
فیزیک I
درس پنجم
صحرایی
گروه فیزیک دانشگاه رازی
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Vectors and Scalars

A Scalar is a physical quantity with magnitude (and units).
Examples:

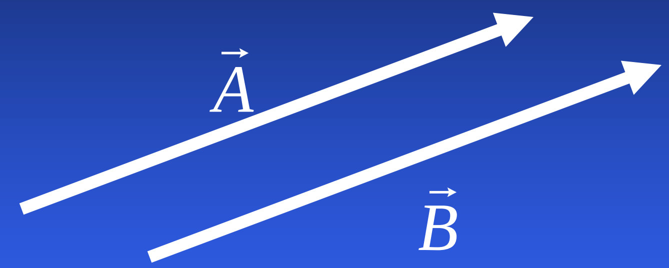
Temperature, Pressure, Distance, Speed

A Vector is a physical quantity with magnitude and direction:

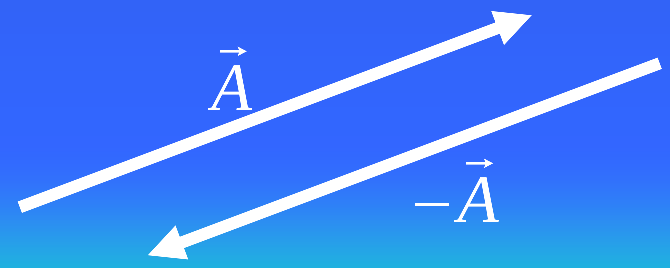
- **Displacement: Washington D.C. is 250 miles N of Norfolk**
- **Wind Velocity : 20mi/hr towards SW**

Fundamental definitions:

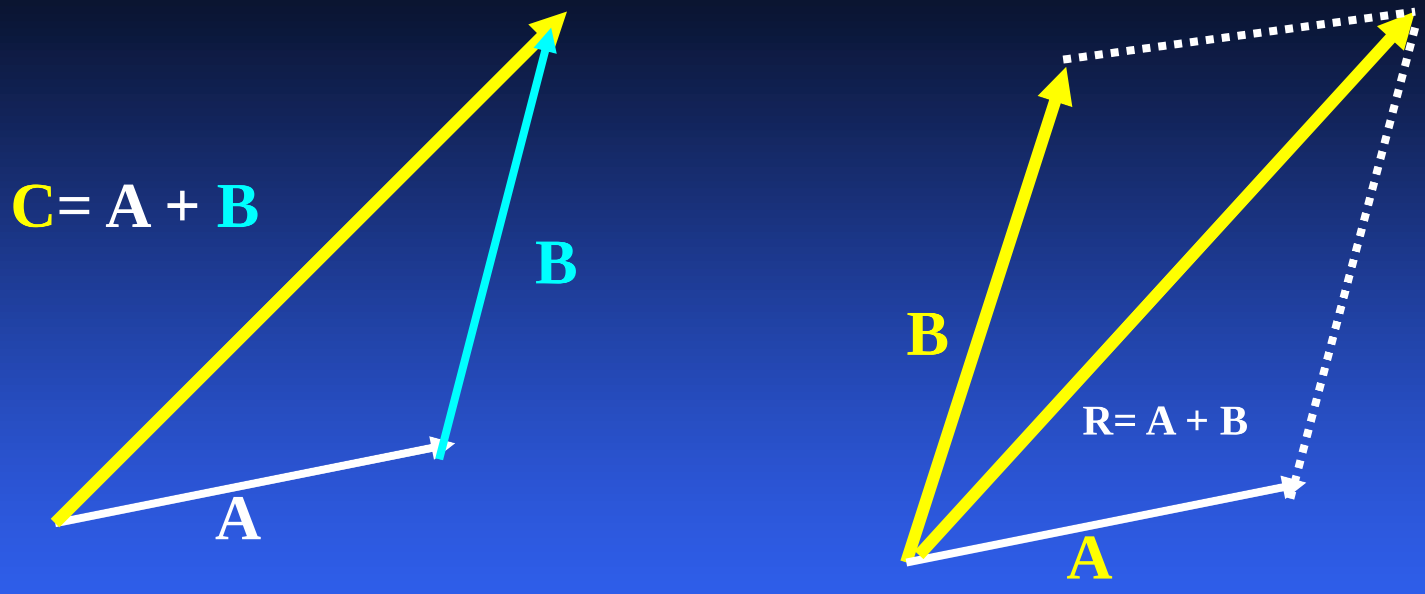
- Two vectors $\vec{A}$ and $\vec{B}$ are equal if they have the same *magnitude* and *direction* regardless of the initial points



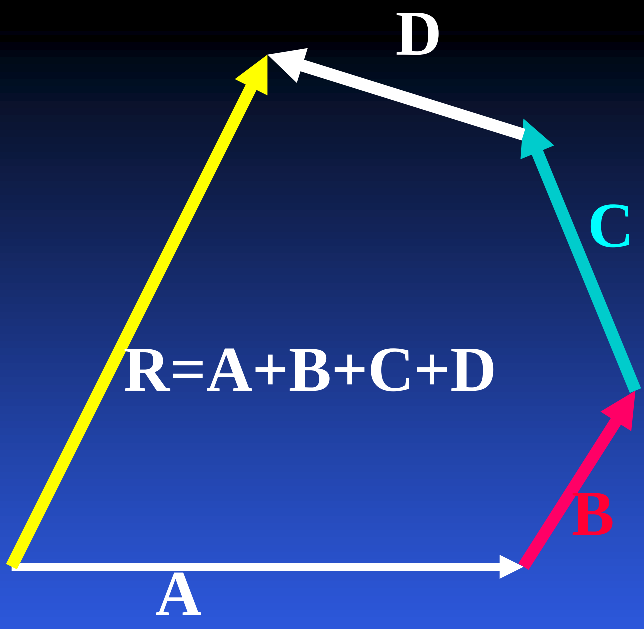
- Having direction opposite to $\vec{A}$ but having the same magnitude

