Physical Meteorology 1

Lecture 11

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In this section we will derive the two most important relationships describing atmospheric structure

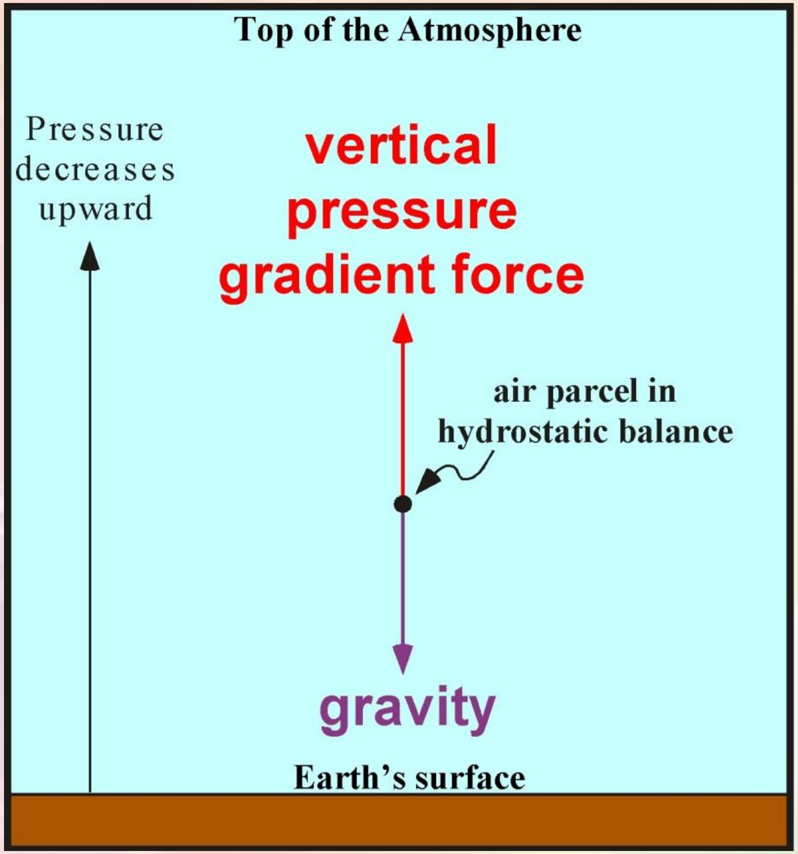
1- Hydrostatic Balance: the vertical force balance in the atmosphere

2-The Hypsometric Equation: the relationship between the virtual temperature of a layer and the layer's thickness

Hydrostatic Equilibrium ترازمندی هیدروستاتیکی

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial z} - g$$

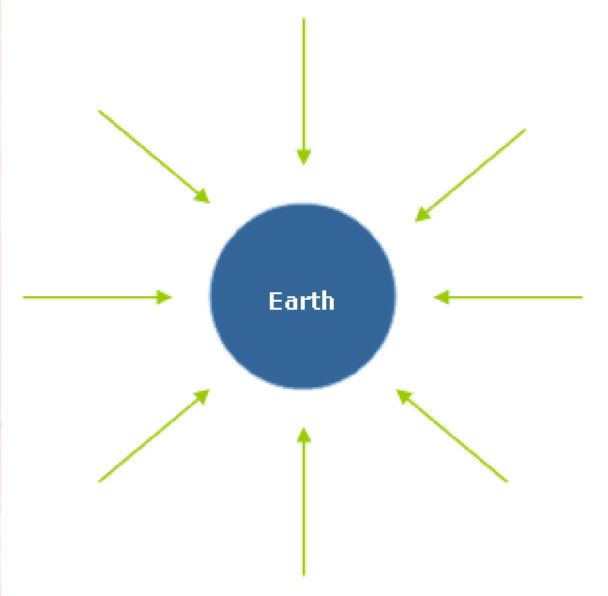
The atmosphere is in hydrostatic balance essentially everywhere except in core regions of significant storms such as hurricanes and thunderstorms



Universal Law of Gravitation

All objects in the Universe attract each other with a force that varies directly as the product of their masses and inversely as the square of their separation from each other.

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r_{21}^{\;2}}\hat{r}_{21}$$



m_1 and m_2 are the masses of the objects 1 and 2, and G is the gravitational constant.

