PhysicsI

Lecture 4

Sahraei

Physics Department

Razi University

http://www.razi.ac.ir/sahraei/

Motion in One Dimension

In this chapter we discuss motion in one dimension.

We introduce definitions for displacement, velocity and acceleration, and derive equations of motion for bodies moving in one dimension with constant acceleration.

We apply these equations to the situation of a body moving under the influence of gravity alone.

Motion in one dimension

Kinematics – describes motion mathematically without regard to cause.

For 1-D motion the coordinate system is linear

- **Frame of reference: a system for specifying the <u>precise</u> location of objects in space and time**

- **The simplest kind of motion**
- **Things can move forward and backward, but not left and right**

Time and Space

Instant ≡ particular point in time

Interval ≡ difference between two instants

$$\Delta t = t_2 - t_1$$

Position ≡ particular point in space

Displacement ≡ difference between two positions

$$\Delta x = x_2 - x_1$$

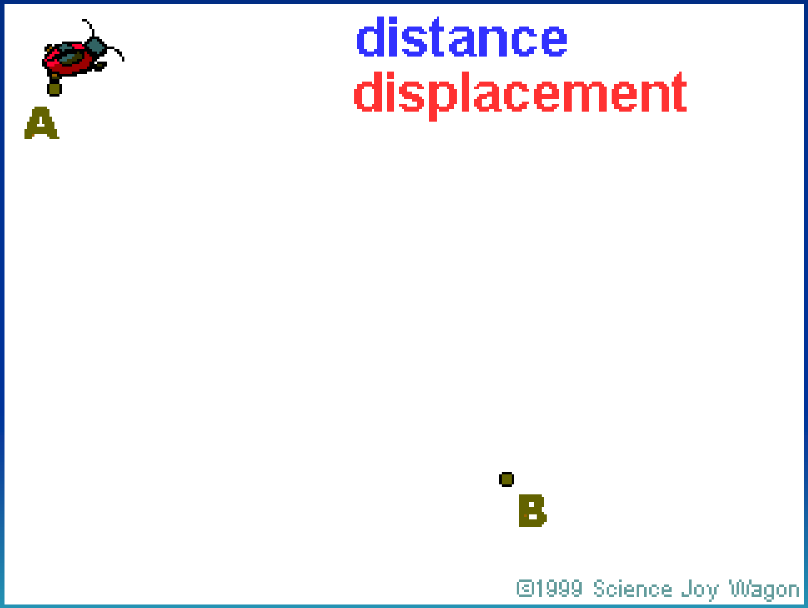
Same concepts apply to any axis

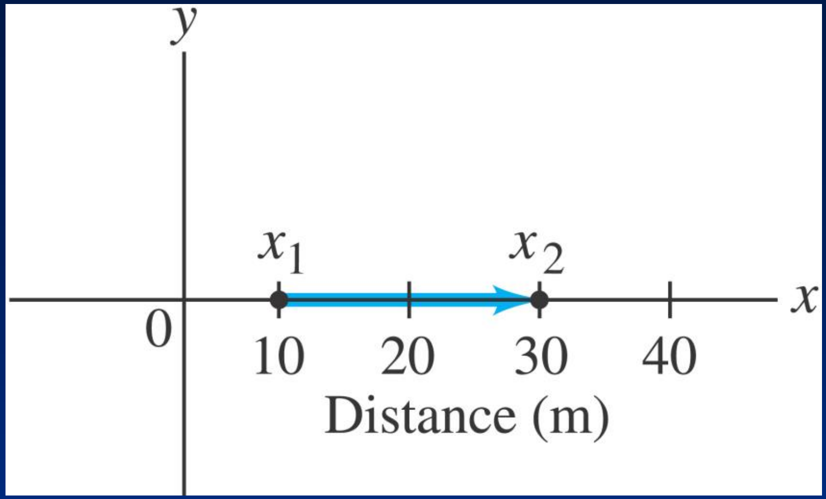
$$\Delta y = y_2 - y_1$$

Displacement

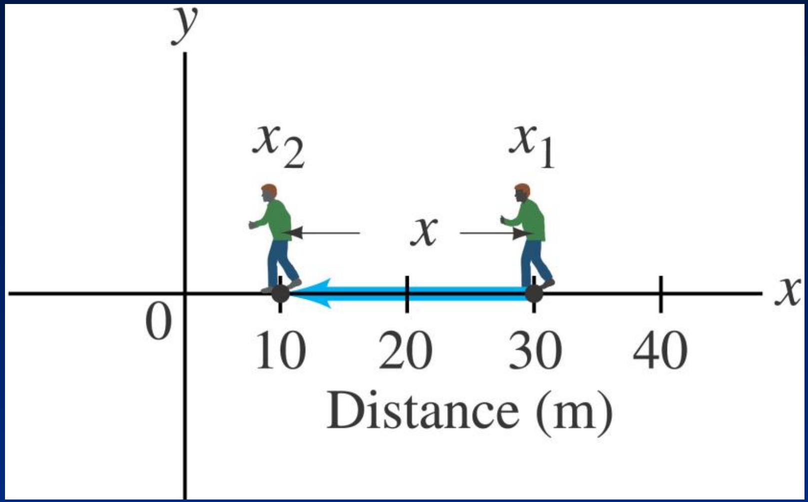
Displacement is a change of position in a certain direction, *not* the total distance traveled

Change in position from initial time to final time



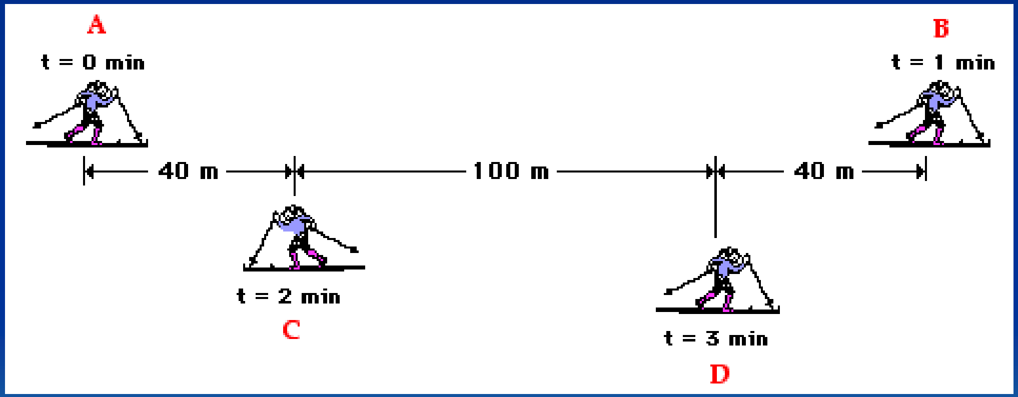


Displacement is positive.



Displacement is negative.

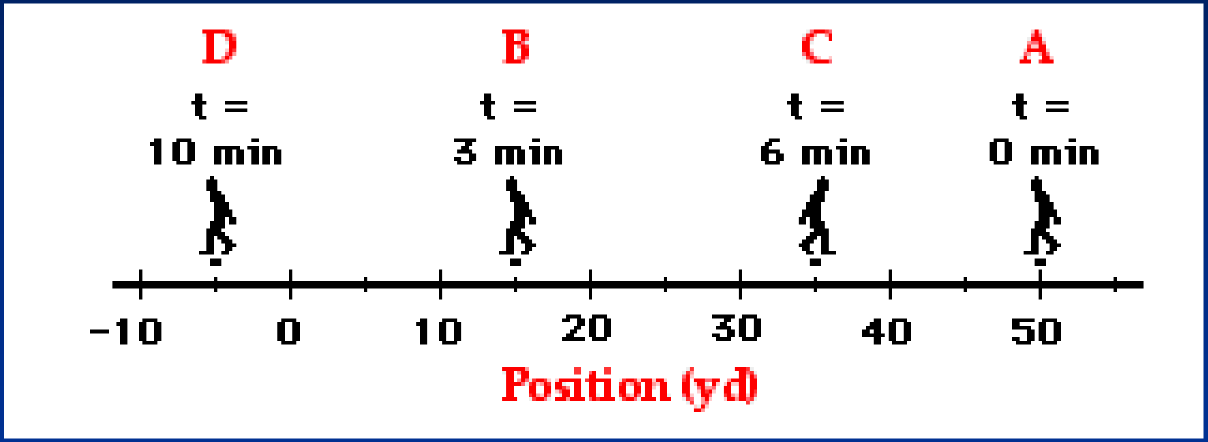
Quick Quiz



The skier covers a distance of (180 m + 140 m + 100 m) = 420 m and has a displacement of 140 m, rightward

Quick Quiz

What is the coach's resulting displacement and distance of travel?



The coach covers a distance of

(35 yds + 20 yds + 40 yds) = 95 yards

and has a displacement of 55 yards, left.

Average Velocity

Average velocity is the total displacement divided by the time interval during which the displacement occurred.

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

$$\text{average velocity} = \frac{\text{change in position}}{\text{change in time}} = \frac{\text{displacement}}{\text{time interval}}$$

Velocity

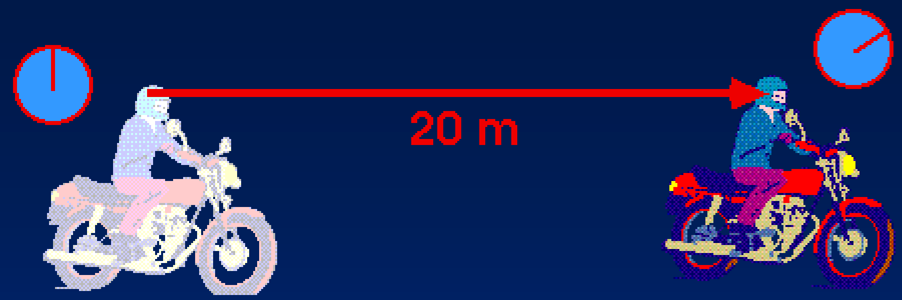
Since displacement is a vector quantity, then velocity is also a vector quantity. It has both magnitude and direction.

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Velocity and Speed

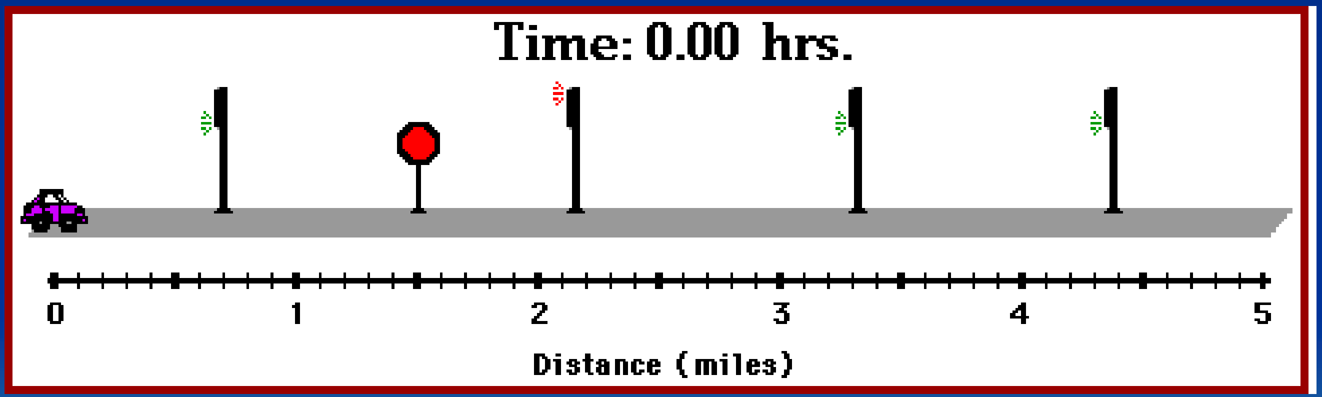
Velocity describes motion with both a direction and a numerical value (a magnitude).