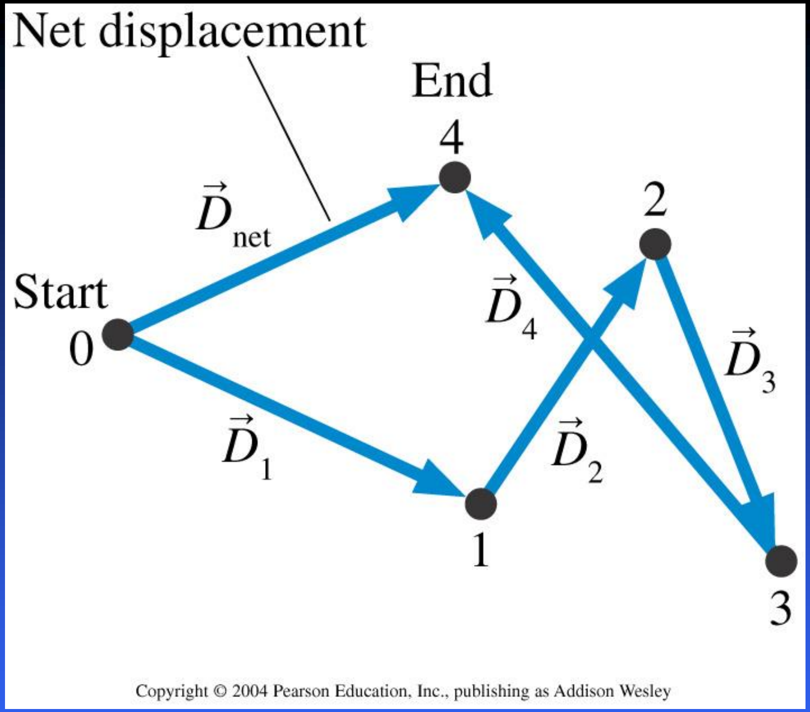


- Addition: $\vec{C} = \vec{A} + \vec{B}$

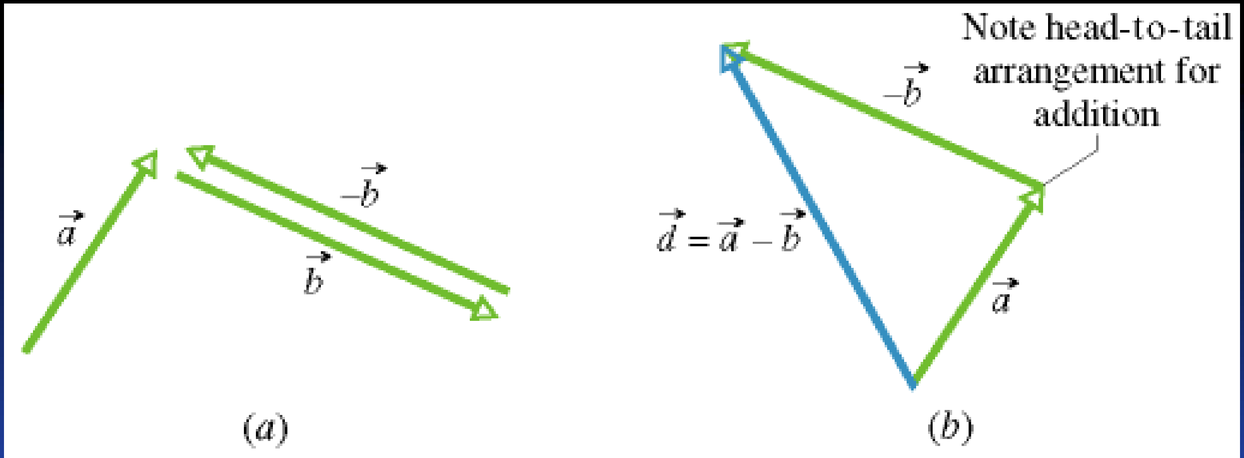


$$\vec{D}_{net} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4$$

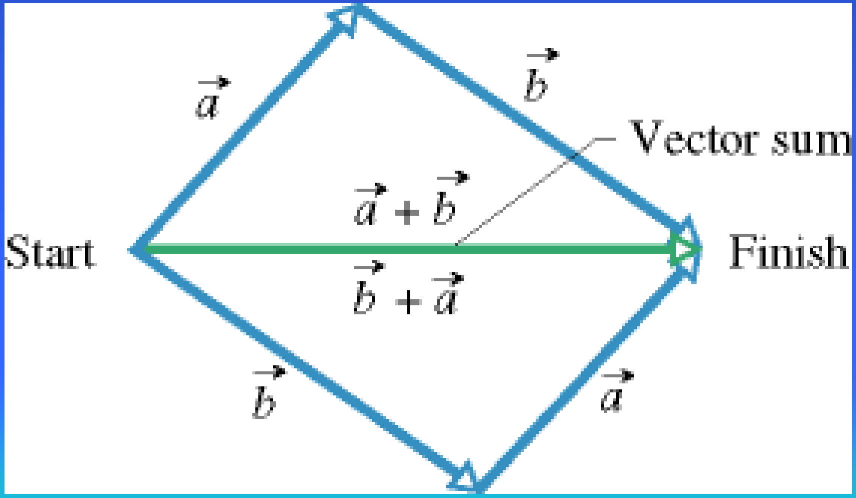




Note head-to-tail arrangement for addition

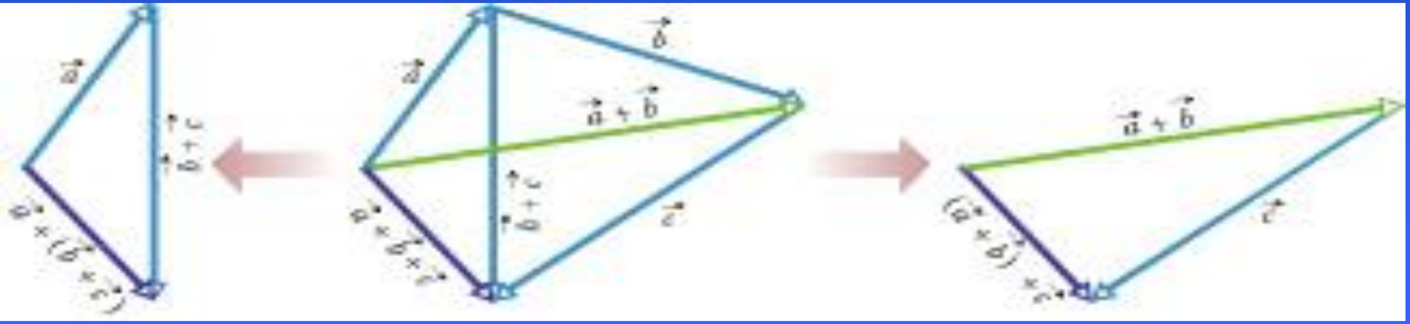


Adding Vectors Geometrically



Associative Law

- Sum obeys associative law
- $$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$$

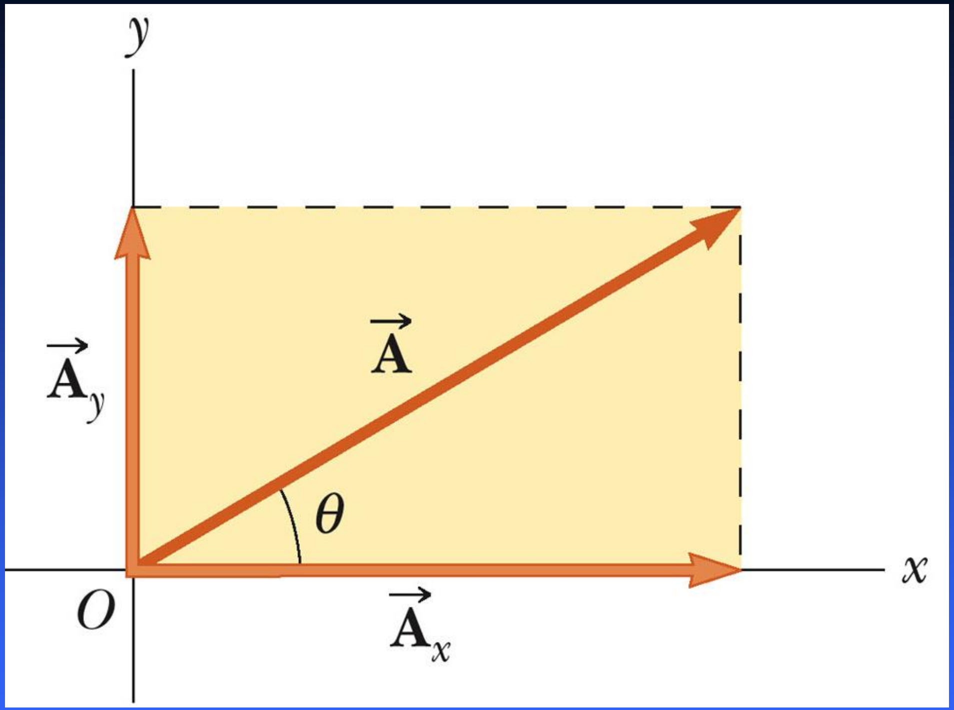


Laws of vector

1. $\vec{A}+\vec{B}=\vec{B}+\vec{A}$
2. $\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$
3. $\lambda\vec{A}=\vec{A}\lambda$
4. $\mu(\lambda\vec{A})=(\mu\lambda)\vec{A}$
5. $(\mu+\lambda)\vec{A}=\mu\vec{A}+\lambda\vec{A}$
6. $\lambda(\vec{A}+\vec{B})=\lambda\vec{A}+\lambda\vec{B}$

Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components**
  - These are the **projections** of the vector along the x- and y-axes



- An angle is positive if the rotation is counter clockwise from the positive x-direction.
- An angle is negative if the rotation is clockwise from the positive x-direction.

Vector Component Terminology

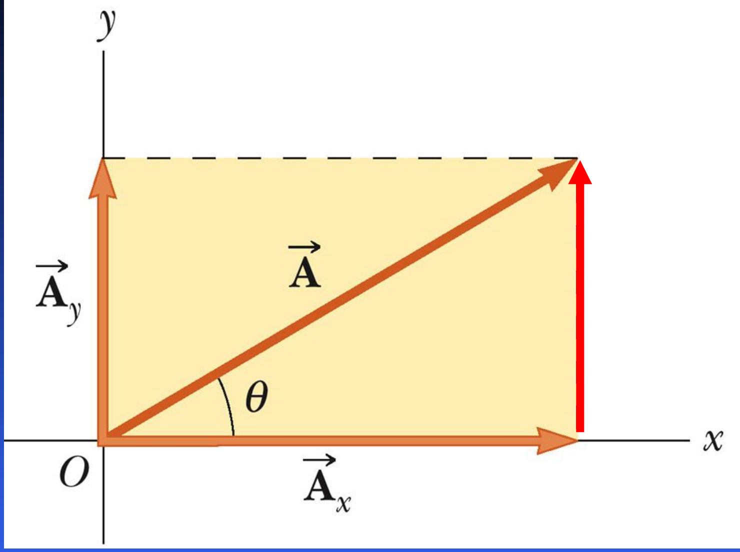
- $\vec{\mathbf{A}}_x$ *and* $\vec{\mathbf{A}}_y$ are the ***component vectors*** of $\vec{\mathbf{A}}$
  - They are vectors and follow all the rules for vectors

$$\vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y = \vec{\mathbf{A}}$$

- A_x and A_y are scalars, and will be referred to as the ***components*** of $\vec{\mathbf{A}}$
- The combination of the component vectors is a valid substitution for the actual vector

- The x-component of a vector is the projection along the x-axis:

$$A_x = \left|\vec{\mathbf{A}}\right| \cos\theta$$



The y-component of a vector is the projection along the y-axis:

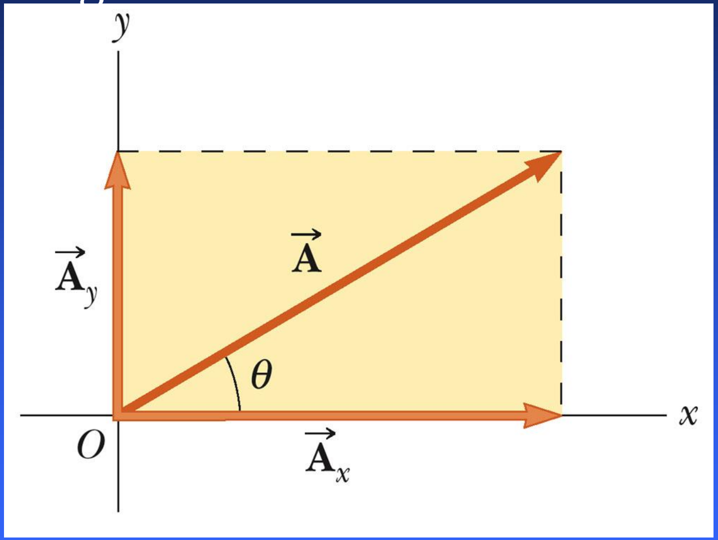
$$A_y = \left|\vec{\mathbf{A}}\right| \sin\theta$$

- When using this form of the equations, θ must be measured ccw from the positive x-axis (mathematical standard definition)

- The components are the legs of the right triangle whose hypotenuse is $\vec{\mathbf{A}}$

$$\left|\vec{\mathbf{A}}\right| = \sqrt{A_x^2 + A_y^2}$$

The Pythagorean theorem

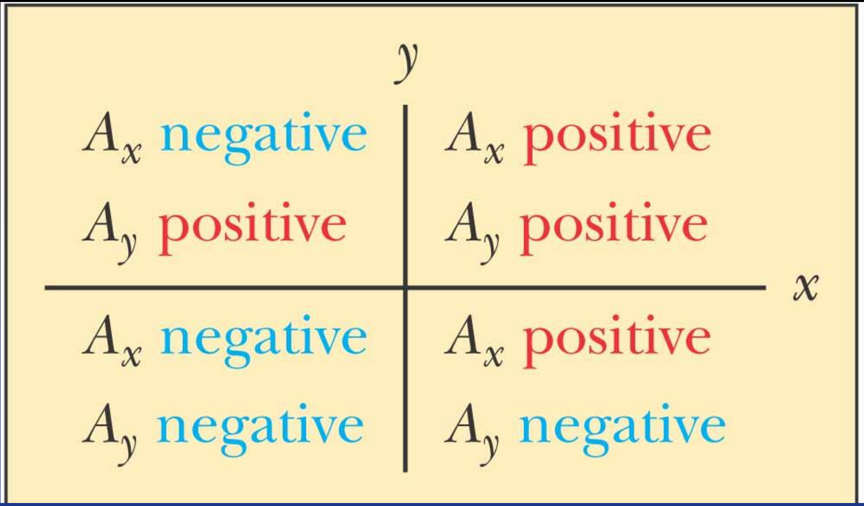


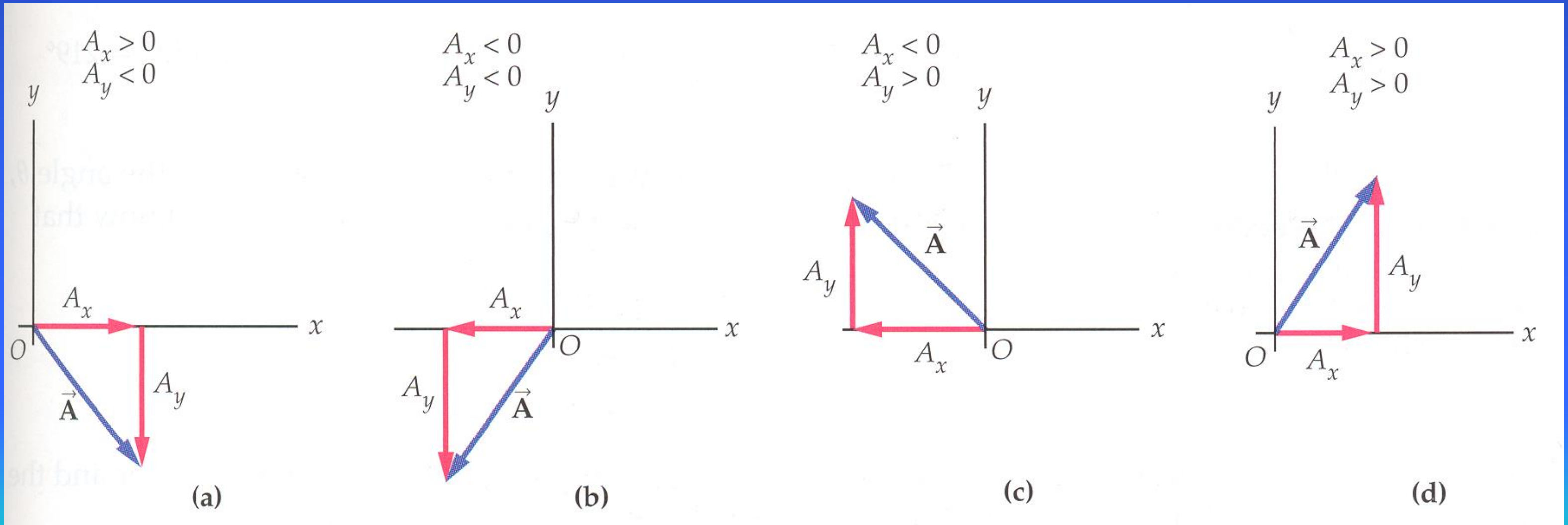
$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

$$\left(\frac{A_y}{A_x} = \frac{A\sin\theta}{A\cos\theta} = \tan\theta\right)$$

- – Must find θ with respect to the positive x-axis
- – Use the signs of A_x and A_y and a sketch to track the correct value of θ

- The components can be positive or negative and will have the same units as the original vector
- The signs of the components will depend on the angle



