخار جزیی وارد آمده توسط بخار آب در دمای آن محیط فشار بخار اشباع نامیده می شود.

فشار بخار اشباع نسبت به سطح آب صاف = e_w

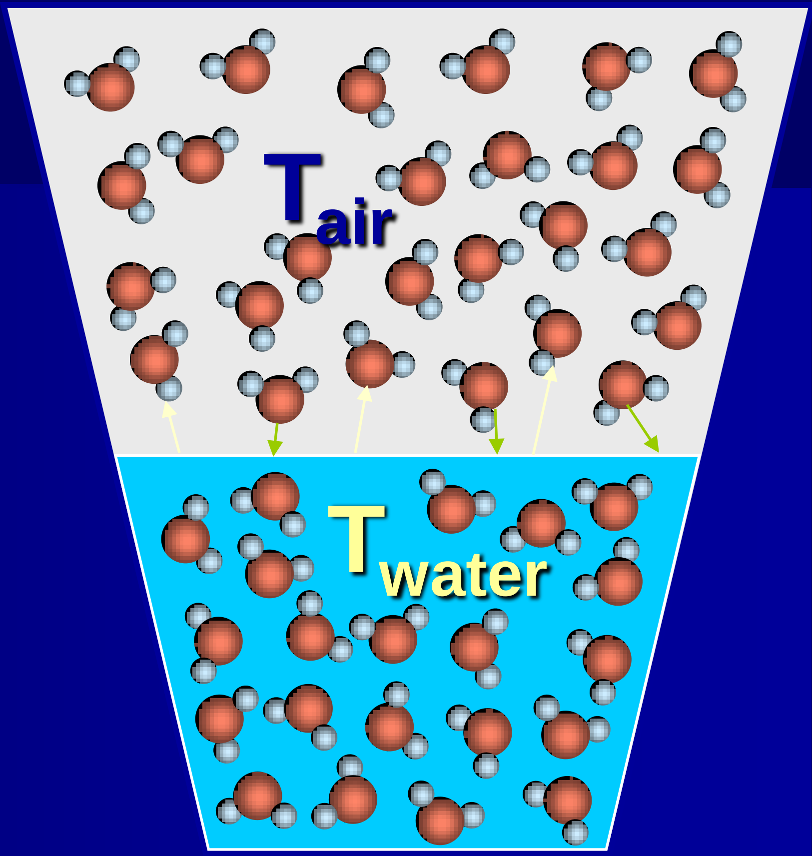
فشار بخار اشباع نسبت به سطح یخ صاف= e_i

Equilibrium

- *At Equilibrium*

$$T_{air} = T_{water}$$

Rate of evaporation = *Rate of condensation*



When air is saturated, evaporation and condensation are in equilibrium.

Mixing Ratio نسبت آمیختگی

mixing ratio (r) – The ratio of the mass of water vapor to the mass of dry air in a sample.

$$r = \frac{m_v}{m_d}$$

Usually expressed in g/kg, but most correct when unitless (i.e. kg/kg). Typical range = 0-25 g/kg.

$$r = \frac{m_v / v}{m_d / v} = \frac{\rho_v}{\rho_d}$$

$$e = \rho_v R_v T$$

$$P - e = \rho_d R_d T$$

$$r = \frac{\dfrac{e}{R_v T}}{\dfrac{P-e}{R_d T}} = \frac{R_d}{R_v}\frac{e}{P-e}$$

$$\frac{R_d}{R_v} = \frac{R^* / M_d}{R^* / M_v} = \frac{M_v}{M_d} = \varepsilon \cong 0.622$$

$$r = \varepsilon \frac{e}{p-e} \approx \varepsilon \frac{e}{p}$$

Saturation Mixing Ratio نسبت آمیختگی اشباع

نسبت آمیختگی اشباع یک نمونه هوای نمناک مقدار نسبت آمیختگی در نمونه ای است که اشباع شده باشد.

$$r_s = \varepsilon \frac{e_s}{p - e_s}$$

Absolute Humidity

mass of water vapor/volume of air

gr/m^3

Relative Humidity رطوبت نسبی

Relative Humidity (RH) -Ratio of mixing ratio to its saturation value. Expressed in percent.