Speed tells how fast an object is moving without saying anything about its direction. Speed is always positive. Speed is the "absolute value" of the velocity. Speed in the velocity information without the sign or direction information.



If a motorcycle travels 20 m in 2 s, then its average velocity is

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{20\ m}{2\ s} = 10\ \frac{m}{s}$$



$$\text{Ave. Speed} = \frac{5\ \text{miles}}{0.2\ \text{hours}} = 25\ \text{miles/hour}$$

Instantaneous Velocity

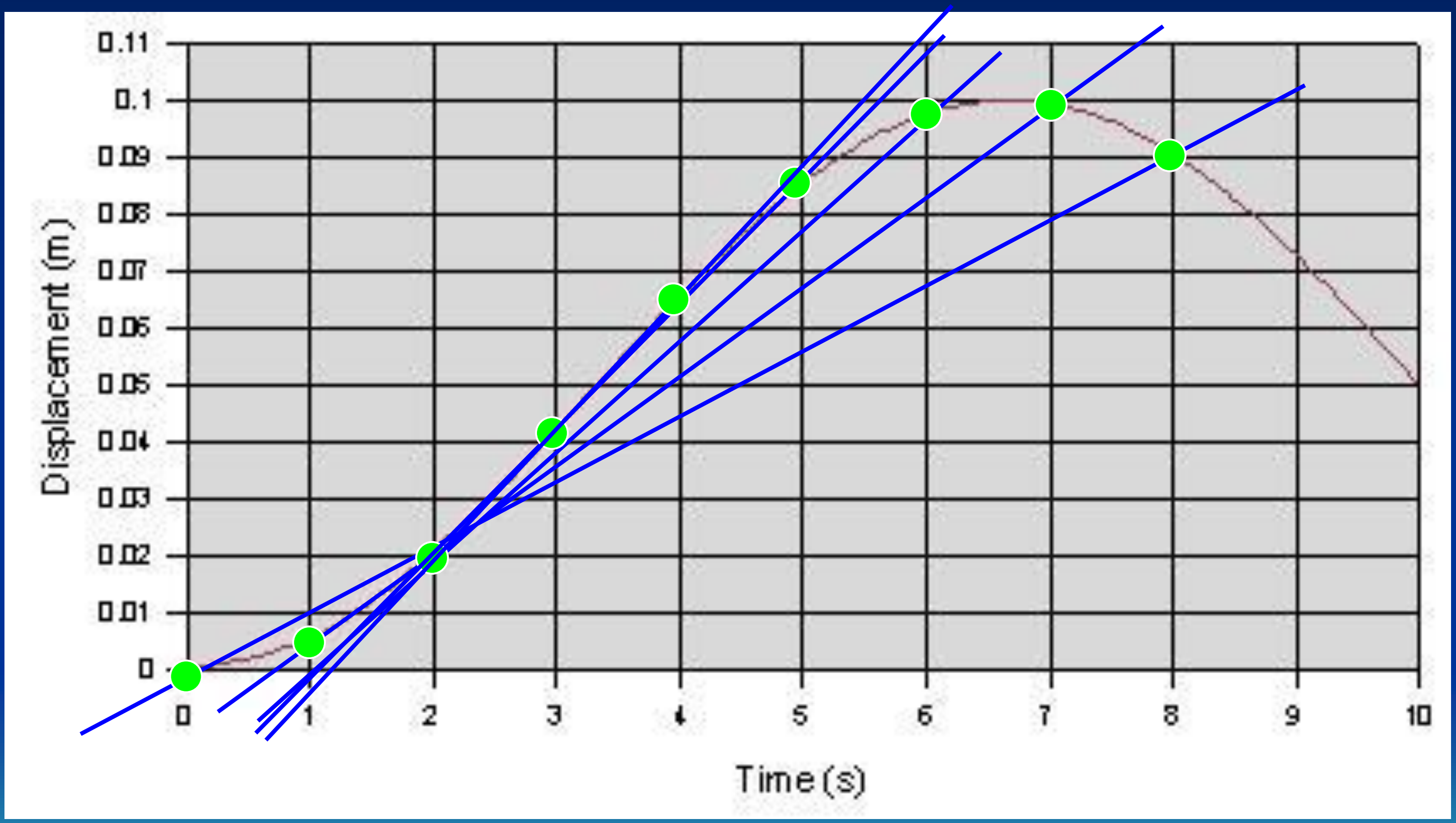
Things do not always move wih a constant velocity. The velocity may change.

The instantaneous velocity is the velocity "right now", the velocity at some particular moment.

$$v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous Velocity – Graphical Interpretation

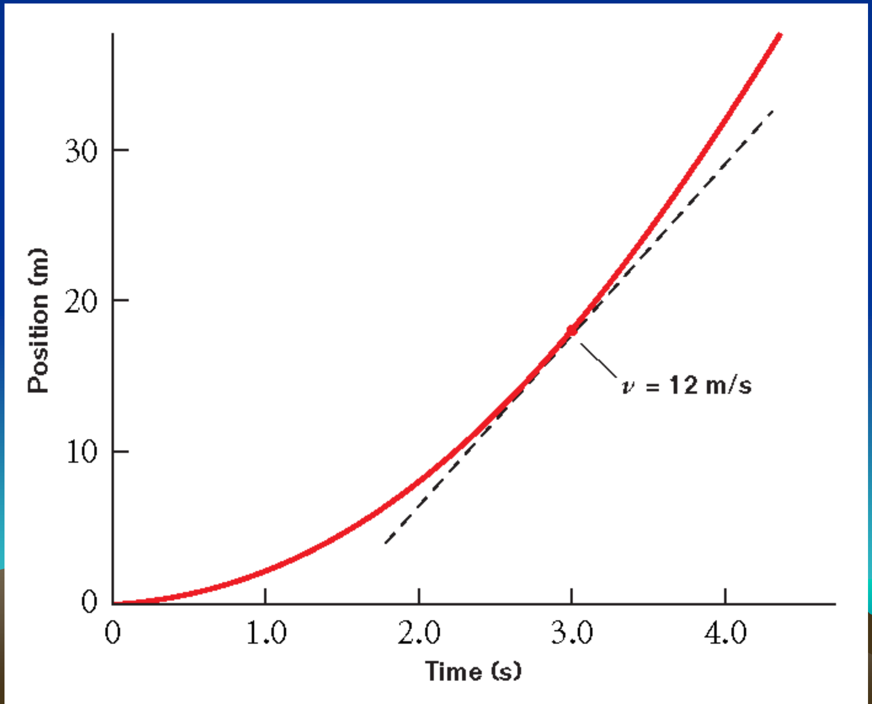
What is v at $t = 4\ s$?



Instantaneous Velocity

The instantaneous velocity is the velocity of an object at some instant or at a specific point in the object's path.

The instantaneous velocity at a given time can be determined by measuring the slope of the line that is tangent to that point on the position-versus-time graph.



Average Acceleration

The rate of change in velocity over a time interval

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

a **is measured in m/s²**

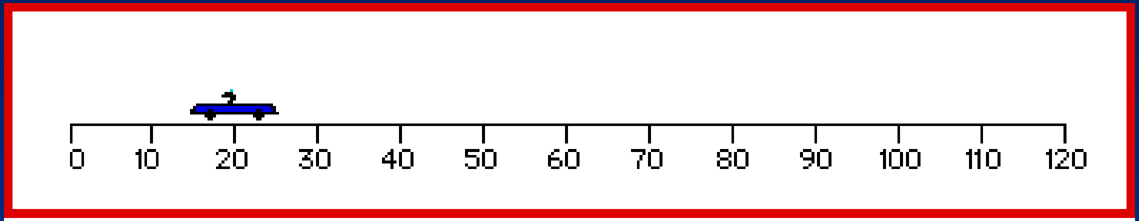
Instantaneous Acceleration

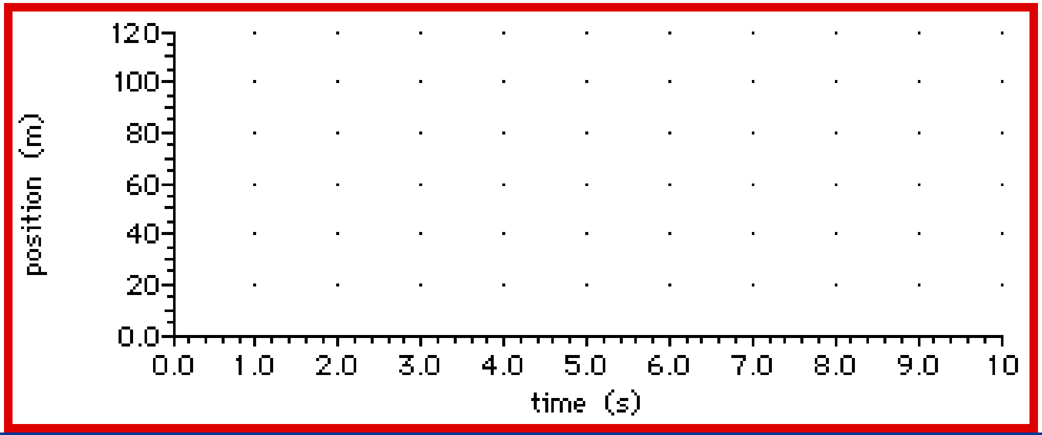
The average acceleration when the time interval approaches zero