$\mathbf{F}_{12}$ is the force on object 1 due to object 2

$r_{21} = |\,\mathbf{r}_1 - \mathbf{r}_2\,|$ is the distance between objects 1 and 2

If the earth is taken to be the mass M and m is taken to be the mass of a fluid parcel or volume element, then we can write the force per unit mass exerted on the fluid by the earth as

$$\frac{\vec{F}_g}{m} \equiv \vec{g}^* = -\frac{GM}{r^2}\left(\frac{\vec{r}}{r}\right) \qquad g^* = -\frac{GM}{(a+z)^2} = -\frac{GM}{a^2\left(1+\frac{z}{a}\right)^2}$$

If a is the radius of the earth and z is the distance above sea level, then

$$\vec{g}^* = \frac{\vec{g}_0^*}{(1+z/a)^2}, \qquad \text{where} \qquad \vec{g}_0^* = -\left(\frac{GM}{a^2}\right)\left(\frac{\vec{r}}{r}\right)$$

Because the depths of the atmosphere is small compared to the radius of the earth ($z \ll a$) we can treat the gravitational force per unit mass as a constant.

$$\vec{g}^* = \vec{g}_0^*$$

The Gravity Forc نیروی گرانی

$$\vec{g}^* = \vec{g}$$

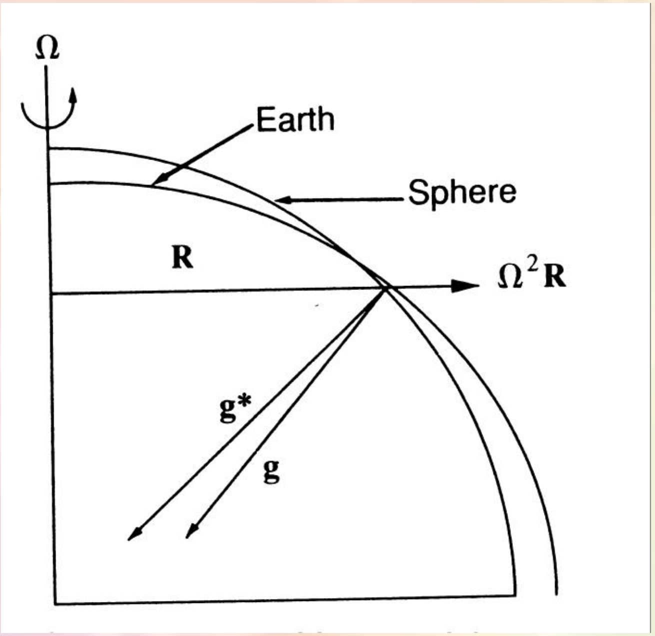
در صورت عدم چرخش زمین

$$\vec{g}^* = \vec{g} \to pole$$

$$g = g^* - \Omega^2 R \to Equator$$

$$\Delta g = g_{pol} - g_{Eq} = 5.2 cm/s^2$$

$$g_x = 0, g_y = 0, g_z = -g$$



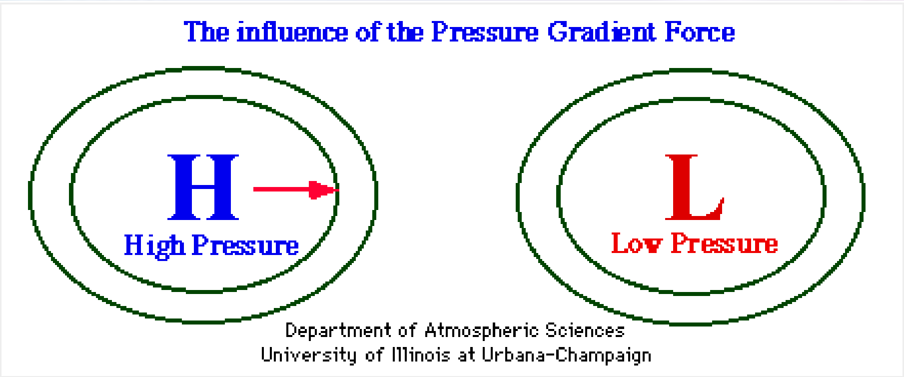
The atmosphere is held to the Earth by the force of gravity, which prevents the gaseous molecules from escaping into space.