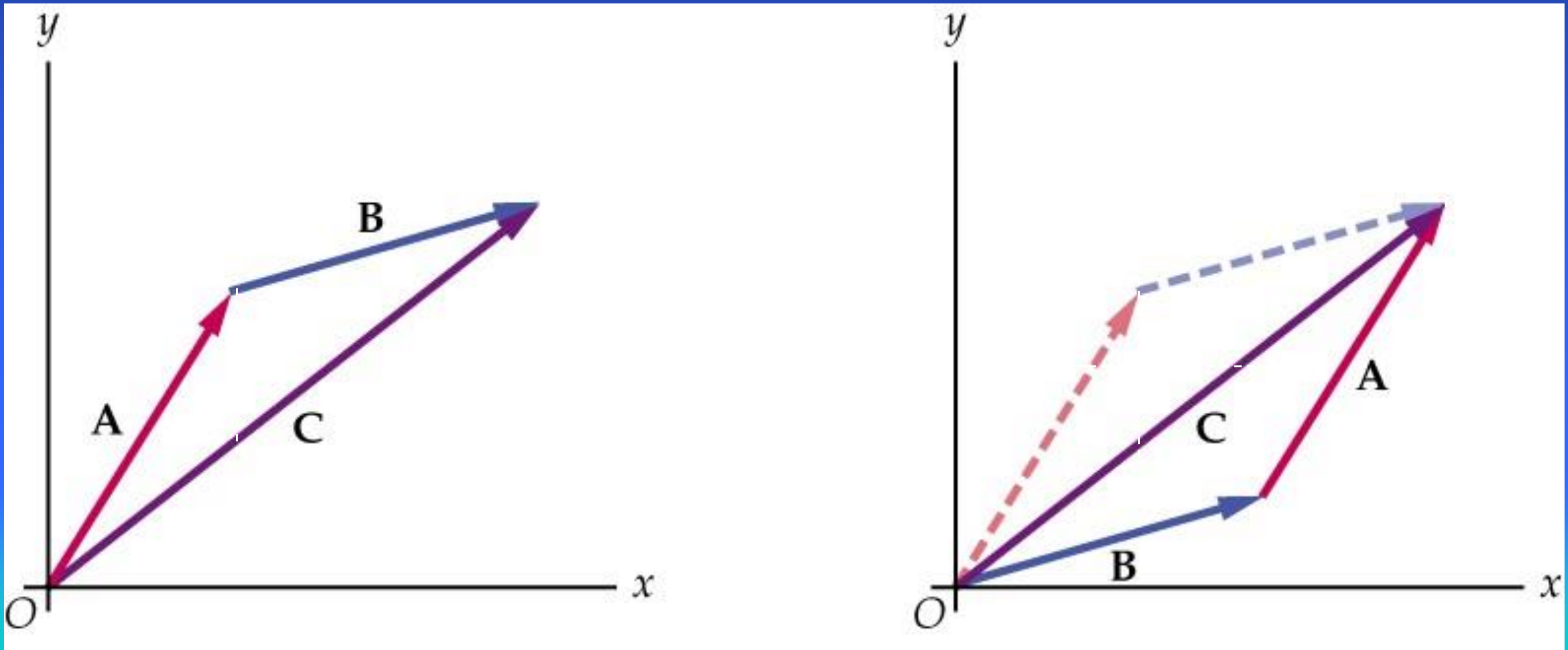


Vector Addition, Components

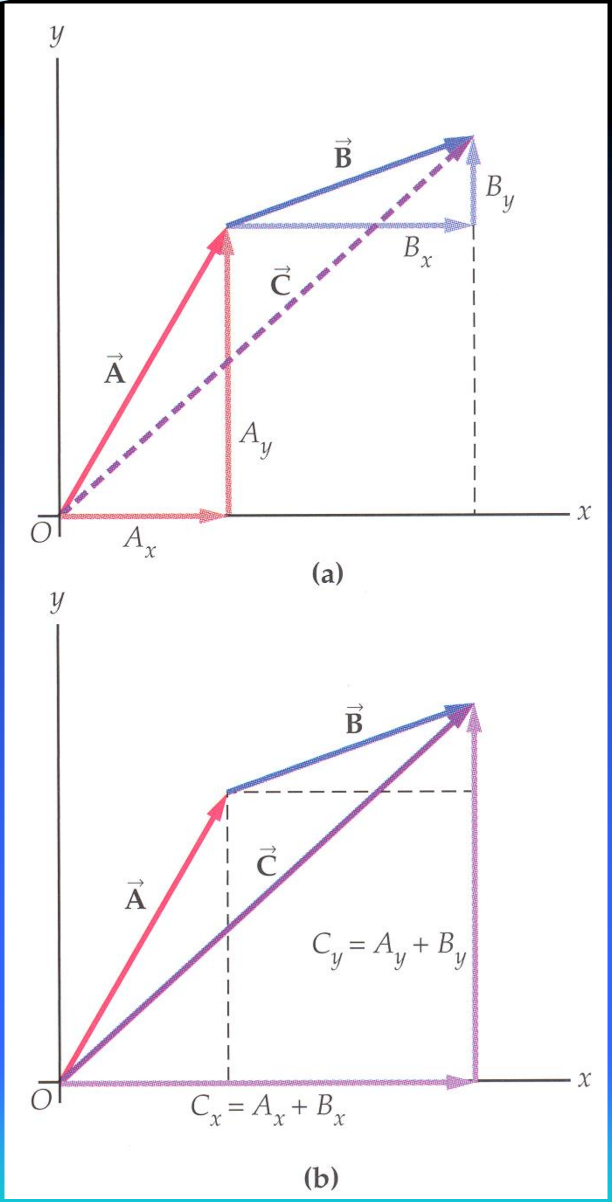
- When we add two vectors, the components add separately:

$C_x = A_x + B_x \qquad = B_x + A_x$

$C_y = A_y + B_y \qquad = B_y + A_y$



Components of a Vector, final



$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$$

$$C_x = A_x + B_x$$

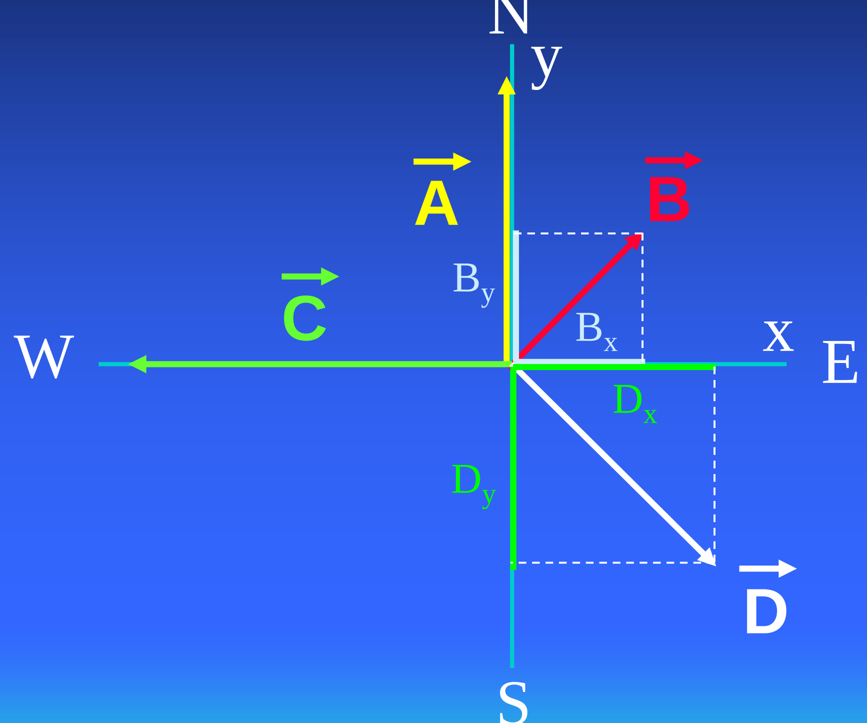
$$C_y = A_y + B_y$$

$$\left|\vec{\mathbf{C}}\right| = C = \sqrt{C_x^2 + C_y^2}$$

$$\theta = \tan^{-1}\frac{C_y}{C_x}$$

Components of a Vector, Example

- **Example: A car travels 3 km North, then 2 km Northeast, then 4 km West, and finally 3 km Southeast. What is the resultant displacement? Use the component method of vector addition.**



X-components

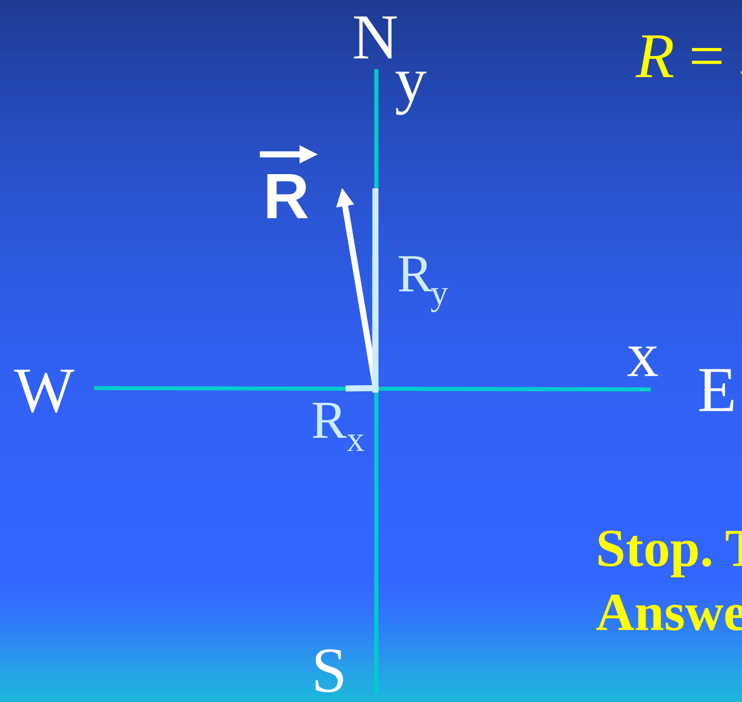
$A_x = 0$ km
$B_x = (2 \text{ km}) \cos 45^\circ = 1.4$ km
$C_x = -4$ km
$D_x = (3\text{km}) \cos 45^\circ = 2.1$ km

Y-components

$A_y = 3$ km
$B_y = (2 \text{ km}) \sin 45^\circ = 1.4$ km
$C_y = 0$ km
$D_y = (3\text{km}) \sin 315^\circ = -2.1$ km

$R_x = A_x + B_x + C_x + D_x$ = 0 km + 1.4 km - 4.0 km + 2.1 km = -0.5 km

$R_y = A_y + B_y + C_y + D_y$ = 3.0 km + 1.4 km + 0 km - 2.1 km = 2.3 km



Magnitude:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.5km)^2 + (2.3km)^2} = 2.4\,km$$

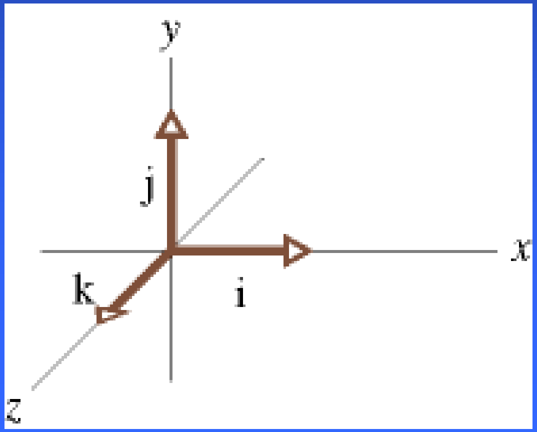
Direction:

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{2.3}{-0.5} = -78^\circ$$

Stop. Think. Is this reasonable? NO! Off by 180º.
Answer: -78º + 180° = 102° from +ve x-axis.