It is the slope of a velocity versus time graph

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The Passing Lane





Types of Motion

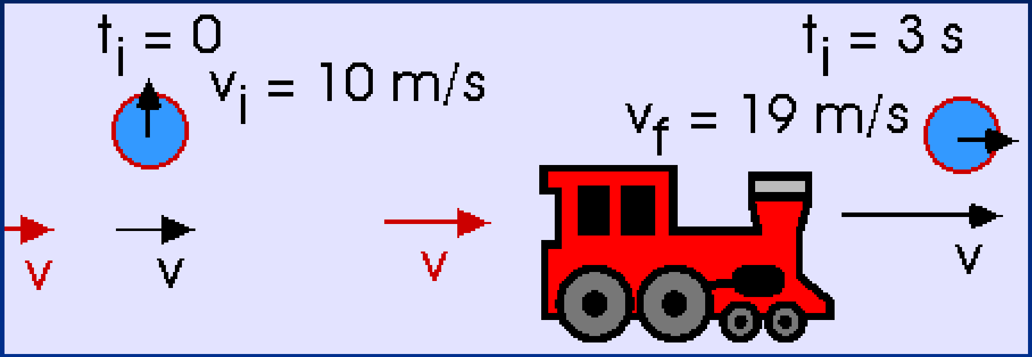
Uniform Motion: v = constant, a = 0

Nonuniform Motion

Uniform acceleration: a = constan

Nonuniform acceleration: a is not constant

a = constant

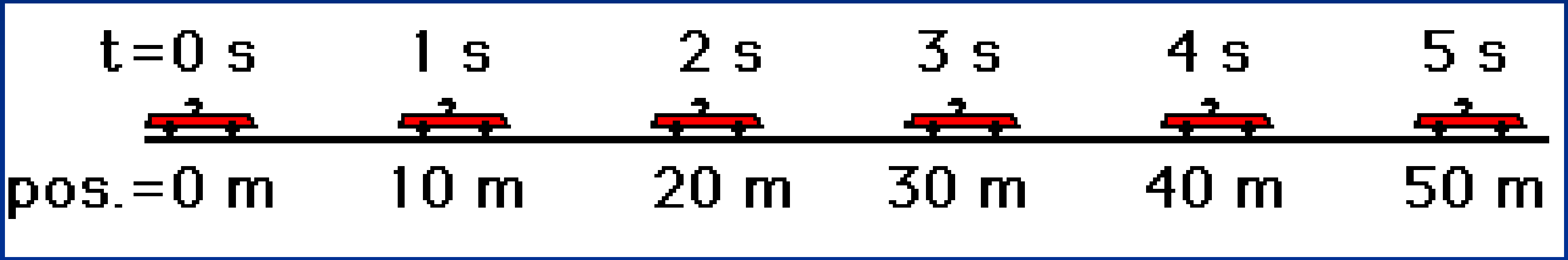


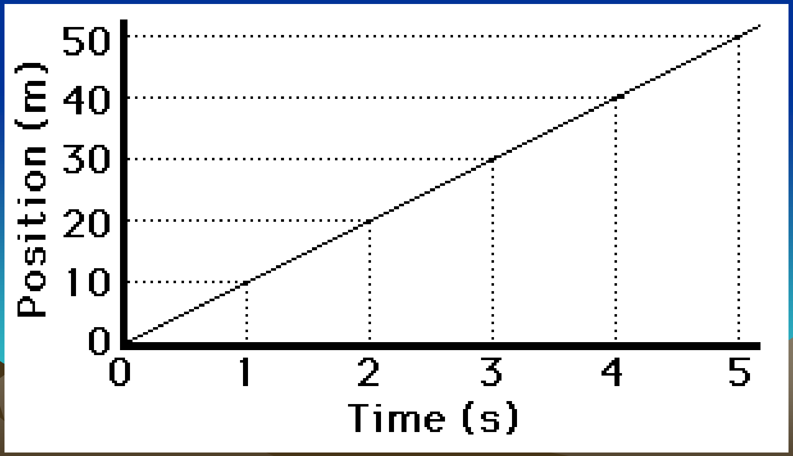
$$a = \frac{\Delta v}{\Delta t} = \frac{(19 - 10)\,\text{m/s}}{3\ \text{s}} = \frac{9\ \text{m/s}}{3\ \text{s}} = 3\ \frac{\text{m/s}}{\text{s}}$$

$$a = 3\ \text{m/s/s} = 3\ \text{m/s}^2$$

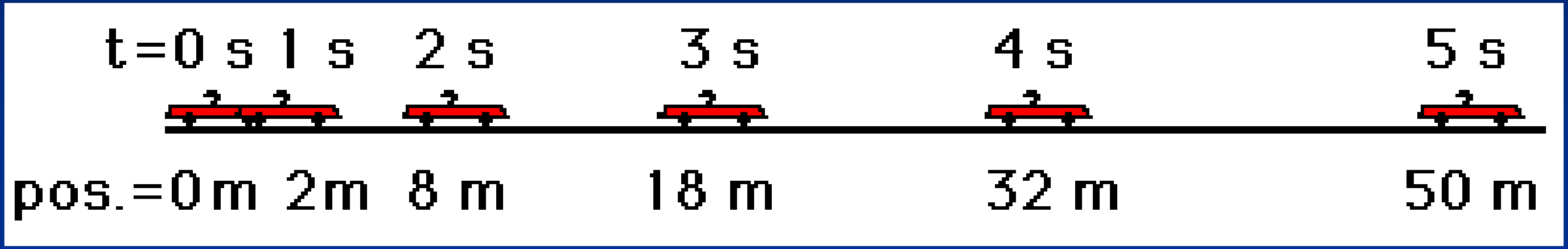
The first part of this lesson involves the study of the relationship between the motion of an object and the shape of its p-t graph.

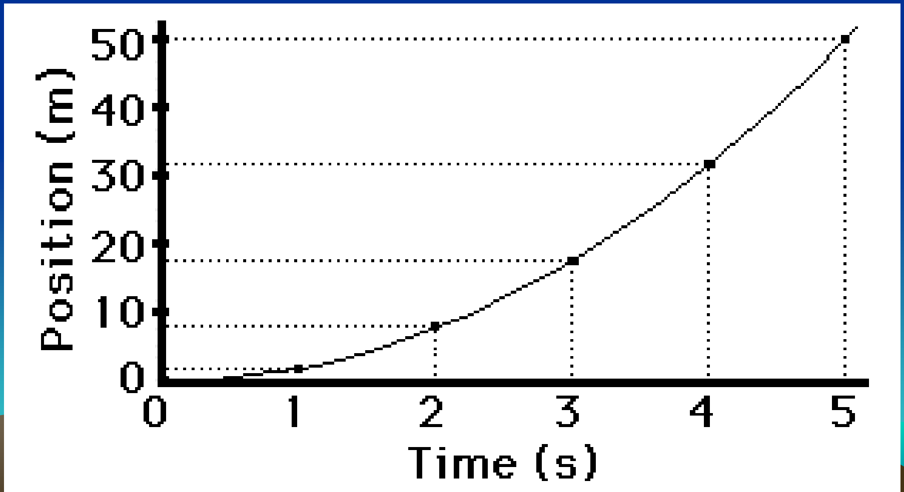
To begin, consider a car moving with a constant, rightward (+) velocity of 10 m/s.



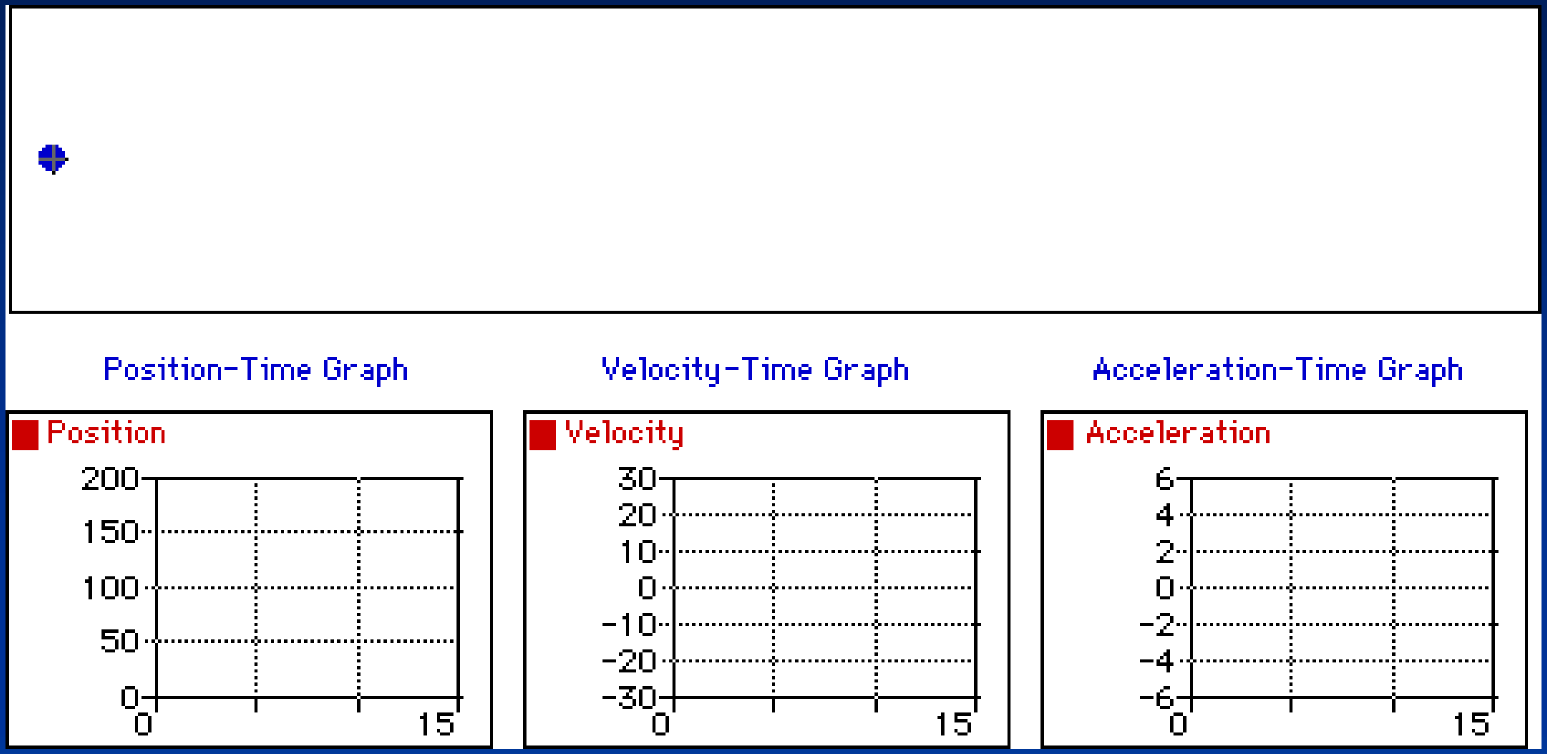


Now consider a car moving with a changing, rightward (+) velocity – that is, a car that is moving rightward and speeding up or accelerating.