The pressure gradient

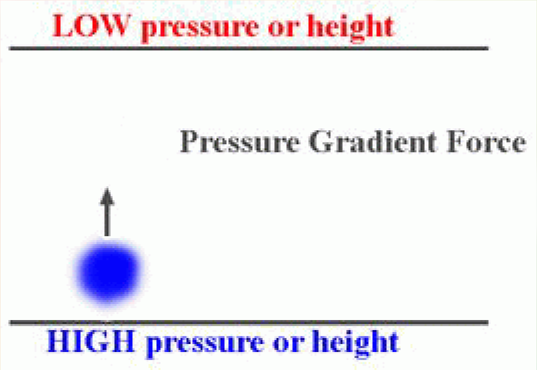
The change in **pressure** measured across a given distance is called a "pressure gradient".

Pressure gradient = $\dfrac{\Delta P}{distance} = \dfrac{P_{high} - P_{low}}{distance}$

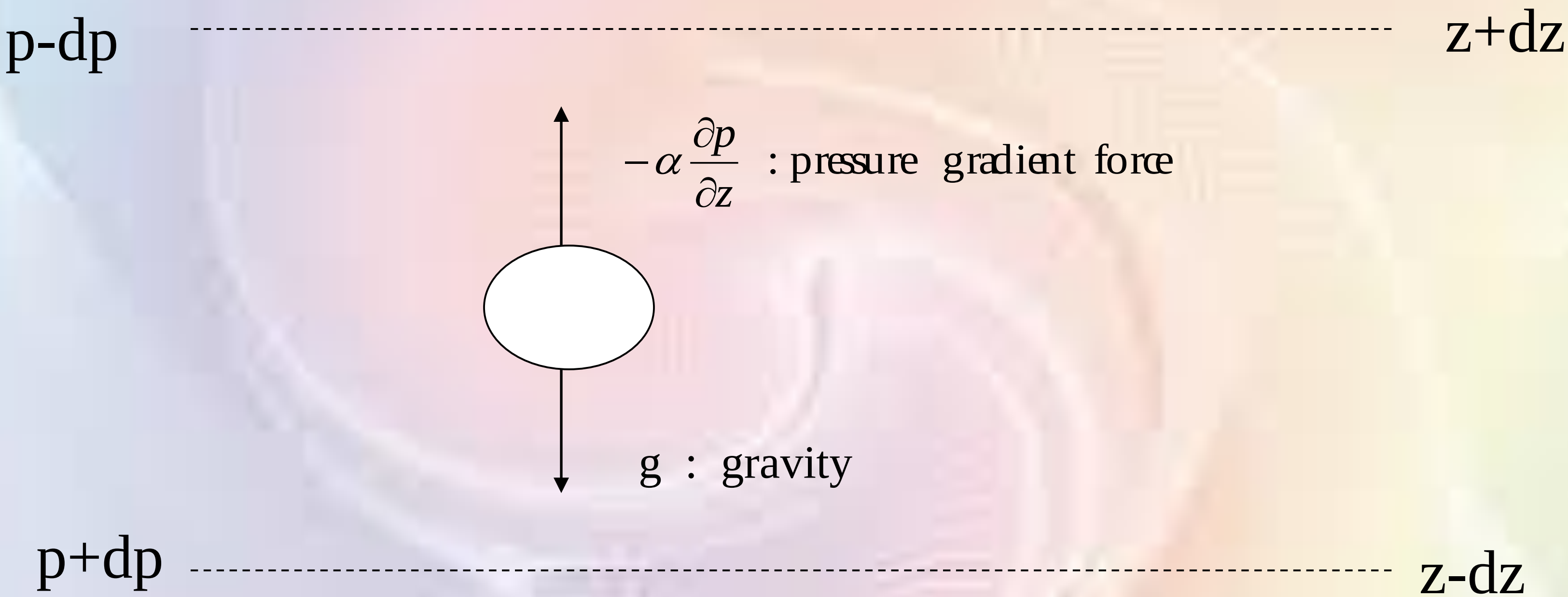
The pressure gradient results in a net force that is directed from high to low pressure and this force is called the "pressure gradient force".