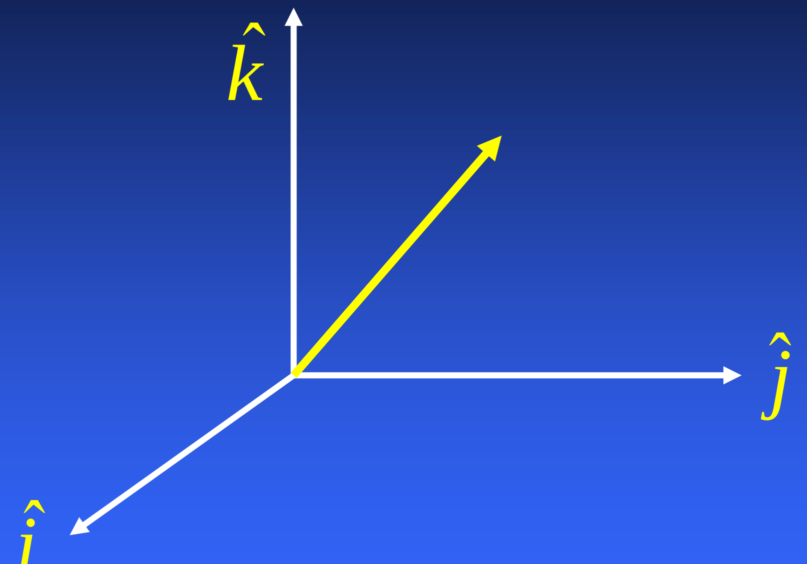
Unit Vectors

- A **unit vector** has a magnitude of 1 and points in a particular direction
- The unit vectors in the positive directions of the *x*, *y*, and *z* axes are labeled $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$ and $\hat{\mathrm{k}}$.



The arrangement of axes is a right-handed coordinate system.

$\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$, (x, y, z) are different components of the vector $\vec{A}$.



- Magnitude of $\vec{A}$:

$$A = \sqrt{x^2 + y^2 + z^2}$$

Example: Find the magnitude and the unit vector of a vector $\vec{A} = -\hat{i} + 2\hat{j} - \hat{k}$

Write: $\vec{A} = A\hat{a}$, where

Magnitude: $A = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{6}$

Unit vector: $\hat{a} = \dfrac{\vec{A}}{A} = -\dfrac{1}{\sqrt{6}}\hat{i} + \dfrac{2}{\sqrt{6}}\hat{j} - \dfrac{1}{\sqrt{6}}\hat{k}$

Adding Vectors by Components

- **Figure shows the following three vectors:**

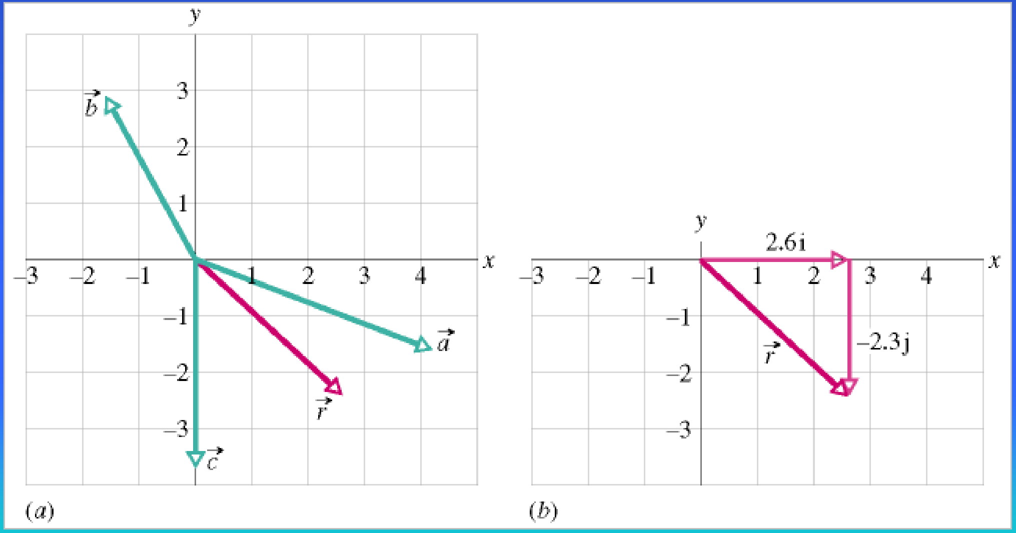
$$\vec{a} = (4.2\,\mathrm{m})\,\hat{i} - (1.5\,\mathrm{m})\,\hat{j},$$

and

$$\vec{b} = (-1.6\,\mathrm{m})\,\hat{i} + (2.9\,\mathrm{m})\,\hat{j},$$

$$\vec{c} = (-3.7\,\mathrm{m})\,\hat{j}.$$

- What is their vector sum $\vec{r}$, which is also shown?



Solution

$$r_x = a_x + b_x + c_x$$

$$= 4.2\text{ m} - 1.6\text{ m} + 0 = 2.6\text{ m}$$

$$r_y = a_y + b_y + c_y$$

$$= -1.5\text{ m} + 2.9\text{ m} - 3.7\text{ m} = -2.3\text{ m}$$

$$\vec{r} = (2.6\text{ m})\,\hat{i} - (2.3\text{ m})\,\hat{j},$$

The magnitude is $r = \sqrt{(2.6\text{ m})^2 + (-2.3\text{ m})^2} \approx 3.5\text{m}$

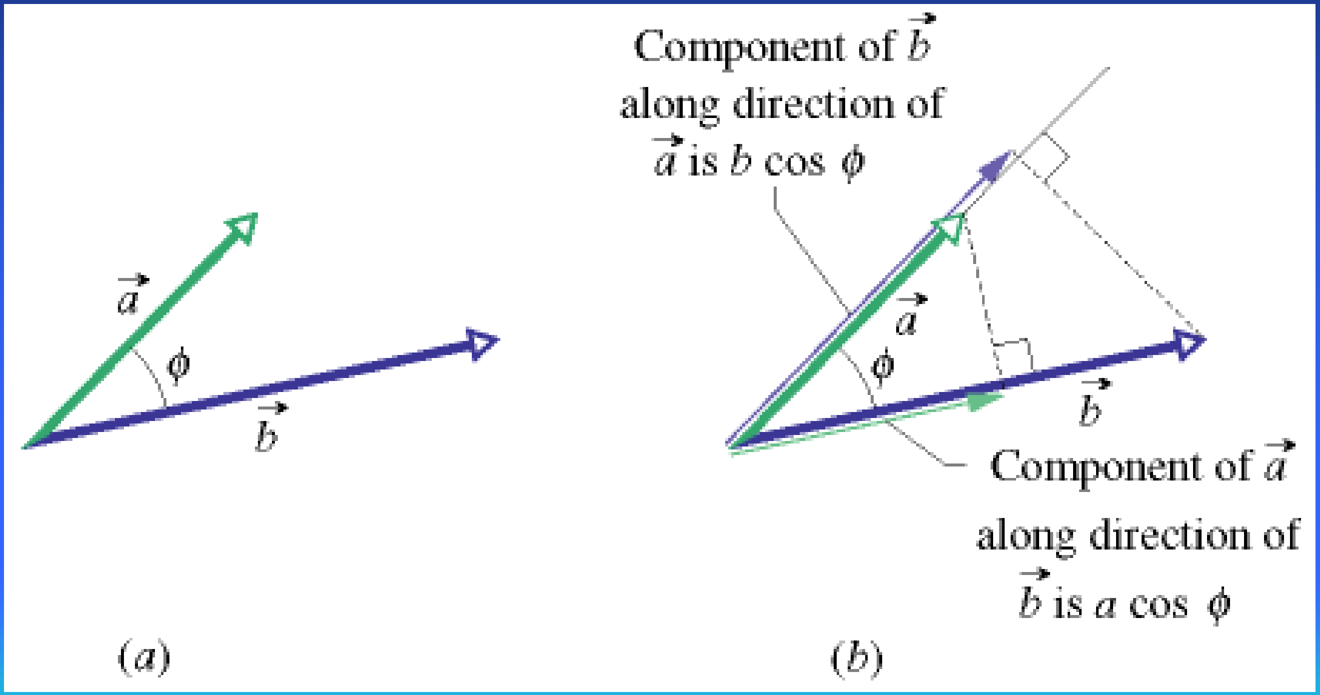
The angle from the positive x-direction is

$$\theta = \tan^{-1}\left(\frac{-2.3\text{ m}}{2.6\text{ m}}\right) = -41^\circ$$

Multiplying Vectors

- The scalar product (also known as the **dot product**)

$$\vec{a} \cdot \vec{b} = a\,b\cos\phi$$



Laws of dot product:

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$$

$$\hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = 0$$

$$\vec{A}\cdot\vec{B} = (A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k})\cdot(B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}),$$

$$\vec{A}\cdot\vec{B} = A_x\,B_x + A_y\,B_y + A_z\,B_z,$$

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

A = 2i + 3j
B = -i + 2j

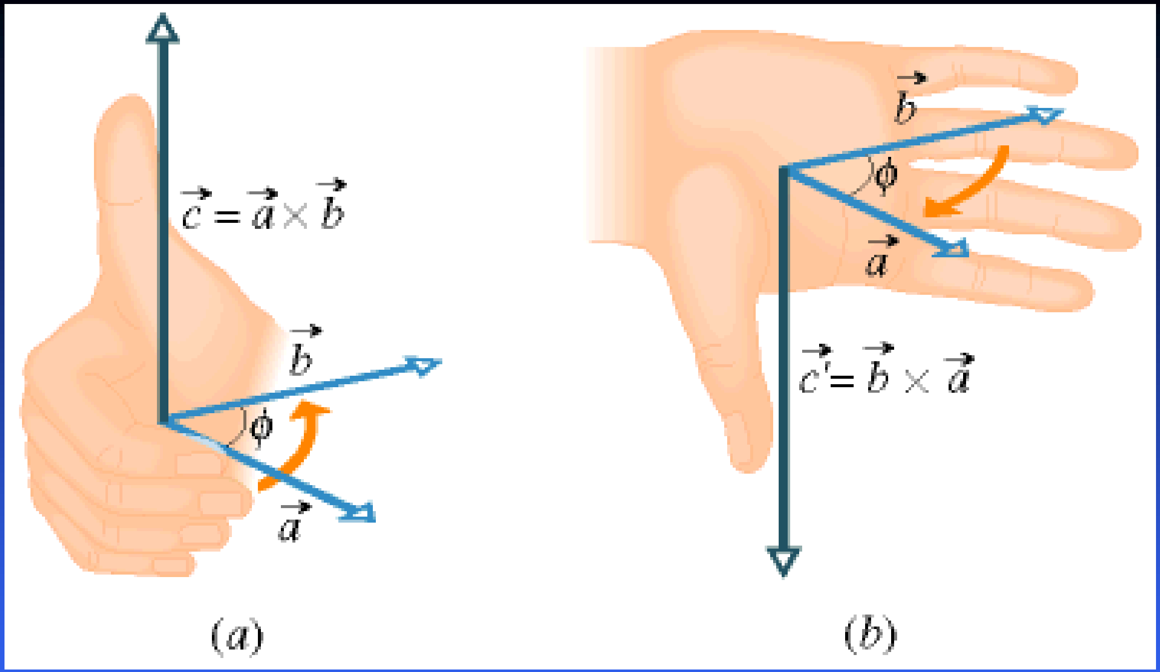
A.B= (2i + 3j) . (-i + 2j)

= -2i . i + 2i . j -3j . i +3j . 2j = -2 + 6 = 4

$$\vec{A} = -\hat{i} + 2\hat{j} - \hat{k} \qquad \vec{B} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{A}.\vec{B} = -2 + 6 + 1 = 5$$

The vector product



$$\vec{a} \times \vec{b} = a\,b\,\sin\phi$$

The right-handed rule

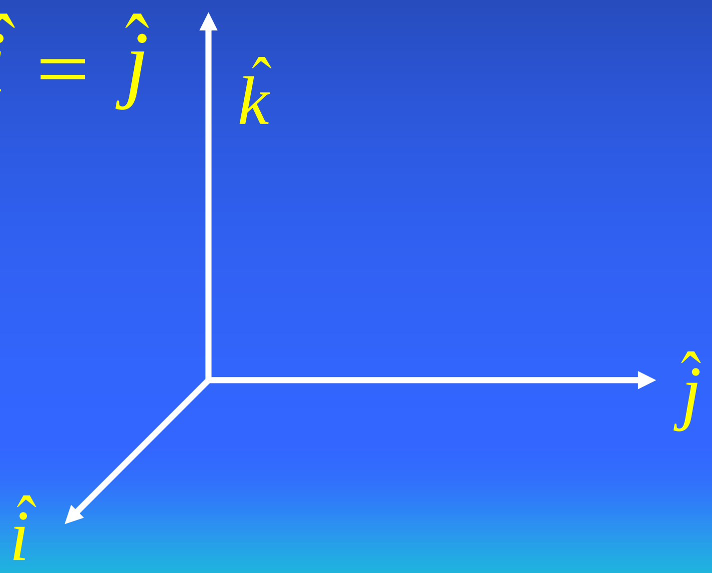
Laws of cross product:

1. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

2. $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

3. $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$;

$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $= \hat{k} \times \hat{i} = \hat{j}$



$$\vec{A}\times\vec{B}=(A_x\,\hat{i}+A_y\,\hat{j}+A_z\,\hat{k})\times(B_x\,\hat{i}+B_y\,\hat{j}+B_z\,\hat{k}),$$

$$\vec{A}\times\vec{B}=(A_yB_z-B_yA_z)\hat{i}-(A_xB_z-A_zB_x)\hat{j}+(A_xB_y-B_xA_y)\hat{k}.$$

Example: Evaluate the cross product of vectors

$$\vec{A}=-\hat{i}+2\hat{j}-\hat{k} \qquad \vec{B}=2\hat{i}+3\hat{j}-\hat{k}$$

$$\vec{A}\times\vec{B}=\hat{i}-3\hat{j}-7\hat{k}$$

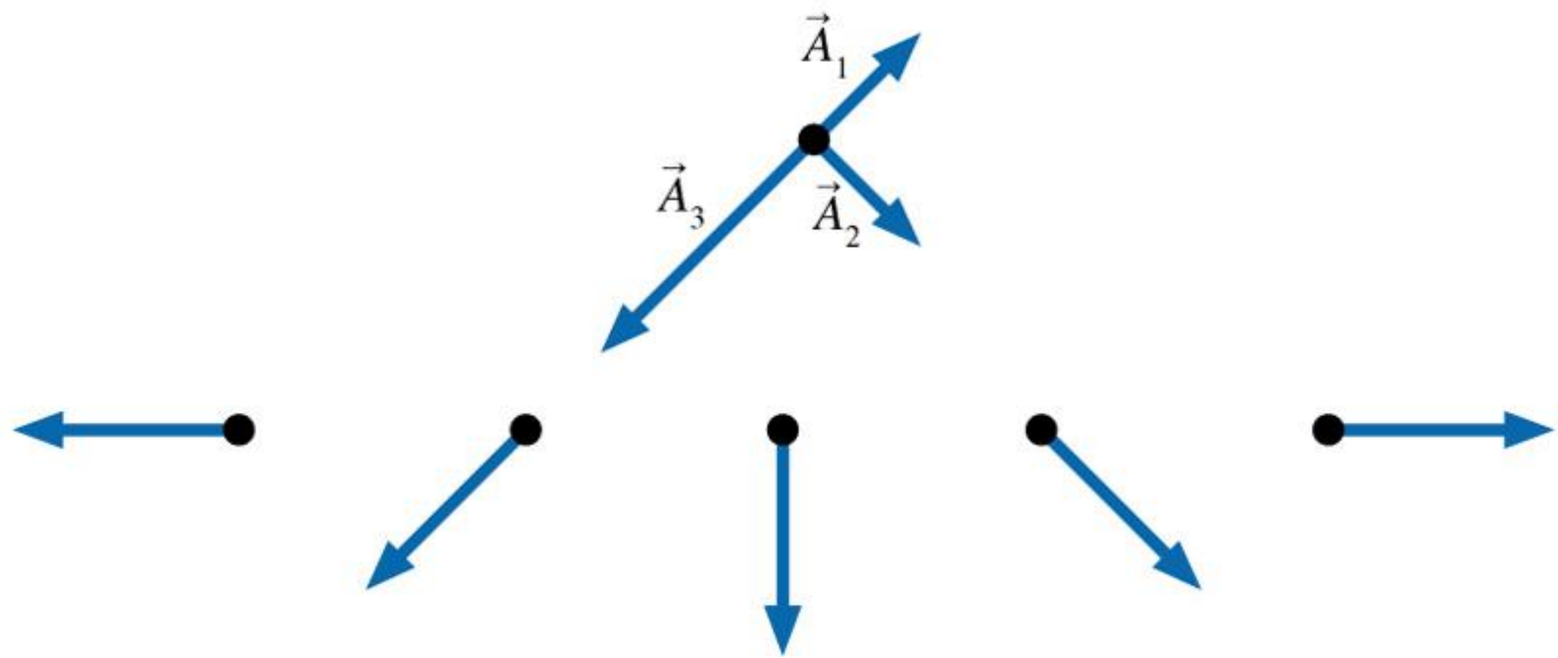
- If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

$$\vec{c} = (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k})$$

$$= -12\hat{i} - 9\hat{j} - 8\hat{k}$$

The vector $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$. We can show that $\vec{c} \cdot \vec{a} = 0$ and $\vec{c} \cdot \vec{b} = 0$

Which figure shows $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?