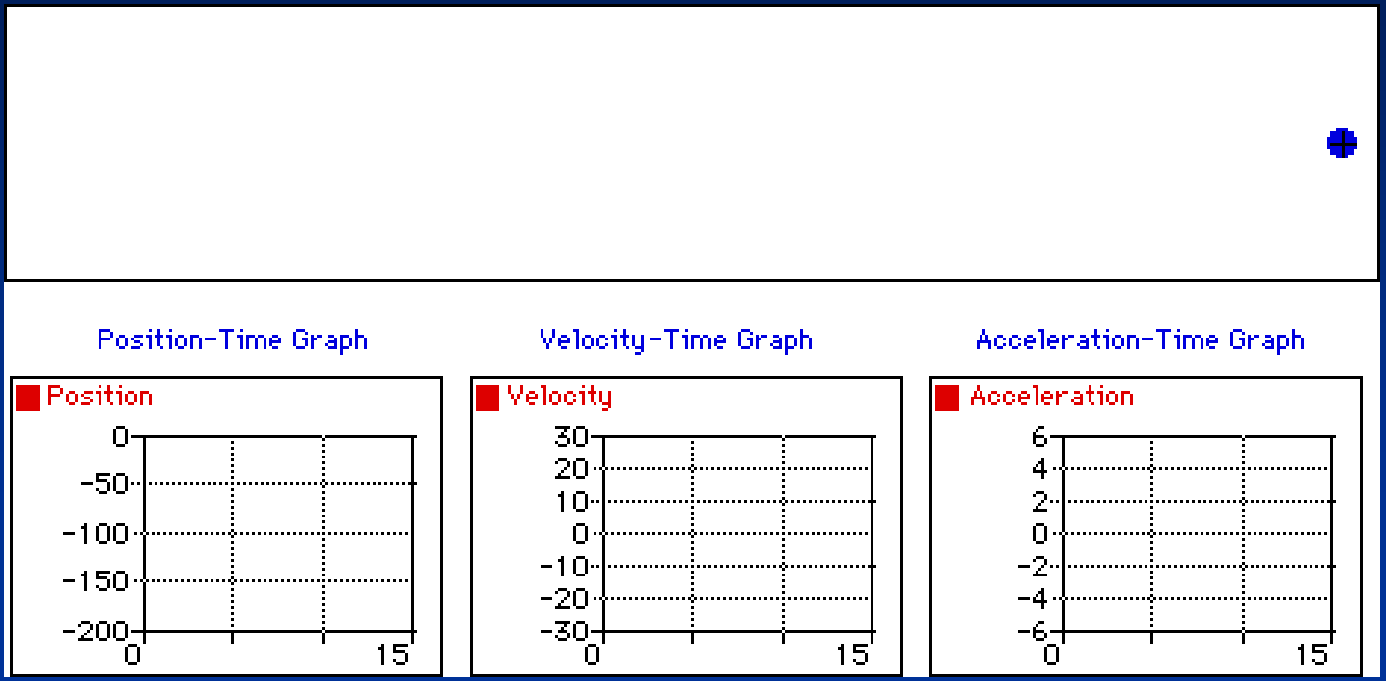




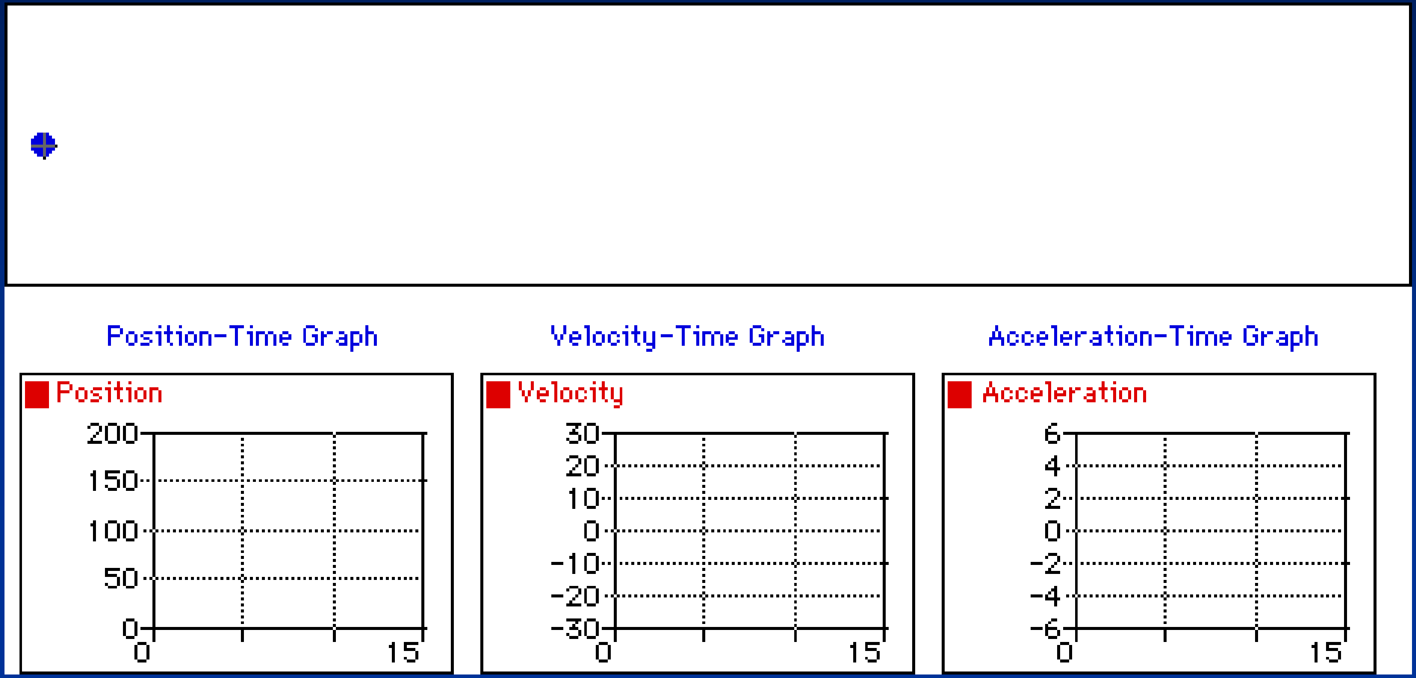
Constant Positve Velocity



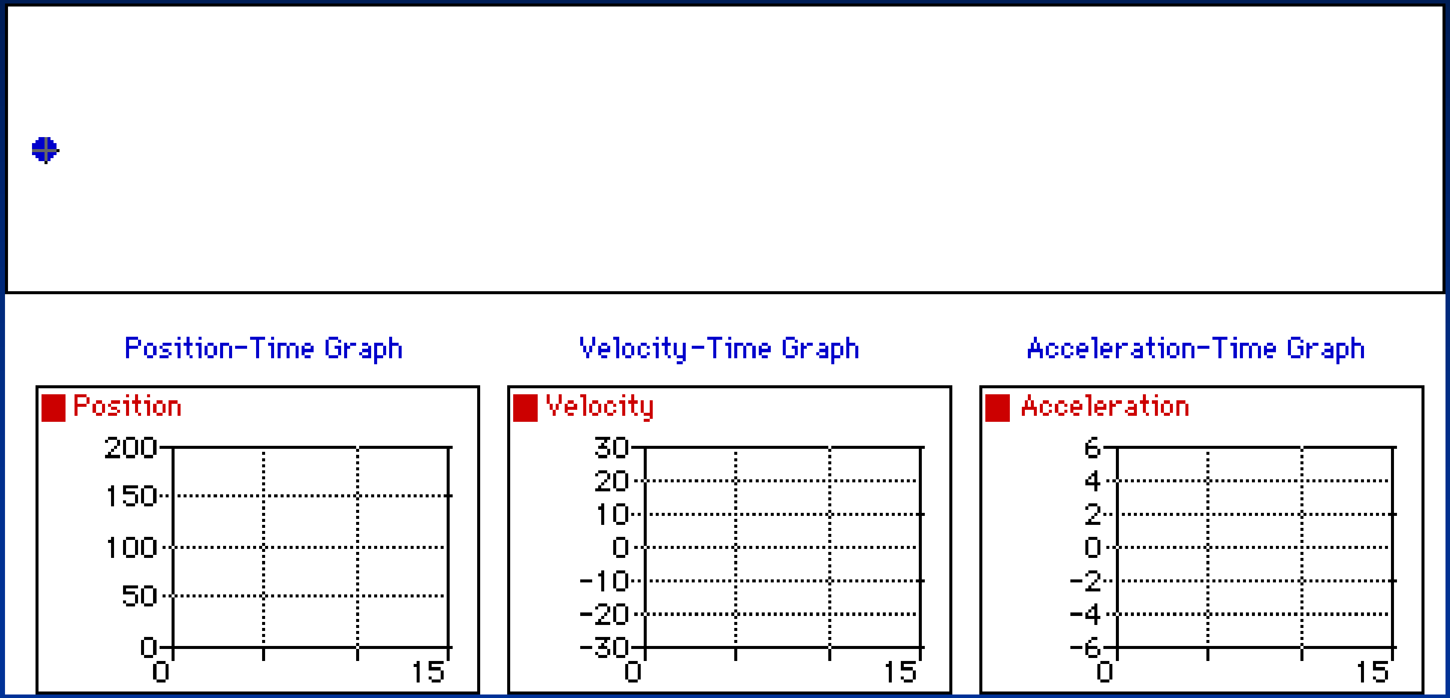
Constant Negative Velocity



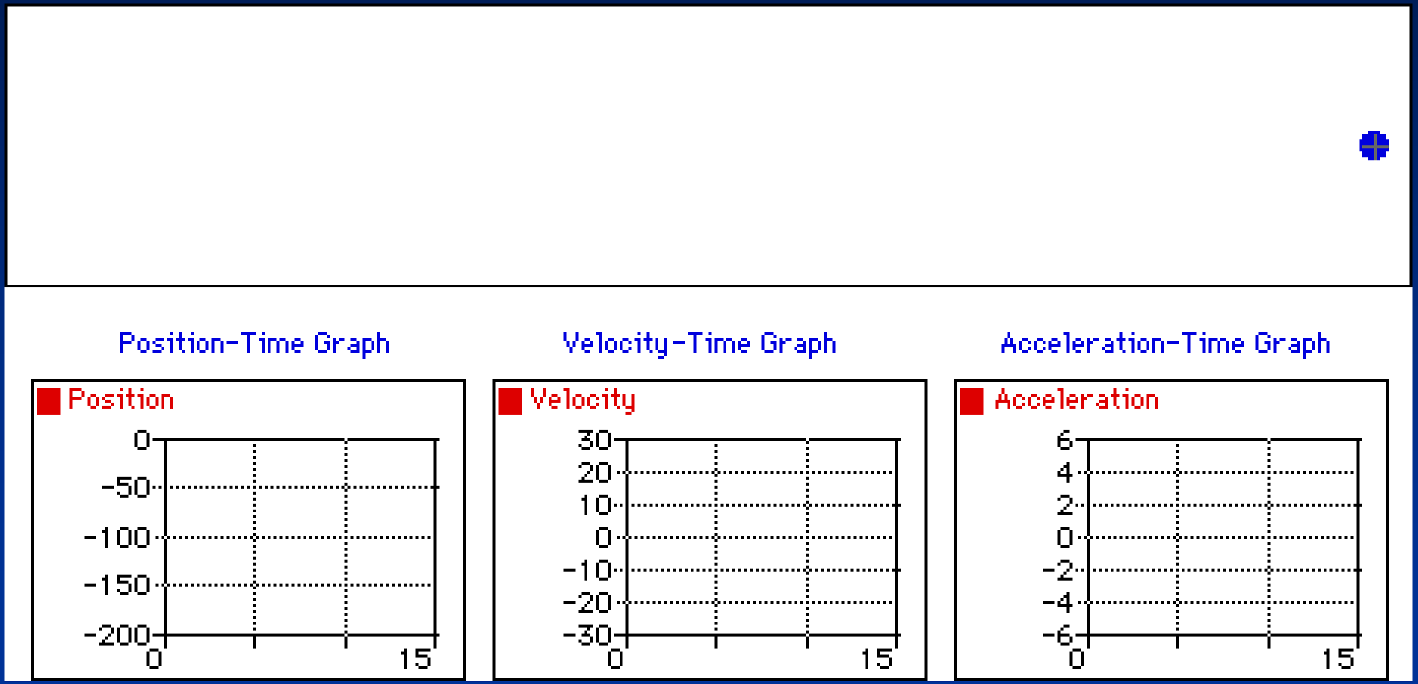
Positive Velocity and Positive Acceleration



Positive Velocity and Negative Acceleration



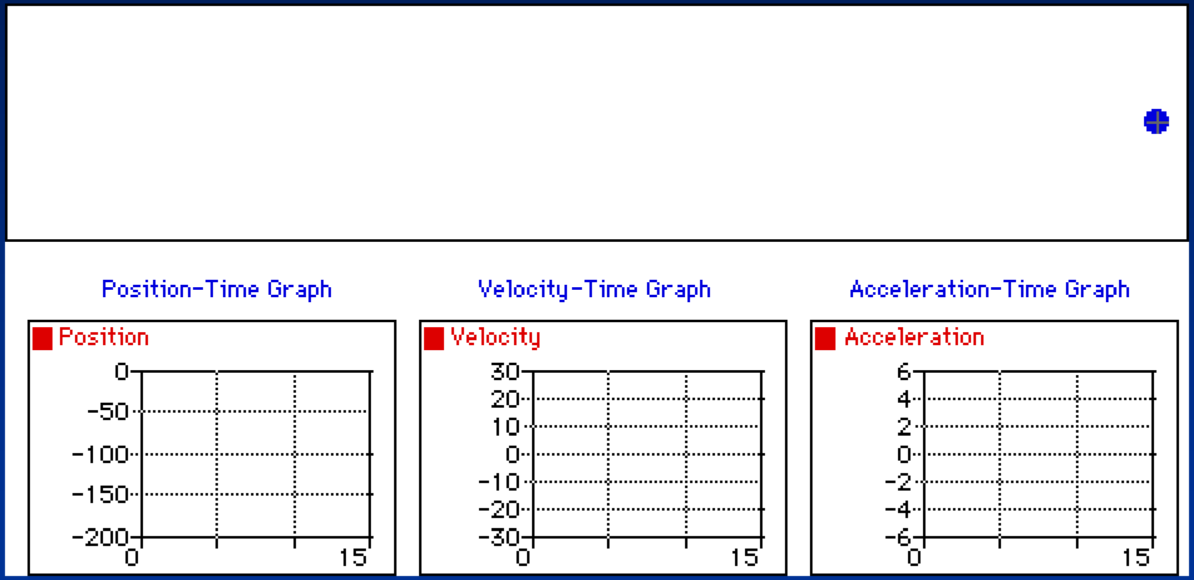
Negative Velocity and Negative Acceleration



Leftward (–) Velocity; Slow to Fast

negative acceleration – moving in the negative direction and speeding up.