Wind blows from high to low pressure



Hydrostatic Equilibrium



hydrostatic balance:

gravity = pressure gradient

$$\text{in equilibrium} \quad -\alpha \frac{dp}{dz} = g$$

(one of the best approximations in meteorology)

$$\frac{\partial p}{\partial z} = \frac{dp}{dz} = -\rho g$$

but $\rho = p / RT_v$

$$or \quad \frac{dp}{p} = -\frac{g}{RT_v} dz$$

$$RT_v \frac{dp}{p} = -g dz = -d\Phi$$

$$\Phi = \int_0^z g dz$$

ژئوپتانسیل: عبارت است ازانرژی پتانسیل گرانشی بر واحد جرم نمونه هوا .

$$\Phi = gz \qquad (J / kg)$$

Geopotential

$$d\Phi \equiv g\,dz \quad \text{(energy/mass)}$$

g has a slight variation with latitude and altitude which can usually be ignored.

$$1\,gpm = 9.8\,J/kg$$

یک ژئوپتانسیل متر انرژی گرانشی نمونه ای از هوا به جرم یک کیلوگرم است که
در ارتفاع یک متر از سطح آزاد دریا که $g = 9.8ms^{-2}$ قرار گرفته باشد.

$$\Phi = \frac{g}{9.8} z \quad gpm$$

Pressure Variation with z for Atmospheres

Consider the case for g constant (good assumption)
Integrate the hydrostatic equation from p = p_o to p and z_o to z.

$$\frac{dp}{p} = -\frac{g}{RT_v}dz$$

$$p(z) = p_o \exp\left(\frac{-g}{RT_v}(z - z_o)\right)$$

معادله بارومتریک

1. Constant Density $\rho = \rho_o$ (Homogenous Atmosphere)

$$dP = -\rho g dz = \rho_o g dz$$

$$\int_{p_o}^{0} dP = -\rho_o g \int_{0}^{H} dz \qquad H = \frac{P_o}{\rho_o g} = \frac{\rho_o RT}{\rho_o g} = \frac{RT_o}{g}$$

H => "Scale Height" : height of homogenous atmosphere ~ 8 km

The height above a reference height at which pressure decreases to 1/e of its value at the reference height.

Since temperature changes with altitude in the atmosphere, scale height also change with altitude.

2. Constant Lapse Rate Atmosphere

Define Lapse Rate $$\gamma \equiv -\frac{\partial T}{\partial z}$$

$$dP = -\rho g dz = -\frac{P}{RT} g dz$$

$$\frac{dP}{P} = -\frac{g}{RT} dz = -\frac{g dz}{R(T_o + \gamma z)}$$

$$\int_{p_0}^{p} dp / p = -\int_{z_0}^{z} \frac{g dz}{R(T_0 - \gamma z)}$$

$$\ln p / p_0 = \frac{g}{R\gamma} \ln \frac{T_0 - \gamma z}{T_0 - \gamma z_0}$$

$$p = p_0 \left(\frac{T_0 - \gamma z}{T_0} \right)^{g/R\gamma}$$

$$z = T_0 / \gamma \left[1 - (p_0 / p)^{-R\gamma / g} \right]$$

3. Isothermal Atmosphere : $\gamma = 0$

$$\frac{dP}{P} = -\frac{g}{RT_o}dz = -\frac{dz}{H}$$

$$P(z) = P_o e^{-z/H}$$

با مشخص شدن ارتفاع می توان فشار را بدست آورد

The Standard Atmosphere جو معيار

a) Air is ideal dry gas

b) The physical constant are:

1. MSL mean molecular weight M_o=28.96644 kg mol^{-1}

2. MSL atmosphere pressure P_o =1013.250 mb

3. MSL temperature T_o=288.15 K

4. MSL density ρ_o=1.250 kg m^{-3}

5. Temperature of the ice point T_i=273.15

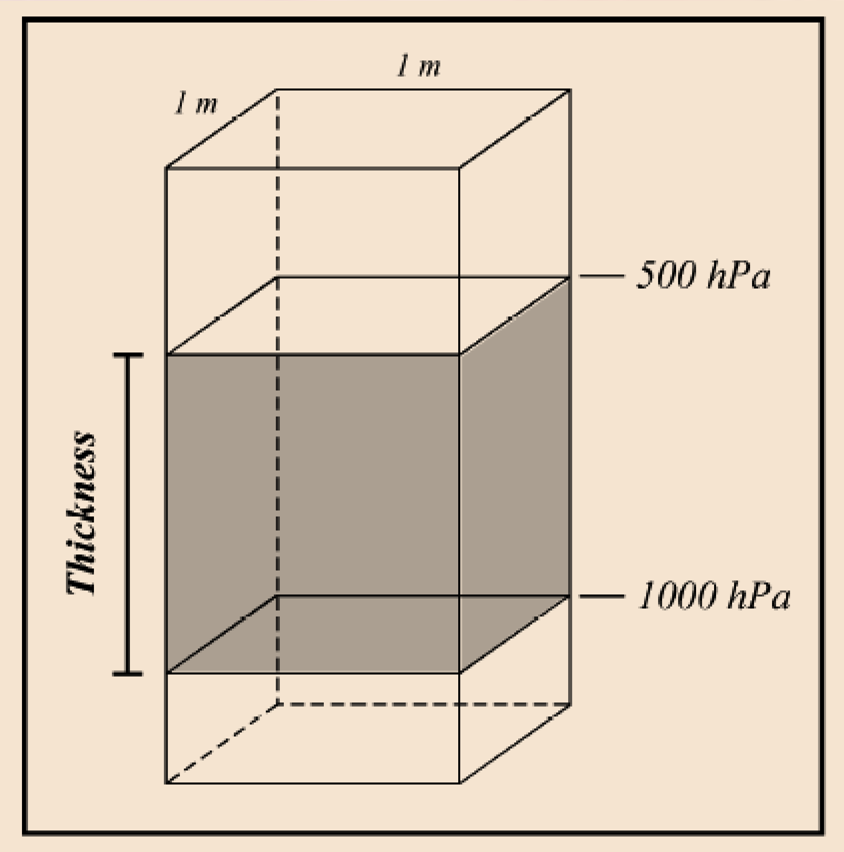
6. Universal gas constant R^*=8314.32 Jmol^{-1}K^{-1}

The Hypsometric Equation

Consider a column of atmosphere that is 1 m by 1 m in area and extends from sea level to space

Let's isolate the part of this column that extends between the 1000 hPa surface and the 500 hPa surface

How much mass is in the column?



$$Mass = (1000 - 500)\ hPa \times \left(\frac{100\ N\ m^{-2}}{hPa}\right) \times 1\ m^2 \times \left(\frac{1}{9.81\ m\ s^{-2}}\right) = 510204\ kg$$

$$\frac{dP}{dz} = -g\rho$$

Hydrostatic equation

$$p = -\rho R_d T_v$$

Ideal Gas Law

Substitute ideal gas law into hydrostatic equation

$$dz = -\frac{R_d T_v}{g}\frac{dp}{p}$$

Integrate this equation between two levels (p_2, z_2) and (p_1, z_1)

$$\int_{z_1}^{z_2} dz = -\int_{p_1}^{p_2} \frac{R_d T_v}{g}\frac{dp}{p}$$

$$\int_{z_1}^{z_2} dz = -\int_{p_1}^{p_2} \frac{R_d T_v}{g} \frac{dp}{p}$$

$$\int_{z_1}^{z_2} dz = \int_{p_2}^{p_1} \frac{R_d T_v}{g} d \ln p$$

Problem: T_v varies with altitude. To perform the integral on the right we have to consider the pressure weighted column average virtual temperature given by:

$$\overline{T_v} = \frac{\int_{p_2}^{p_1} T_v d \ln p}{\int_{p_2}^{p_1} d \ln p}$$

We can then integrate to give

$$\boxed{z_2 - z_1 = \frac{R_d \overline{T_v}}{g} \ln\left(\frac{p_1}{p_2}\right)}$$

This equation is called the Hypsometric Equation

The equation relates the thickness of a layer of air between two pressure levels to the average virtual temperature of the layer

Geopotential Height

We can express the hypsometric (and hydrostatic) equation in terms of a quantity called the geopotential height