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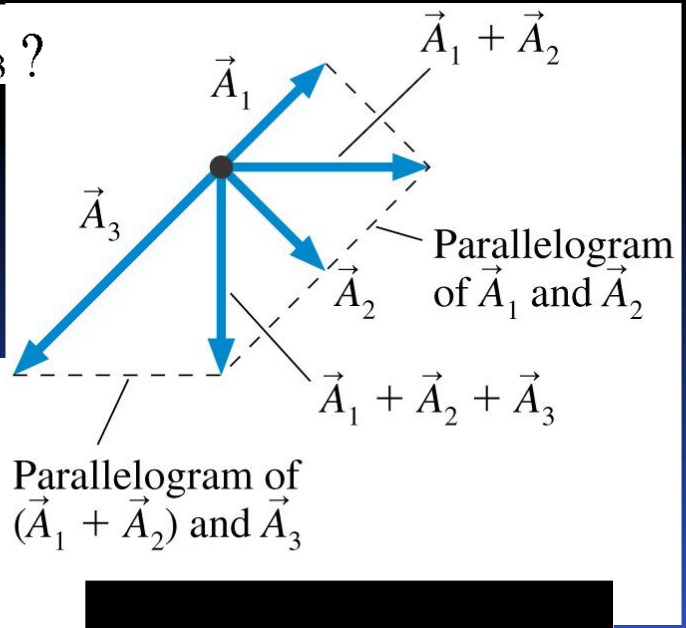
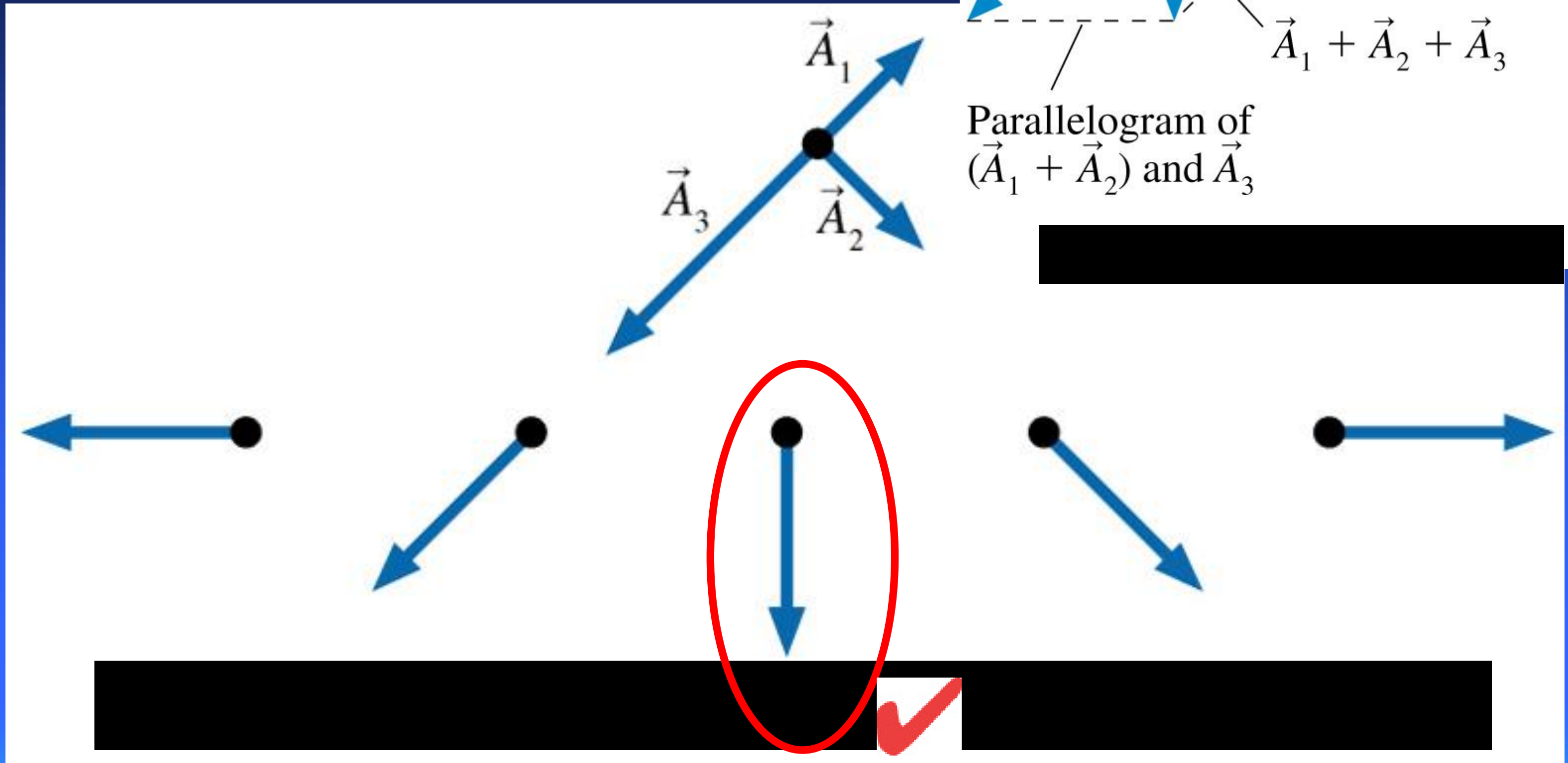
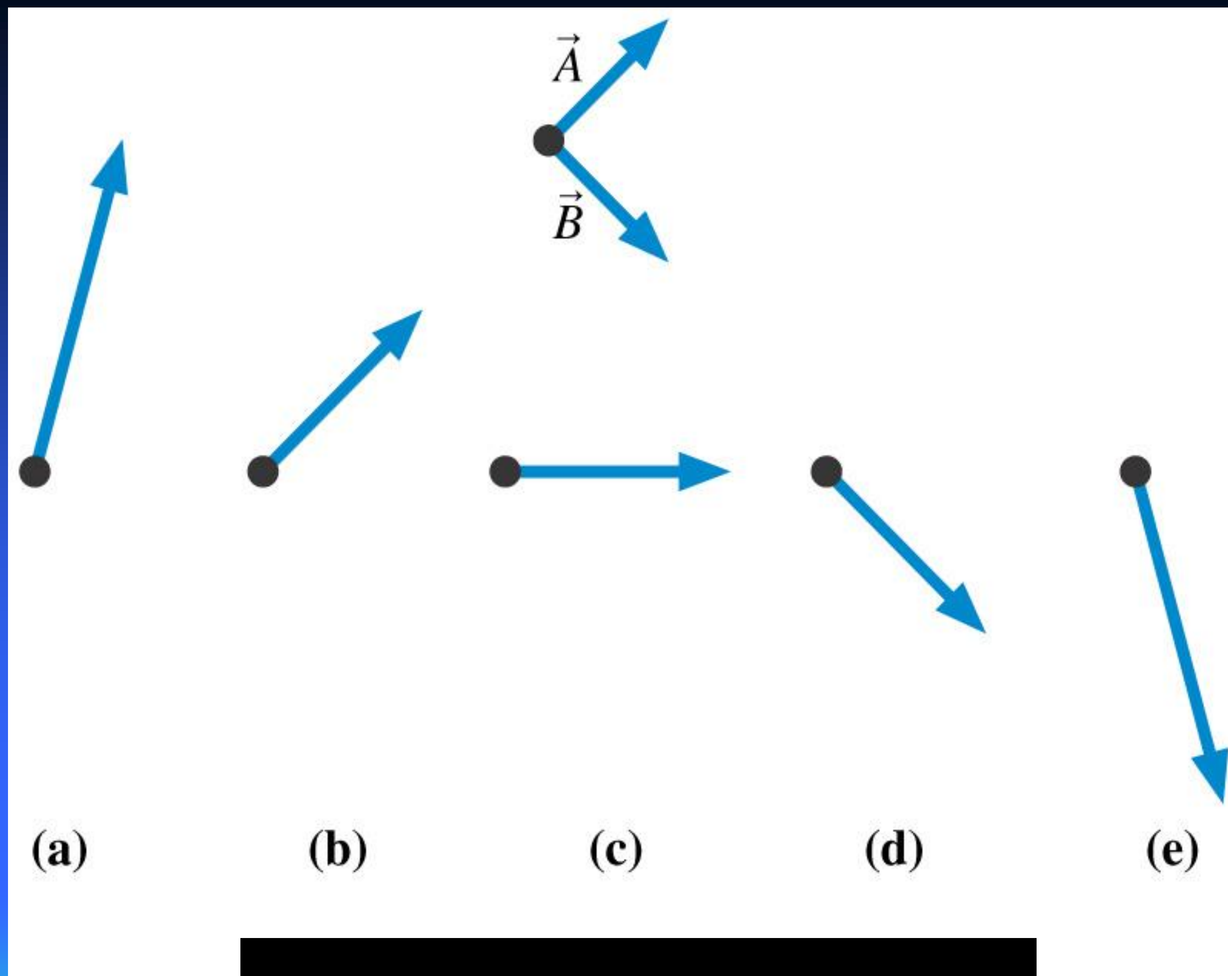
Which figure shows $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?





Which figure shows $2\vec{A} - \vec{B}$?