Negative Velocity and Positive Acceleration



Leftward (–) Velocity; Fast to Slow

positive acceleration – moving in a negative direction and slowing down.

حرکت با شتاب ثابت

- Consider uniform acceleration (a = constant)
- Goal is finding equations for:
  - Velocity as a function of time
  - Displacement as a function of time
  - Velocity as a function of displacement

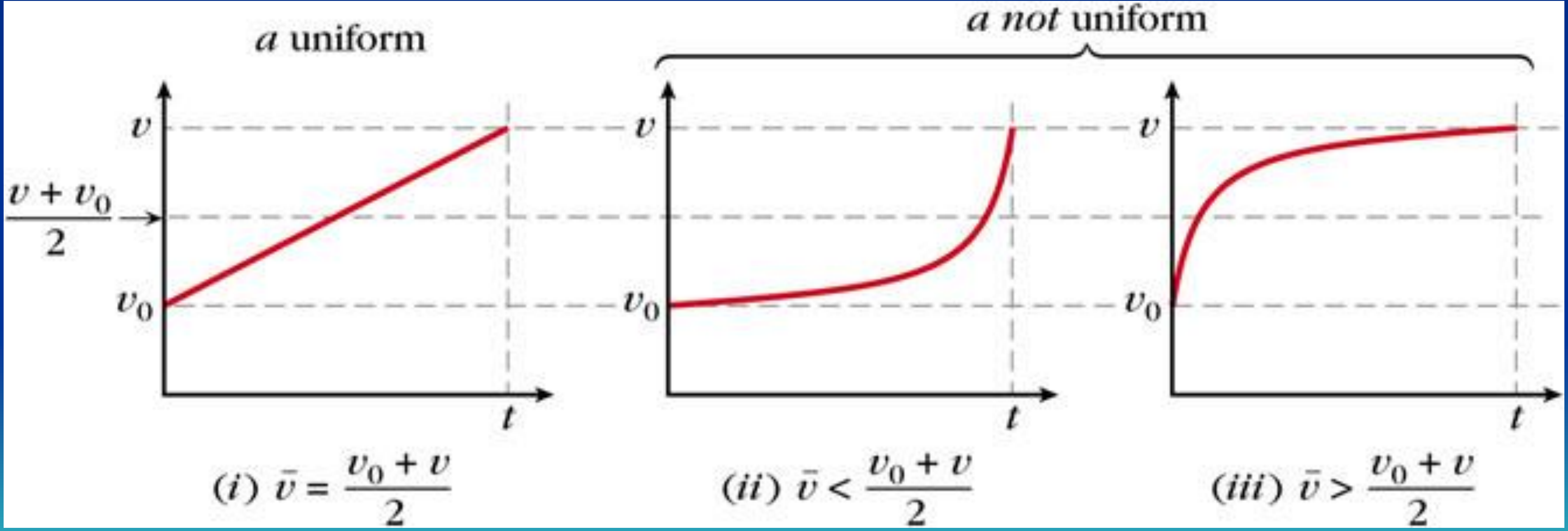
- **Procedure:**
  - Algebraically manipulate equations
  - Remove all 2 subscripts
  - Change all 1 subscripts to 0
  - Let $t_0 = 0$
- **First: derive expression *v* as function of *t***

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}$$

$$v = v_0 + at$$

Evaluate Velocity for Uniform *a*

- Velocity is linear function of time ($v = v_0 + at$)
- Average velocity is midway between v and v_0



Displacement for Uniform *a*

Start with definition of average *v*

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} = \frac{x - x_0}{t}$$

$$x - x_0 = \bar{v}t$$

$$x - x_0 = \frac{v + v_0}{2} t \qquad x - x_0 = \frac{1}{2}\left(v + v_0\right)t$$

Displacement for Uniform *a*

Substitute in expression for v

$$x - x_0 = \frac{1}{2}\left(v_0 + at + v_0\right)t$$

$$x - x_0 = \frac{1}{2}\left(2v_0 + at\right)t$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

Velocity and Displacement

- Combine
  - Displacement as a function of time
  - Velocity as a function of time
  - Expression for average velocity

$$x - x_0 = \bar{v}t$$

$$v - v_0 = at$$

$$\bar{v} = \frac{v + v_0}{2}$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

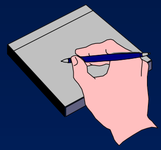
Summary of Motion in 1-D with Constant *a*

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

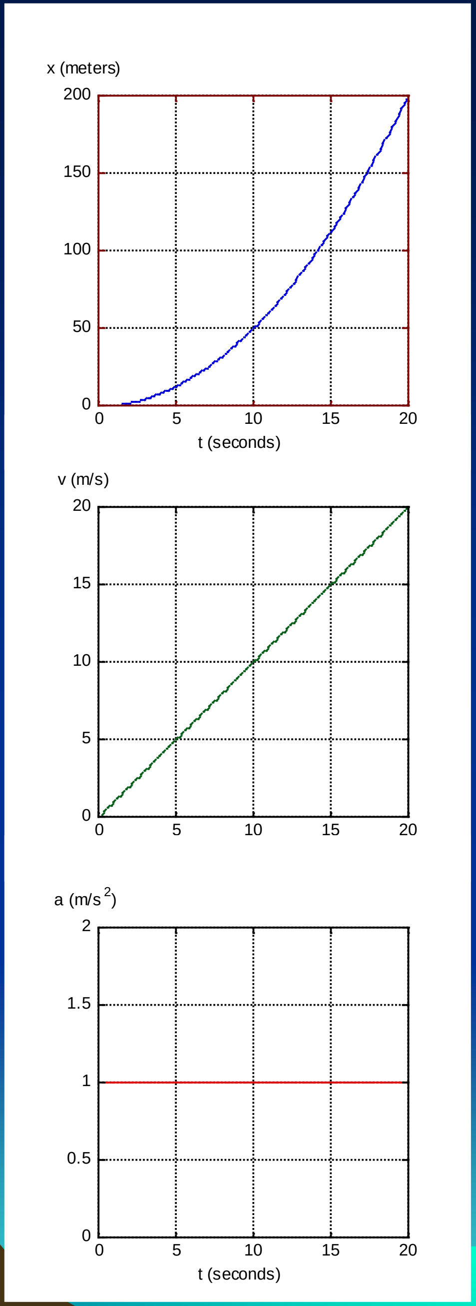
When $a = 0$ these equations correspond to *uniform motion* (v = constant)



Equations for Constant Acceleration

- $x = x_0 + v_0 t + 1/2\ at^2$
- $v = v_0 + at$
- $v^2 = v_0^2 + 2a(x-x_0)$

- $\Delta x = v_0 t + 1/2\ at^2$
- $\Delta v = at$
- $v^2 = v_0^2 + 2a\ \Delta x$



Freely Falling Objects

- The only acceleration is due to gravity
- There is no air resistance to impede the motion
- Gravity provides a constant (uniform) acceleration near the surface of the earth
- Replace *x* with *y* to indicate vertical position in equations of motion

$$a = -g = -9.8\ \mathrm{m/s^2}$$

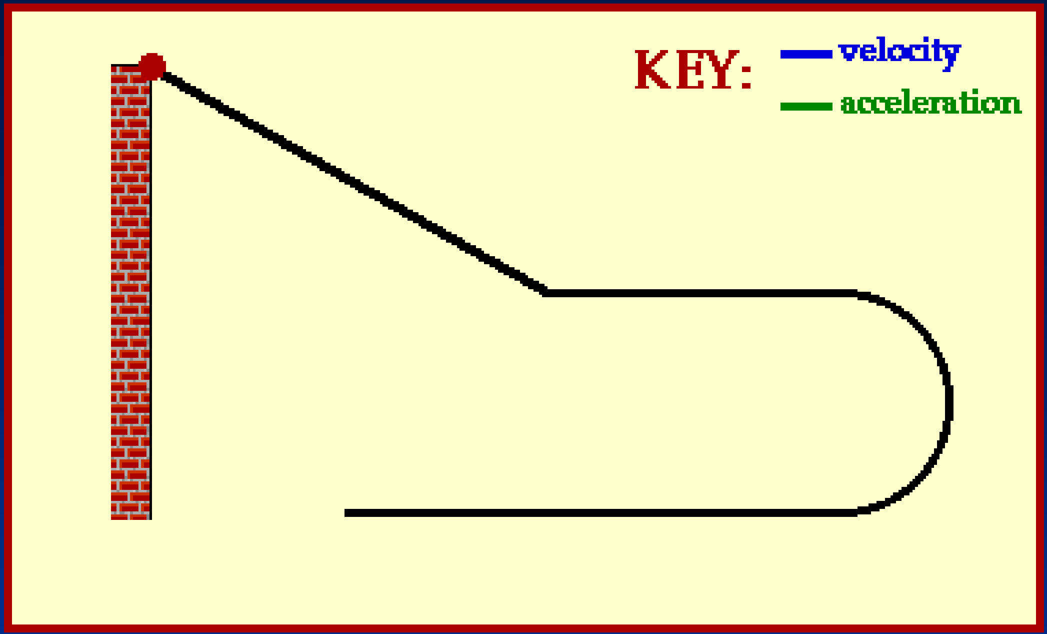
Summary of Free Fall Equations

$$v = v_0 - gt$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

Up is the positive y direction.



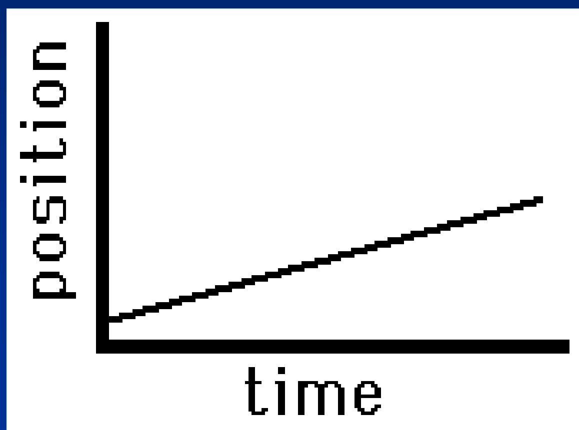
The simple animation above depicts some additional information about the car's motion. The velocity and acceleration of the car are depicted by vector arrows. The direction of these arrows are representative of the direction of the velocity and acceleration vectors. Note that the velocity vector is always directed in the same direction which the car is moving. That is, a car moving eastward would be described as having an eastward velocity; and a car moving westward would be described as having a westward velocity.

