فیزیک 1

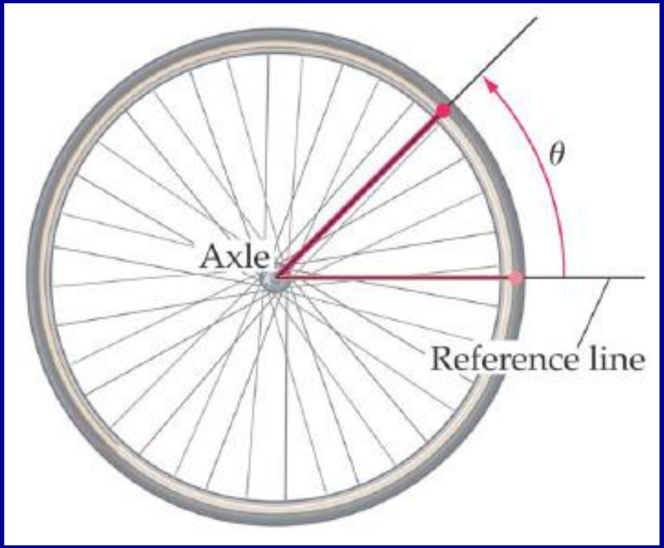
درس 22

صحرایی

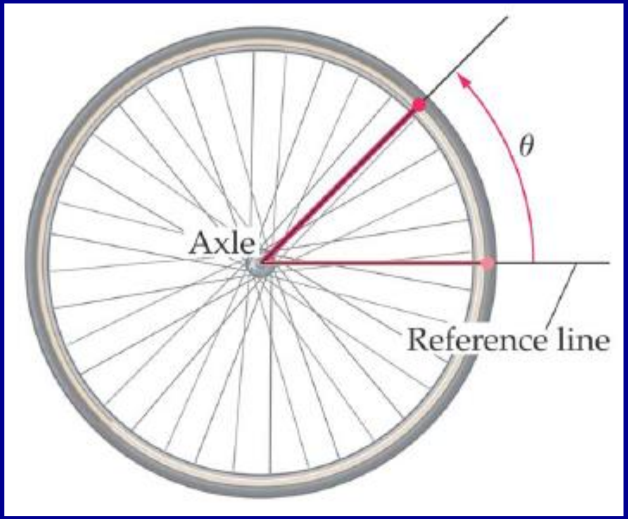
گروه فیزیک دانشگاه رازی

http://www.razi.ac.ir/sahraei

Rotational Kinematics



Rotational kinematics	Linear kinematics
Angular Position θ	x
Angular Velocity ω	v
Angular Acceleration α	a



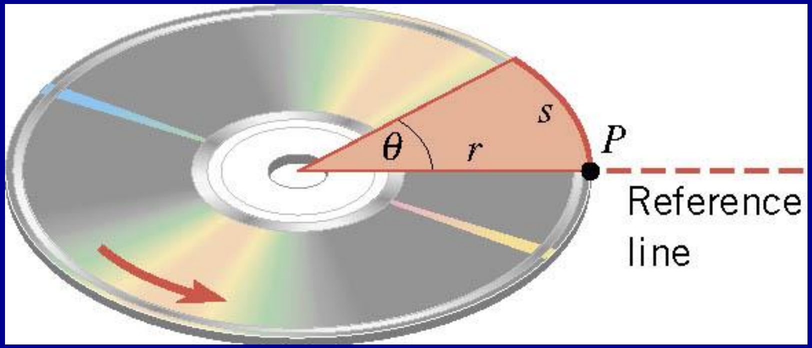
Angular displacement is expressed in one of three units:

1. Degree
2. Revolution (rev)
3. Radian (rad)

$\theta = 0$: Reference line

$\theta > 0$: Counterclockwise

$\theta < 0$: Clockwise



$$\theta \text{ (in radians)} = \frac{arc\ \ length}{Radius} = \frac{s}{r}$$

For 1 full rotation, $\theta = \frac{2\pi r}{r} = 2\pi\ \ rad$

$$\therefore 1\,rev = 360^{\circ}\ \text{degree} = 2\pi\ \ rad$$

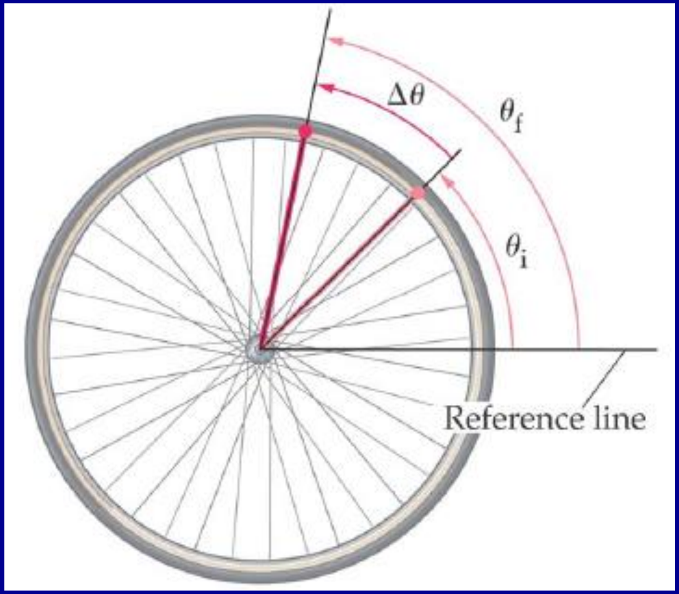
$$1\ \ rad = \frac{360^{\circ}}{2\pi} = 57.3^{\circ}$$

1 rad =1rev/2π = 0.159 rev

Angular Velocity

The **average angular velocity** of an object undergoing rigid rotation is defined as the angular displacement Δθ of any point on the object divided by the time Δt taken for that point to sweep through the displacement Δθ:

$$\overline{\omega} = \frac{\theta_f - \theta_i}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$



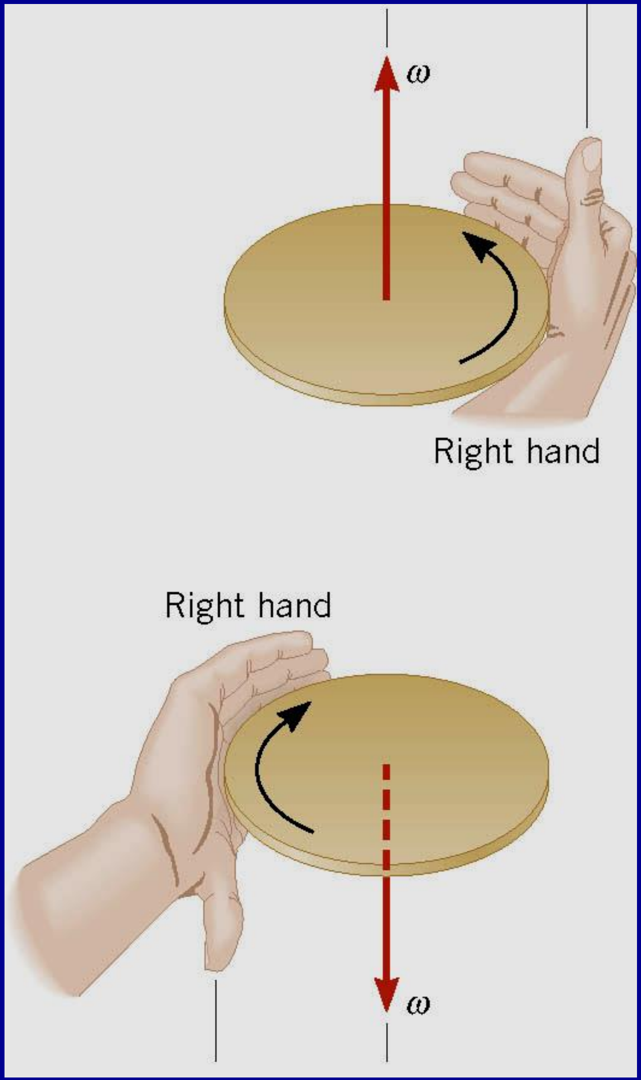
The **instantaneous angular velocity** is defined as the limiting value of the average angular velocity when Δt goes to zero:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \qquad \omega = \frac{d\theta}{dt}$$