Geopotential (Φ): Work (energy) required to raise a unit mass a distance *dz* above sea level

$$d\Phi = g dz$$

$$\boxed{\Delta\Phi = R_d \overline{T}_v \ln\left(\frac{p_1}{p_2}\right)}$$

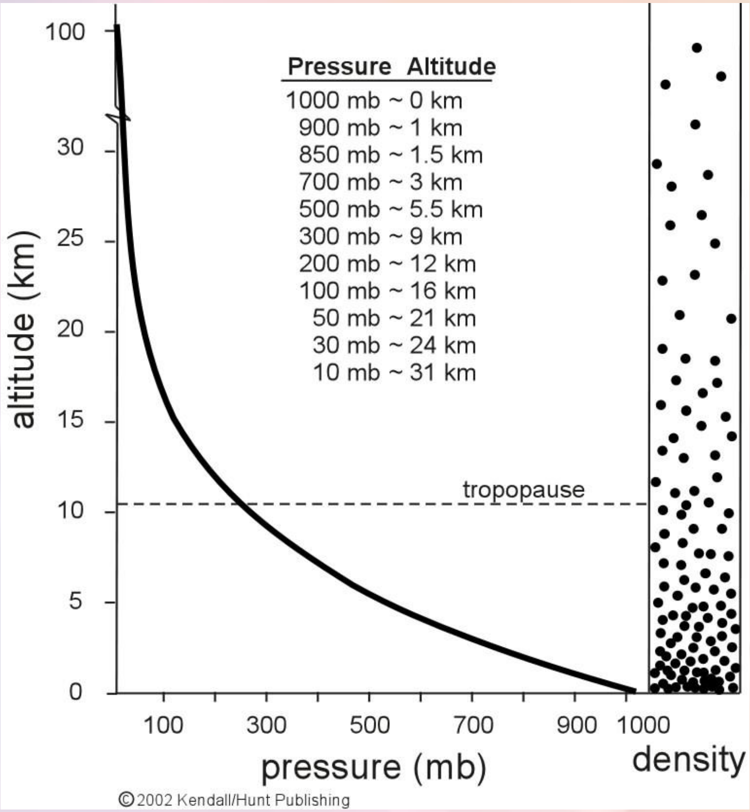
Meteorologists often refer to "geopotential height" because this quantity is directly associated with energy to vertically displace air

Geopotential Height (Z): $$Z = \frac{\Phi}{g_0} = \frac{gz}{g_0}$$

g_0 is the globally averaged Value of gravity at sea level

For practical purposes, Z and z are about the same in the troposphere

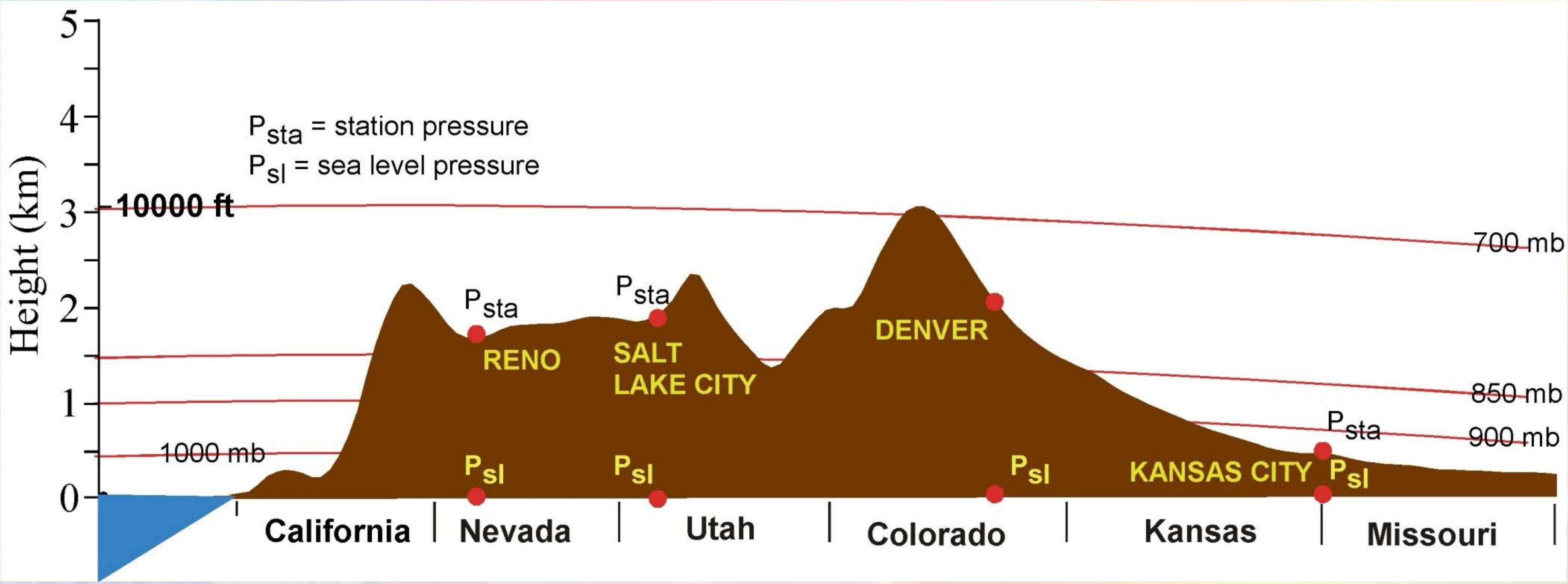
Atmospheric pressure varies exponentially with altitude,
but very slowly on a horizontal plane
as a result, a map of surface (station) pressure looks like
a map of altitude above sea level.