- **The direction of the acceleration vector is not so easily determined. As shown in the animation, an eastward heading car can have a westward directed acceleration vector. And a westward heading car can have an eastward directed acceleration vector. So how can the direction of the acceleration vector be determined? A simple *rule of thumb* for determing the direction of the acceleration is that an object which is slowing down will have an acceleration directed in the direction opposite of its motion. Applying this *rule of thumb* would lead us to conclude that an eastward heading car can have a westward directed acceleration vector only if it is slowing down.**
- **So be careful when discussing the direction of the acceleration of an object; slow down, apply some thought and use the *rule of thumb*.**

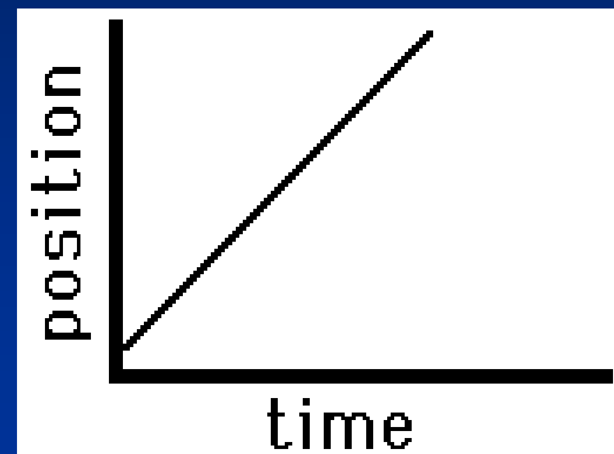
The principle is that the slope of the line on a position-time graph reveals useful information about the velocity of the object. It's often said, "As the slope goes, so goes the velocity."

- Whatever characteristics the velocity has, the slope will exhibit the same (and vice versa). If the velocity is constant, then the slope is constant (i.e., a straight line). If the velocity is changing, then the slope is changing (i.e., a curved line). If the velocity is positive, then the slope is positive (i.e., moving upwards and to the right). This principle can be extended to any motion conceivable.

Slow, Rightward (+) Constant Velocity



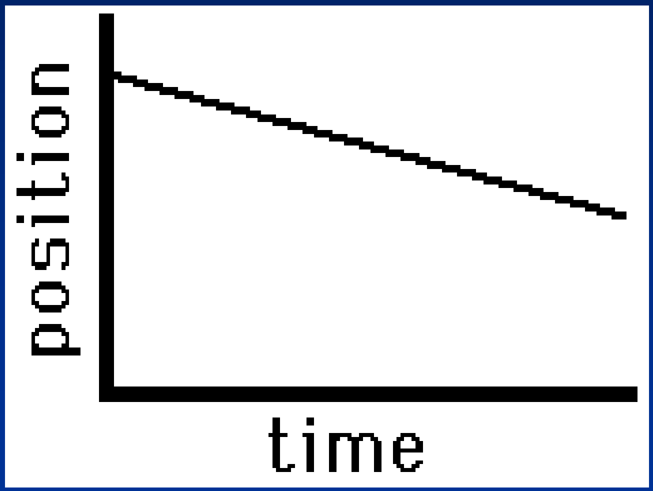
Fast, Rightward (+) Constant Velocity



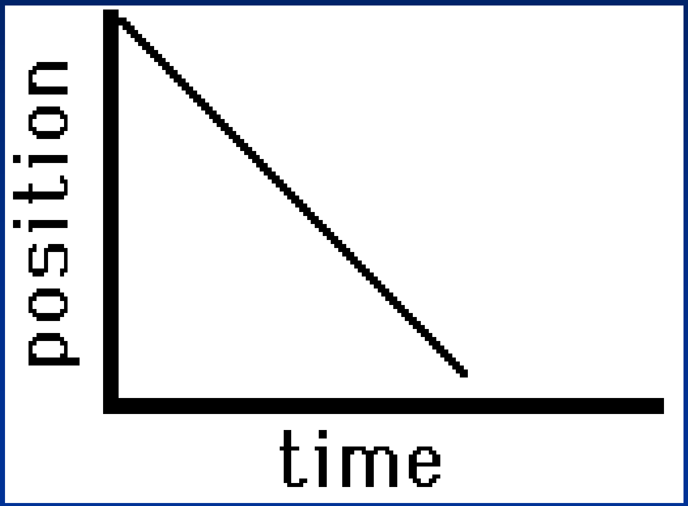
the slope of the graph on the right is larger than that on the left and this larger slope is indicative of a larger velocity.

there is a constant, negative velocity (as denoted by the constant, negative slope)

Slow, Leftward (–) Constant Velocity



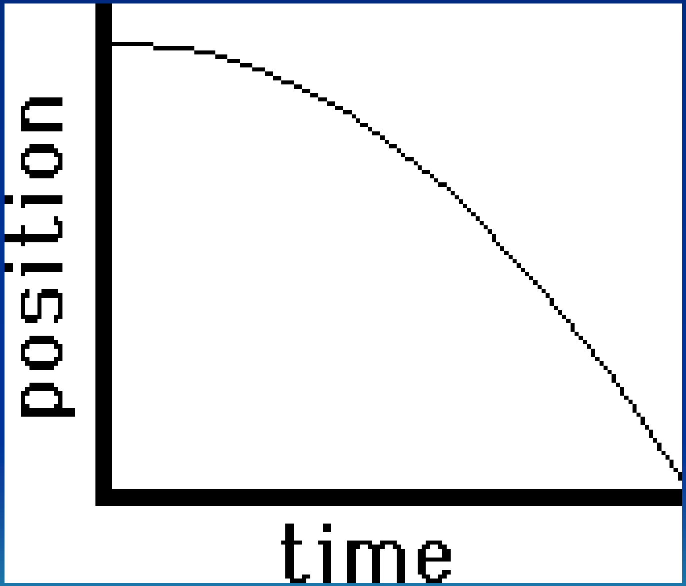
Fast, Leftward (–) Constant Velocity



The object represented by the graph on the right is traveling faster than the object represented by the graph on the left.

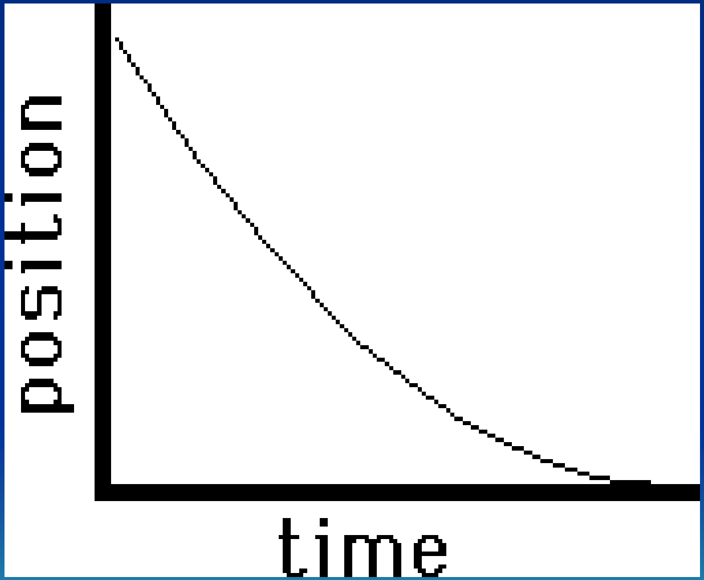
- **Curved lines have changing slope; they may start with a very small slope and begin curving sharply (either upwards or downwards) towards a large slope. In either case, the curved line of changing slope is a sign of accelerated motion (i.e., changing velocity).**

Leftward (–) Velocity; Slow to Fast

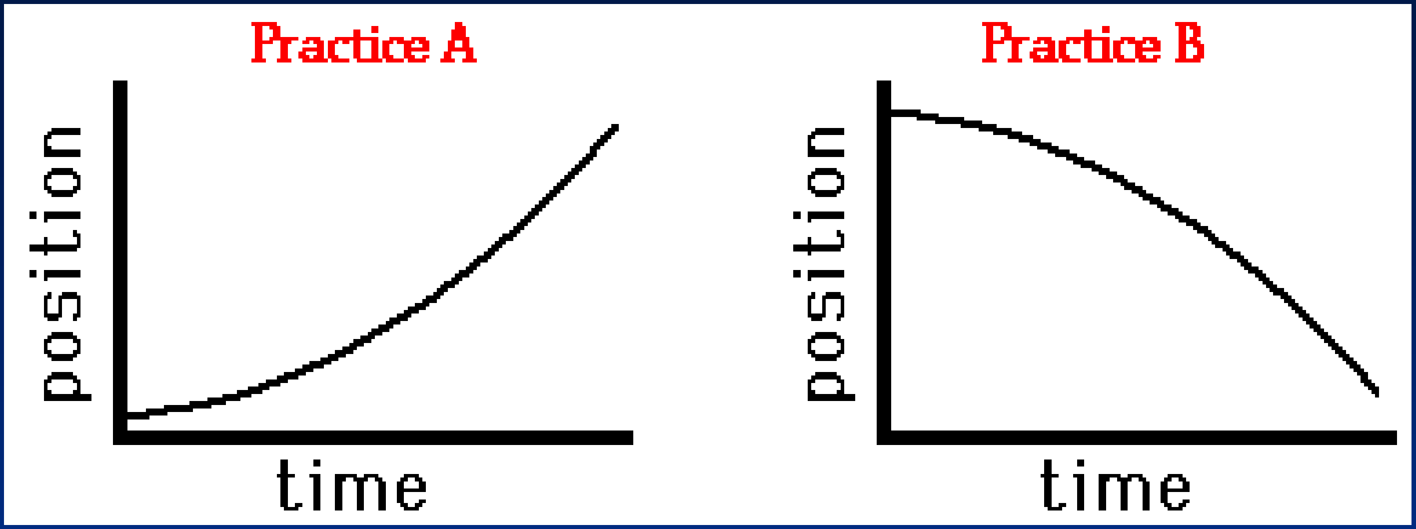


negative acceleration – moving in the negative direction and speeding up.

Leftward (–) Velocity; Fast to Slow



positive acceleration – moving in a negative direction and slowing down.



- **The object has a positive or rightward velocity (note the + slope). The object has a changing velocity (note the changing slope); it is accelerating. The object is moving from slow to fast (since the slope changes from small big).**

The object has a negative or leftward velocity (note the - slope). The object has a changing velocity (note the changing slope); it has an acceleration. The object is moving from slow to fast (since the slope changes from small to big).

Graphs can tell us things

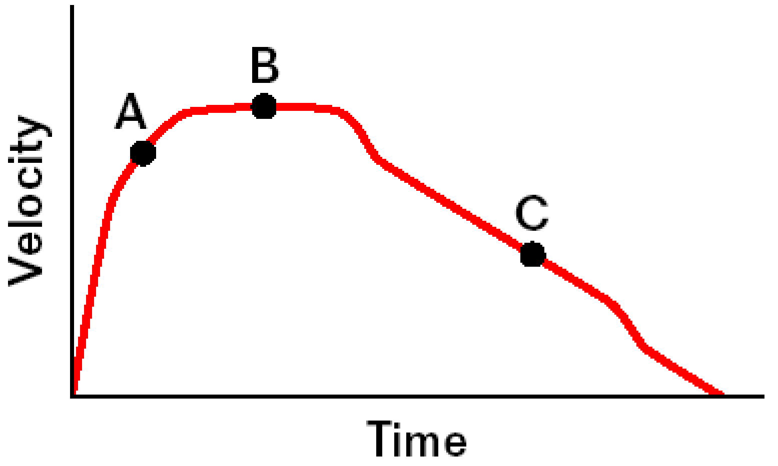


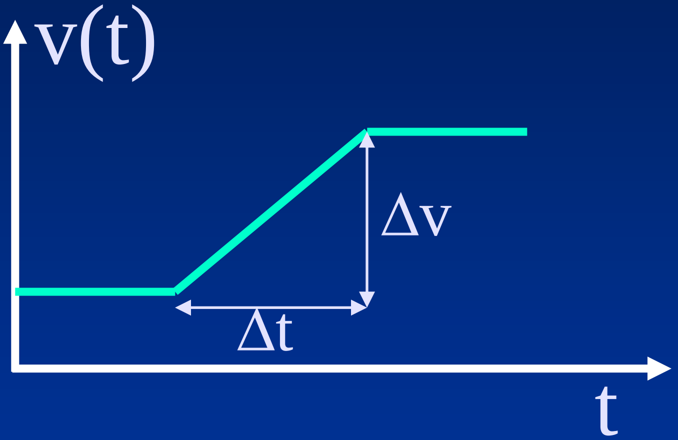
Figure 10

When the velocity in the positive direction is increasing, the acceleration is positive, as at point A. When the velocity is constant, there is no acceleration, as at point B. When the velocity in the positive direction is decreasing, the acceleration is negative, as at point C.