$$RH = \frac{Actual \quad Amount}{Amount \quad if \quad Saturated} \; x\; 100\ \%$$

$$RH = \frac{r}{r_s} \times 100\,\% \approx \frac{e}{e_s} \times 100\,\%$$

RH = 100% means air is saturated

Specific Humidity رطوبت ویژه

Specific humidity (q) *– mass of water vapor per unit mass of moist air. similar to r, but slightly smaller values. (g/kg)*

$$q = \frac{m_v}{m_v + m_d}$$

$$q = \frac{\rho_v}{\rho_v + \rho_d} = \varepsilon \frac{e}{p - (1-\varepsilon)e} \approx \varepsilon \frac{e}{p}$$

معادله حالت برای هوای نمناک

$$p\alpha = R_m T = \frac{R^*}{M} T$$

$$\frac{1}{\overline{M}} = \frac{\sum \frac{m_n}{M_n}}{\sum m_n}$$

$$\frac{1}{\overline{M}} = \frac{\frac{m_d}{M_d} + \frac{m_v}{M_v}}{m_d + m_v}$$

$$R_d = R^* / M_d \quad , \quad R_v = R^* / M_v$$

$$\frac{1}{\overline{M}} = \frac{m_d \frac{R_d}{R^*} + m_v \frac{R_v}{R^*}}{m_d + m_v}$$

$$\therefore \frac{R^*}{\overline{\overline{M}}} = \frac{m_d R_d + m_v R_v}{m_d + m_v}$$

$$\therefore R_m = \frac{m_d R_d + m_v R_v}{m_d + m_v}$$

$$R_m = \frac{R_d + rR_v}{1+r} = \frac{R_d\left(1+\frac{R_v}{R_d}r\right)}{1+r} \cong \frac{R_d(1+1.61r)}{1+r}$$

$$r = \frac{m_v}{m_d} << 1$$

$$(1+r)^{-1} \approx 1-r$$

$$\therefore R_m \cong R_d(1+1.61r)(1-r)$$

$$R_m \cong R_d\left(1+0.61r\right)$$

Virtual Temperature دمای مجازی

$$p=\rho_m R_m T \qquad , \qquad p=\rho_d R_d T$$

$$\frac{\rho_m}{\rho_d}=\frac{R_d}{R_m}\approx\frac{R_d}{R_d\left(1+0.61r\right)}$$

$$\rho_m \approx \frac{1}{\left(1+0.61r\right)}\rho_d$$

Virtual Temperature

For moist air, the gas constant changes because water vapor *is less dense* than dry air. To simplify things, a virtual temperature, T_v can be defined to include the effects of water vapor on density:

$$T_v = T \cdot (1 + 0.61 \cdot r)$$

where r is the water vapor mixing ratio (g water vapor / kg dry air) and all temperatures are in K.

Moist air of temperature T behaves as dry air of temperature T_v.

Interpretation: Virtual temperature is defined as the temperature that perfectly dry air would need to have to have the same density as the air with moisture.

Logically, as the moisture content of the air increases, the virtual temperature increases since an increase of moisture decreases density.

Moist air is less dense than dry air.

Therefore, air which is perfectly dry needs to have a higher temperature in order to have the same density as the less dense moist air.

You may ask the purpose in needing to know virtual temperature. It is important since it makes meteorological equations MUCH less complicated. If virtual temperature is used, the moisture in the air can be ignored.

*Two air parcels that have the same virtual temperature will also have the same density. By using virtual temperature it can be inferred immediately which parcel is less or more dense.

Example problem: What is the virtual temperature of air that is 50° F and has a mixing ratio of 5g/kg?

Answer: First change mixing ratio of kg/kg.
5g/kg =0.005kg/kg
Second, convert temperature to Kelvins. 50° F = 10° C = 283.0 K
The mixing ratio is UNITLESS since kg/kg cancels.

$$T_v = 283.0\ K(1 + (0.61 \times 0.005)) = 283.9\ K$$

دمای مجازی یک نمونه هوا نمناک را به عنوان دمایی تعریف می کنیم که در آن دما هوای خشک باید با همان فشار کل همان چگالی نمونه را داراست.

$$p = \rho_m R_m T \quad , \quad p = \rho_d R_d T_v$$

$$\text{since} \quad \rho_{\mathrm{m}} = \rho_d$$

$$R_m T = R_d T_v$$

$$p\alpha = R_d T_v$$

محاسبه دمای مجازی

$$R_m \cong R_d\left(1 + 0.61r\right)$$

$$R_m T = R_d T_v$$

$$T_v \approx T\left(1 + 0.61r\right)$$