variation of pressure
with altitude

Station pressures are measured at locations worldwide

Analysis of horizontal pressure fields, which are responsible for the earth's winds and are critical to analysis of weather systems, requires that station pressures be converted to a common level, which, by convention, is mean sea level.



Reduction of station pressure to sea level pressure:

Integrate hypsometric equation

$$\int_{p_{STA}}^{p_{SL}} \frac{dp}{p} = \int_{z_{STA}}^{0} \frac{-g}{RT_v} dz$$

sea level pressure (P_{SL})
to station pressure (P_{STN})
and right side from z = 0
to station altitude (Z_{STN}).

$$\ln(p_{SL}) = \ln(p_{STN}) + \frac{g}{R_d \overline{T}_v} z_{STN}$$

BUT WHAT IS $\overline{T}_v$? WE HAVE ASSUMED A FICTICIOUS ATMOSPHERE THAT IS BELOW GROUND!!!

National Weather Service Procedure to estimate T_v

1. Assume $T_v = T$
2. Assume a mean surface temperature = average ofcurrent temperature and temperature 12 hours earlier to eliminate diurnal effects.
3. Assume temperature increases between the station and sea level of 6.5°C/km to determine T_{SL}.
4. Determine average T and then P_{SL}.

In practice, PSL is determined using a table of "R" values, where R is the ratio of station pressure to sea level pressure, and the table contains station pressures and average temperatures.

Table contains a "plateau correction" to try to compensate for variations in annual mean sea level pressures calculated for nearby stations.

Implications of hypsometric equation $\Delta z = \frac{R_d \bar{T}_v}{g} \ln\left(\frac{p_1}{p_2}\right)$

Consider the 1000—500 hPa thickness field. Using $R_d = 287\, J\, kg^{-1}\, K^{-1}$