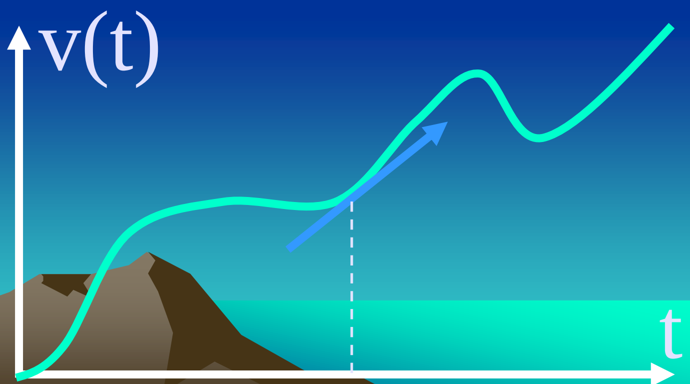
Acceleration (m/s²)

- The average acceleration is the change in velocity divided by the change in time.

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

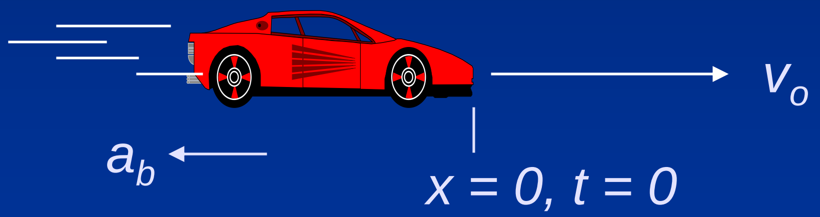


- Instantaneous acceleration is limit of average velocity as Δt gets small. It is the slope of the v(t) plot



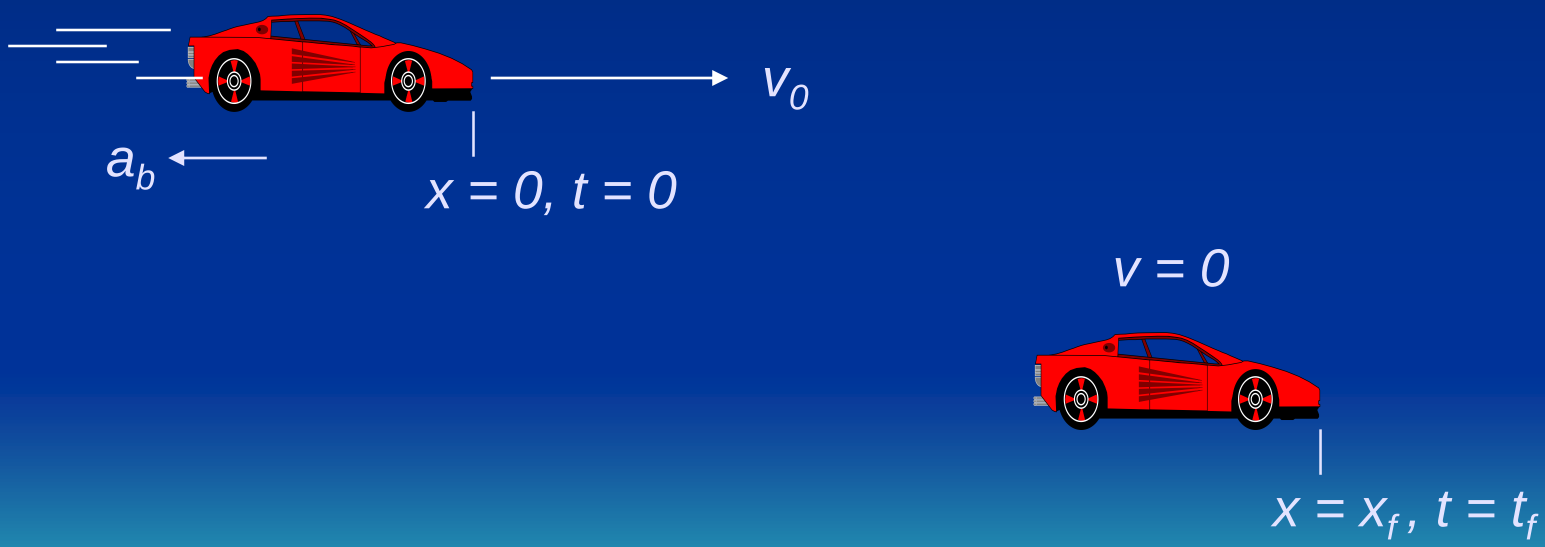
Problem 1

- **A car is traveling with an initial velocity v_0. At $t = 0$, the driver puts on the brakes, which slows the car at a rate of a_b**



Problem 1...

- **A car is traveling with an initial velocity v_0. At $t = 0$, the driver puts on the brakes, which slows the car at a rate of a_b. At what time t_f does the car stop, and how much farther x_f does it travel?**



Problem 1...

- Above, we derived: $v = v_0 + at$
- Realize that $a = -a_b$
- Also realizing that $v = 0$ at $t = t_f$:

  find $0 = v_0 - a_b t_f$ or

$$t_f = v_0 / a_b$$

Problem 1...

- To find stopping distance we use:

$$v^2 - v_0^2 = 2a(x - x_0)$$

- In this case $v = v_f = 0$, $x_0 = 0$ and $x = x_f$

$$-v_0^{\,2} = 2(-a_b)x_f$$

$$x_f = \frac{v_0^{\,2}}{2a_b}$$

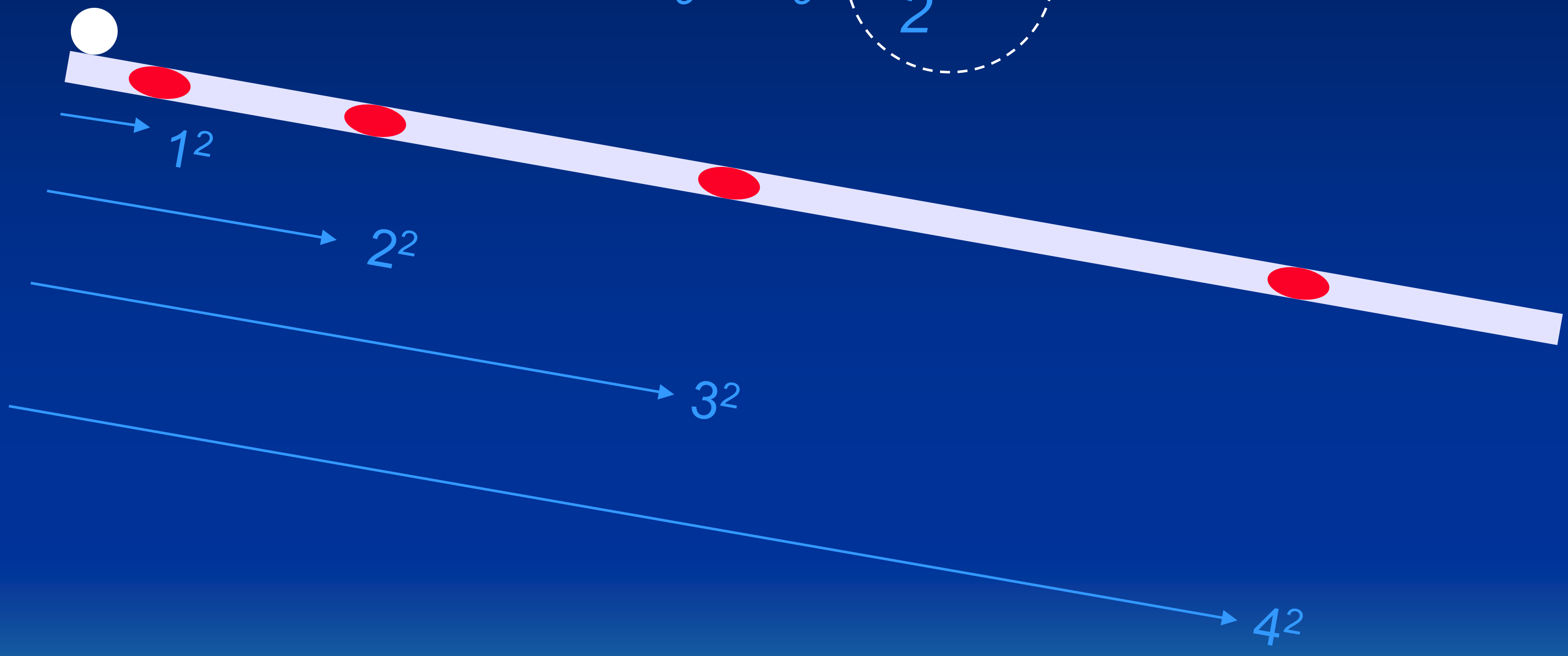
Problem 1...

- So we found that $$t_f = \frac{v_0}{a_b}, \; x_f = \frac{1}{2}\frac{v_0^2}{a_b}$$

- Suppose that $v_o = 65\ mi/hr\ = 29\ m/s$
- Suppose also that $a_b = g = 9.81\ m/s^2$
  - Find that $t_f = 3\ s$ and $x_f = 43\ m$

Recall what you saw:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$



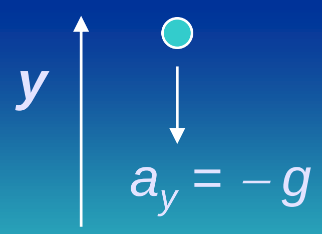
1-D Free-Fall

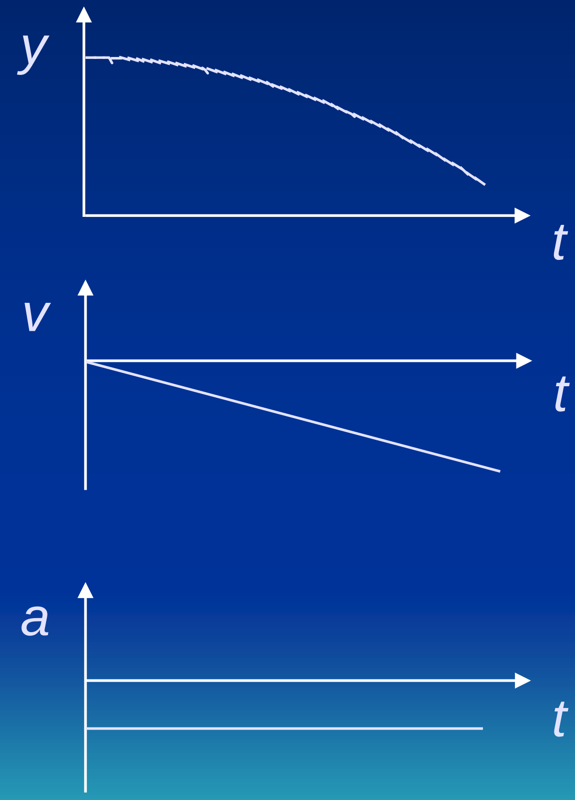
- **This is a nice example of constant acceleration (gravity):**
- **In this case, acceleration is caused by the force of gravity:**
  - **Usually pick y-axis "upward"**
  - **Acceleration of gravity is "down":**

$$a_y = -g$$

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}\, t - \frac{1}{2} g\, t^2$$