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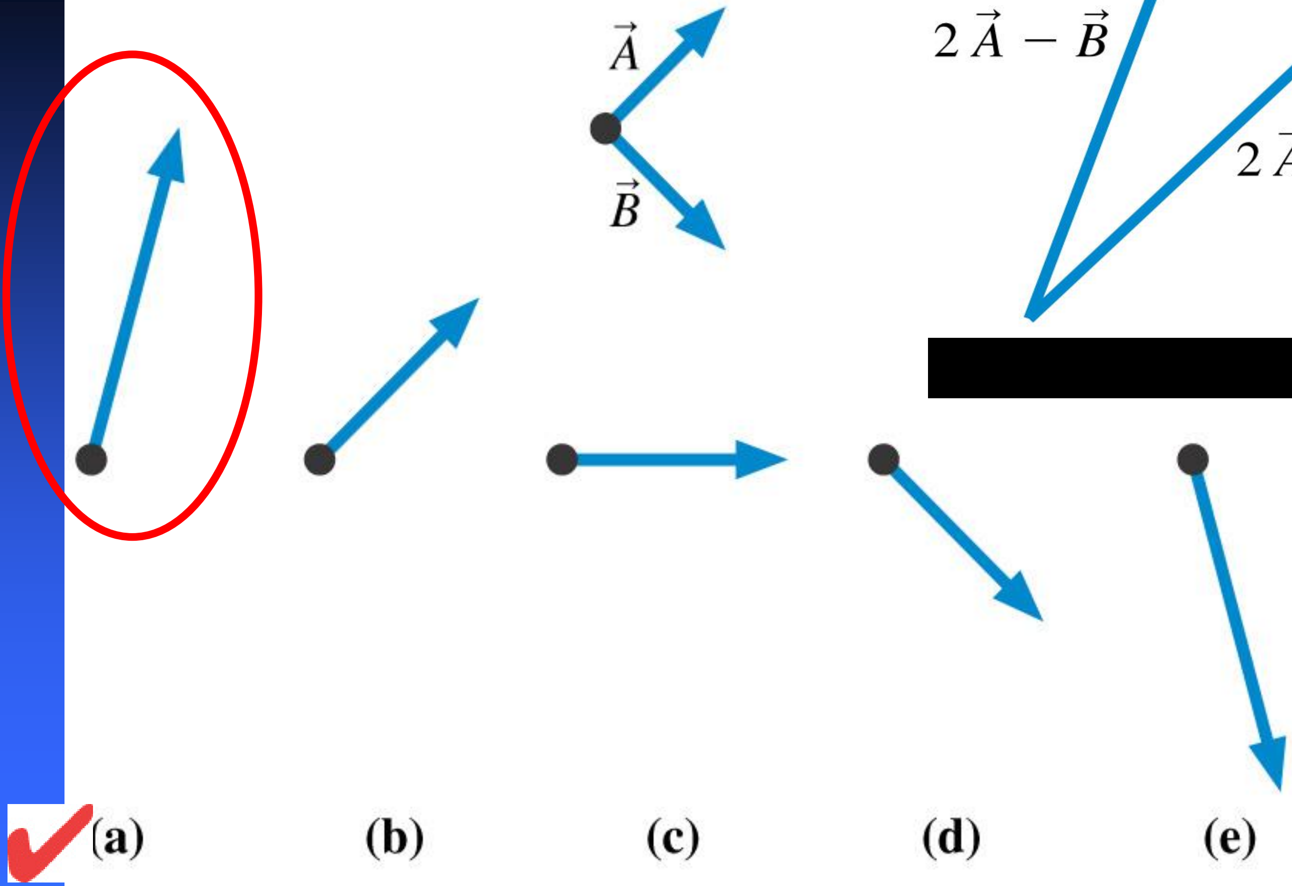

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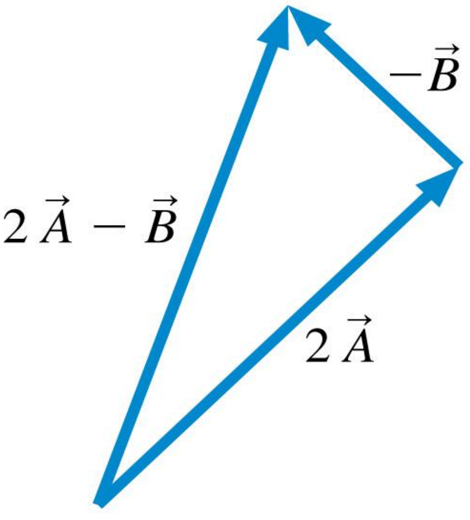

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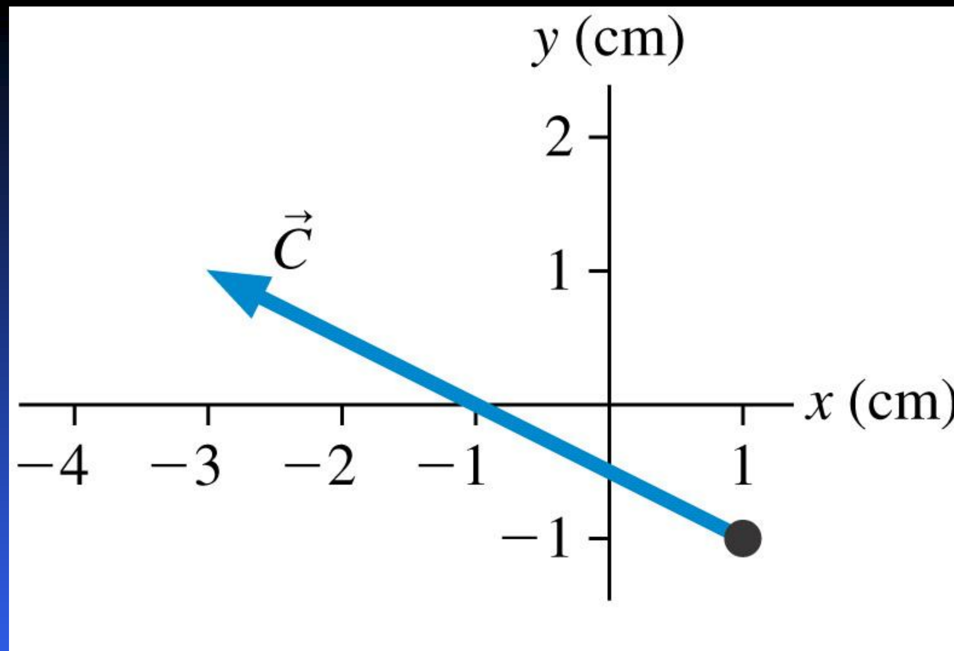
What are the x- and y-components C_x and C_y of vector $\vec{C}$?



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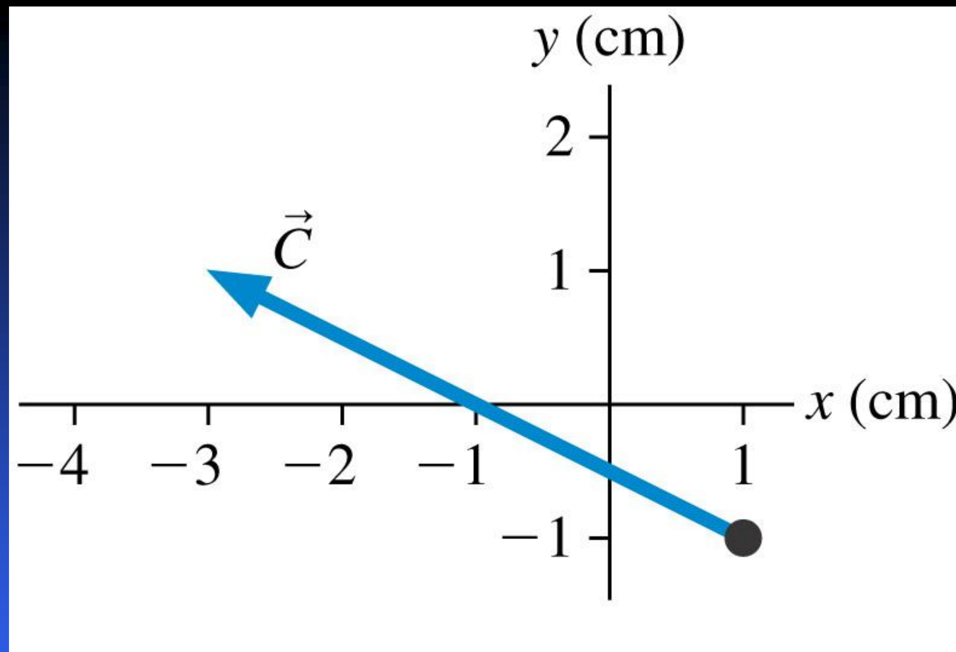
1) $C_x = -3$ cm, $C_y = 1$ cm
2) $C_x = -4$ cm, $C_y = 2$ cm
3) $C_x = -2$ cm, $C_y = 1$ cm
4) $C_x = -3$ cm, $C_y = -1$ cm
5) $C_x = 1$ cm, $C_y = -1$ cm

What are the x- and y-components C_x and C_y of vector $\vec{C}$?



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