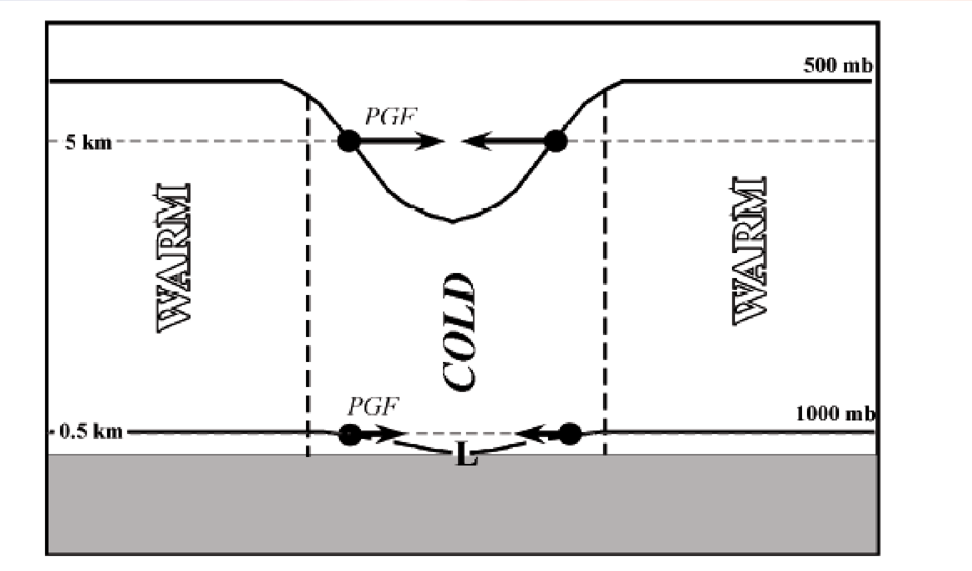
$$g = 9.81\, m\, s^{-2}$$

$$Layer\ Thickness = 20.3\, \bar{T}_v\ meters$$

$$\Delta(Layer\ Thickness) = 20.3\, \Delta\bar{T}_v\ meters$$

Ballpark number: A decrease in average 1000-500 hPa layer temperature of 1K leads to a reduction in thickness of the layer of 60 hPa

Implications of Hypsometric Equation



A cold core weather system (one which has lower temperature at its center) will winds that increase with altitude

Sea-level Pressure Reduction

Near the bottom of the troposphere, pressure gradients are large in the vertical (order of 10 kPa km–1) but small in the horizontal (order of 0.001 kPa km–1). As a result, pressure differences between neighboring surface weather stations are dominated by their relative station elevations z_{stn} (m) above sea level. However, horizontal pressure variations are important for weather forecasting, because they drive horizontal winds. To remove the dominating influence of station elevation via the vertical pressure gradient, the reported station pressure P_{stn} is extrapolated to a constant altitude such as mean sea level (MSL). Weather maps of mean-sea-level pressure (P_{MSL}) are frequently used to locate high- and lowpressure centers at the bottom of the atmosphere. The extrapolation procedure is called sea-level pressure reduction, and is made using the hypsometric equation:

$$P_{MSL} = P_{stn} \cdot \exp\left(\frac{z_{stn}}{a \cdot \overline{T_v^*}} \right) \tag{9.1}$$

where $a = \Re_d/|g| = 29.3$ m K^{-1}, and the average air virtual temperature $\overline{T_v}$ is in Kelvin.

A difficulty is that $\overline{T_v}$ is undefined below ground. Instead, a fictitious average virtual temperature is invented:

$$\overline{T_v^*} = 0.5 \cdot [T_v(t_o) + T_v(t_o - 12\text{ h}) + \gamma_{sa} \cdot z_{stn}] \tag{9.2}$$

where $\gamma_{sa} = 0.0065$ K m^{-1} is the standard-atmosphere lapse rate for the troposphere, and t_o is the time of the observations at the weather station. Eq. (9.2) attempts to average out the diurnal cycle, and it also extrapolates from the station to halfway toward sea level to try to get a reasonable temperature.

Sample Application

Phoenix Arizona (elevation 346 m MSL) reports dry air with T = 36°C now and 20°C half-a-day ago. P_{stn} = 96.4 kPa now. Find P_{MSL} (kPa) at Phoenix now.

Find the Answer

Given: T(now) = 36°C, T(12 h ago) = 20°C,
z_{stn} = 346 m, P(now) = 96.4 kPa. Dry air.
Find: P_{MSL} = ? kPa

$T_v = T$, because air is dry. Use eq. (9.2) : $\overline{T_v^*}$ =
= $0.5\cdot[\,(36°C) + (20°C) + (0.0065\ K\ m^{-1})\cdot(346\ m)]$
= 29.16°C (+ 273.15) = 302.3 K

Use eq. (9.1):
P_{MSL} = (96.4 kPa)·exp[(346 m)/((29.3 m K^{-1})·(302.3 K))]
= (96.4 kPa)·(1.03984) = **<u>100.24 kPa</u>**

Check: Units OK. Physics OK. Magnitude OK.
Discus.: P_{MSL} can be significantly different from P_{stn}

Virtual temperature is a fictitious temperature that takes into account moisture in the air.

The formal definition of virtual temperature is the temperature that dry air would have if its pressure and specific volume were equal to those of a given sample of moist air.

Virtual temperature allows meteorologists to use the equation of state for dry air even though moisture is present.