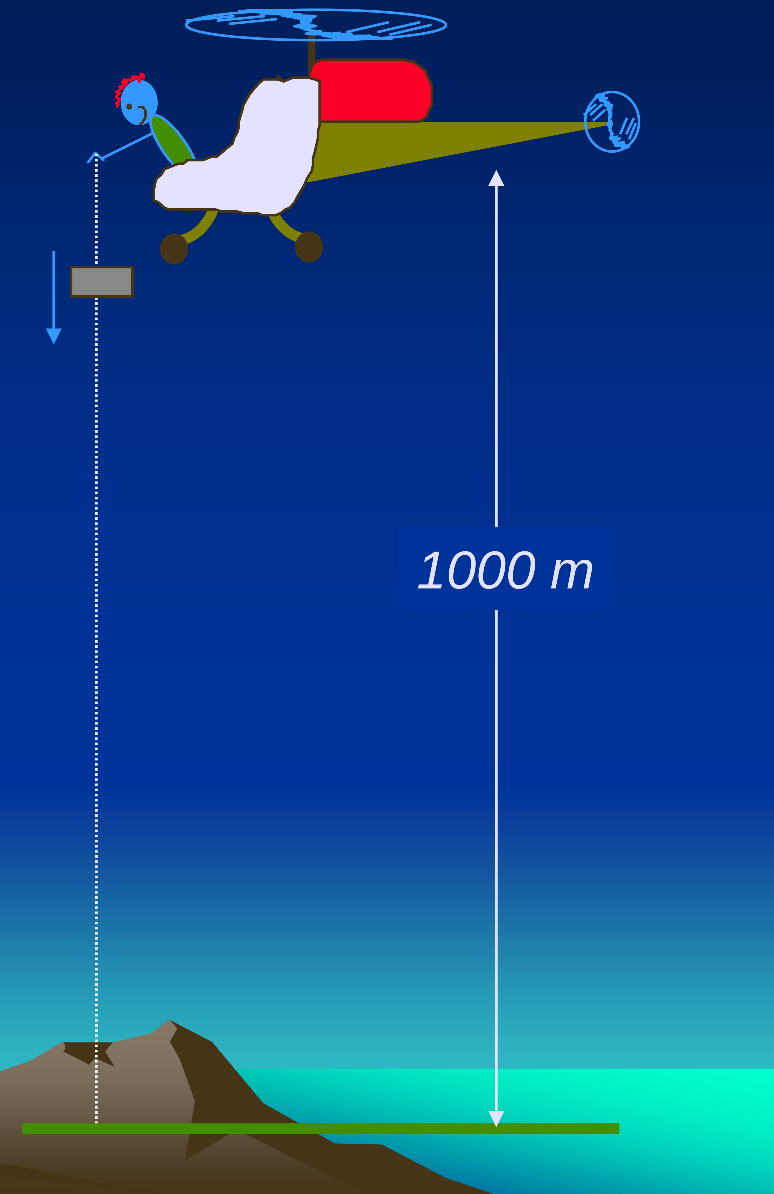


Gravity facts:

- g does not depend on the nature of the material!
  - Galileo (1564-1642) figured this out without fancy clocks & rulers!
- demo - feather & penny in vacuum
- Nominally, $g = 9.81$ m/s^2
  - At the equator $g = 9.78$ m/s^2
  - At the North pole $g = 9.83$ m/s^2
- More on gravity in a few lectures!

Problem:

- The pilot of a hovering helicopter drops a lead brick from a height of 1000 m. How long does it take to reach the ground and how fast is it moving when it gets there? (neglect air resistance)



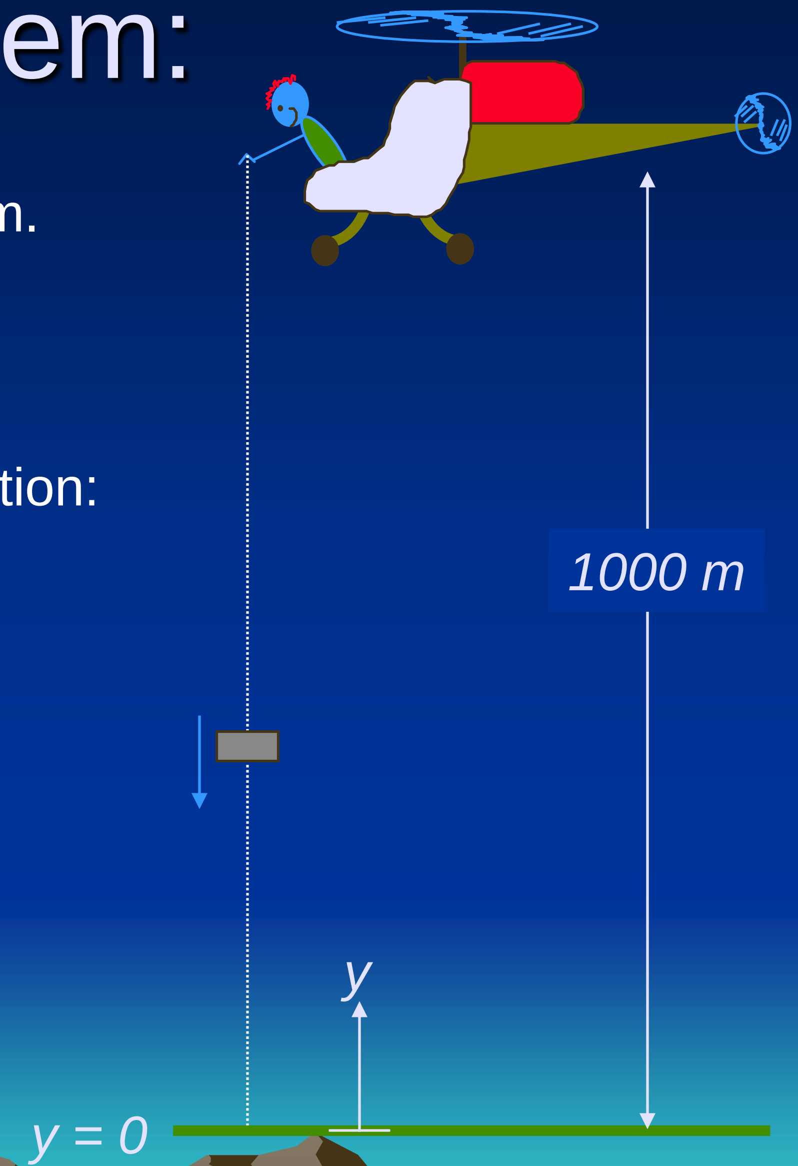
Problem:

- First choose coordinate system.
  - Origin and *y*-direction.

- Next write down position equation:

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

- Realize that $v_{0y} = 0$.



$$y = y_0 - \frac{1}{2} g t^2$$

$$y = y_0 - \frac{1}{2} g t^2$$

Problem:

- Solve for time t when $y = 0$ given that $y_0 = 1000\ m.$

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2 \times 1000 m}{9.81 m/s^2}} = 14.3 s$$

- Recall:

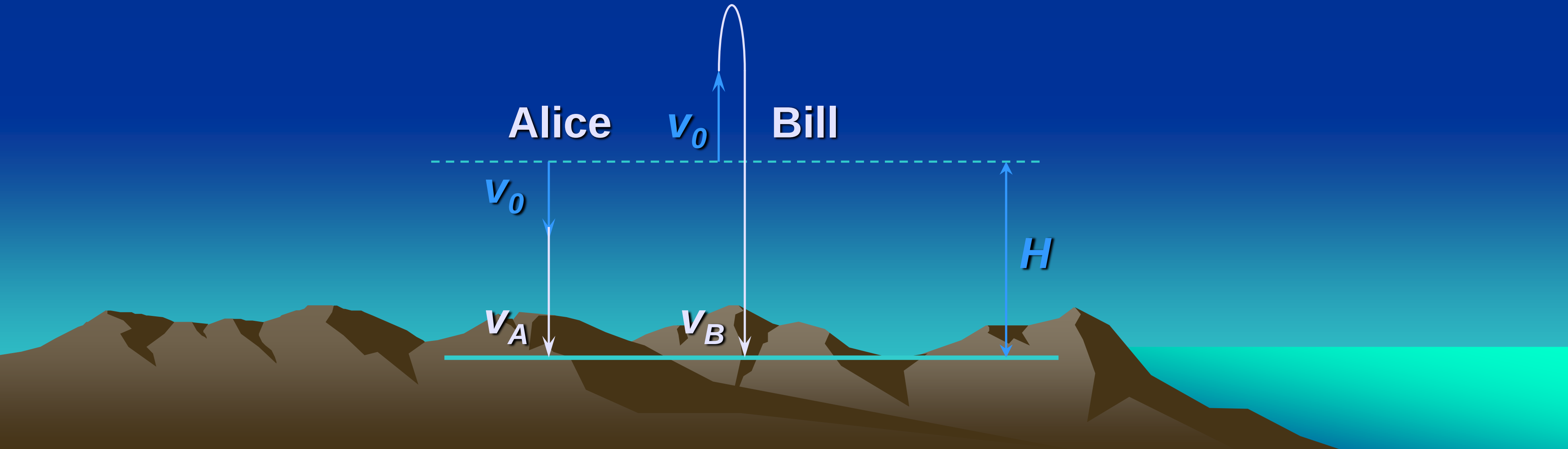
$$v_y^2 - v_{0y}^2 = 2a(y - y_0)$$

- Solve for v_y:

$$v_y = \pm\sqrt{2gy_0}$$
$$= -140\ m/s$$

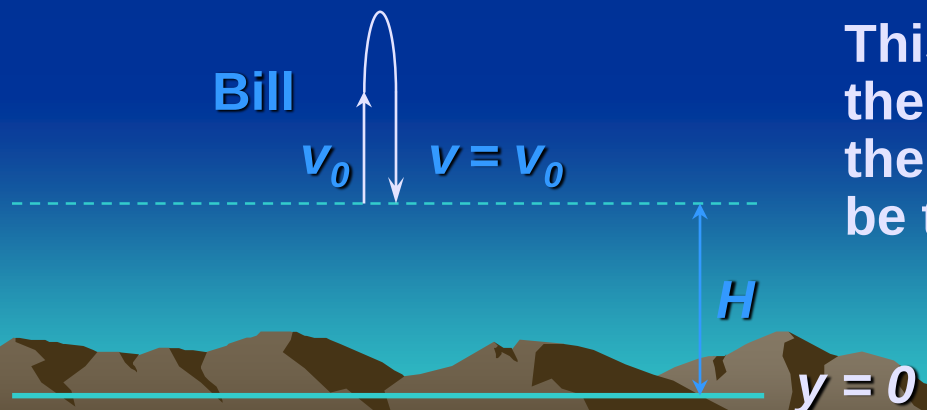
- Alice and Bill are standing at the top of a cliff of height H. Both throw a ball with initial speed v_0, Alice straight down and Bill straight up. The speed of the balls when they hit the ground are v_A and v_B respectively. Which of the following is true:

(a) $v_A < v_B$ **(b)** $v_A = v_B$ **(c)** $v_A > v_B$



- Since the motion up and back down is symmetric, intuition should tell you that $v = v_0$
  - We can prove that your intuition is correct:

Equation: $v^2 - v_0^2 = 2(-g)(H - H) = 0$



This looks just like Bill threw the ball down with speed v_0, so the speed at the bottom should be the same as Alice's ball.

- **We can also just use the equation directly:**

Alice: $v^2 - v_0^2 = 2(-g)(0 - H)$

Bill: $v^2 - v_0^2 = 2(-g)(0 - H)$

same !!