The SI unit of angular velocity is radians per second (rad/s)

The vector nature of angular velocity

- We have treated angular velocity as a scalar, though in truth it is a vector quantity. The angular velocity vector is parallel to the axis of rotation, and the direction it points is given by the **right hand rule:** grasp the axis of rotation with your right hand so that your fingers curl in the direction of rotation; then the direction your extended thumb points in is the direction of the angular velocity vector.



Angular Acceleration

- **The average angular acceleration of an object undergoing rigid rotation is the change in the angular velocity of the object within the time interval Δt:**

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

- **The instantaneous angular acceleration is defined as the limiting value of the average angular acceleration when Dt goes to zero:**

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \qquad \alpha = \frac{d\omega}{dt}$$

- **The SI unit of angular acceleration is radians per second-squared (rad/s^2)**

Symbols Used in Rotational and Linear Kinematics

Rotational Motion	Quantity	LinearMotion
θ	Displacement	x
ω_0	Initial velocity	v_0
ω	Final velocity	v
α	Acceleration	a
t	Time	t

دوران با شتاب زاویه ای ثابت

$$d\omega = \alpha\, dt$$

$$\int_{\omega_0}^{\omega} d\omega = \int_0^{t} \alpha\, dt = \alpha \int_0^{t} dt$$

$$\omega - \omega_0 = \alpha\, t$$

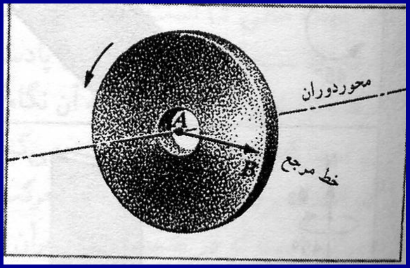
$$\omega = \omega_0 + \alpha\, t$$

$$v = v_0 + at$$

The Equations of Kinematics for Rational and Linear Motion

Linear Motion (a = constant)	**Rotational Motion** (α = constant)
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + \frac{1}{2}(v_0 + v)t$	$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

مثال(1): سنگ سنباده ای در لحظه t=0 با شتاب زاویه ای ثابت α برابر با $3.2\ rad/s^2$ از حالت سکون شروع به حرکت می کند. در t=0 خط مرجع AB افقی است. (الف) جابجایی زاویه ای خط AB (و در نتیجه سنگ سنباده) چقدر است؟



$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$t = 0,\ \theta_0 = 0,\ \omega = \omega_0 = 0$$

$$\theta = 0 + (0)(2.7s) + \frac{1}{2}(3.2rad/s^2)(2.7s)^2$$

$$= 11rad = 1.9rev$$

(ب) سرعت زاویه ای سنگ سنباده در زمان t=2.7 s چقدر است؟

$$\omega = \omega_0 + \alpha t = 0 + (3.2rad/s^2)(2.7s)$$

$$= 8.6rad/s = 1.4rev/s$$

مثال(2): فرض کنید که موتور چرخاننده سنگ سنباده مثال 1 در زمانی که سنگ با سرعت زاویه ای 8.6 rad/s می چرخد خاموش شود. نیروی اصطکاکی کوچک وارد بر محور موجب یک شتاب زاویه ای منفی ثابت می شود و سرانجام چرخ پس از 192s متوقف می شود. (الف) شتاب زاویه ای را تعیین کنید.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 8.6 rad/s}{192 s} = -0.045 rad/s$$

(ب) زاویه ای را که چرخ قبل از توقف پیموده است تعیین کنید.

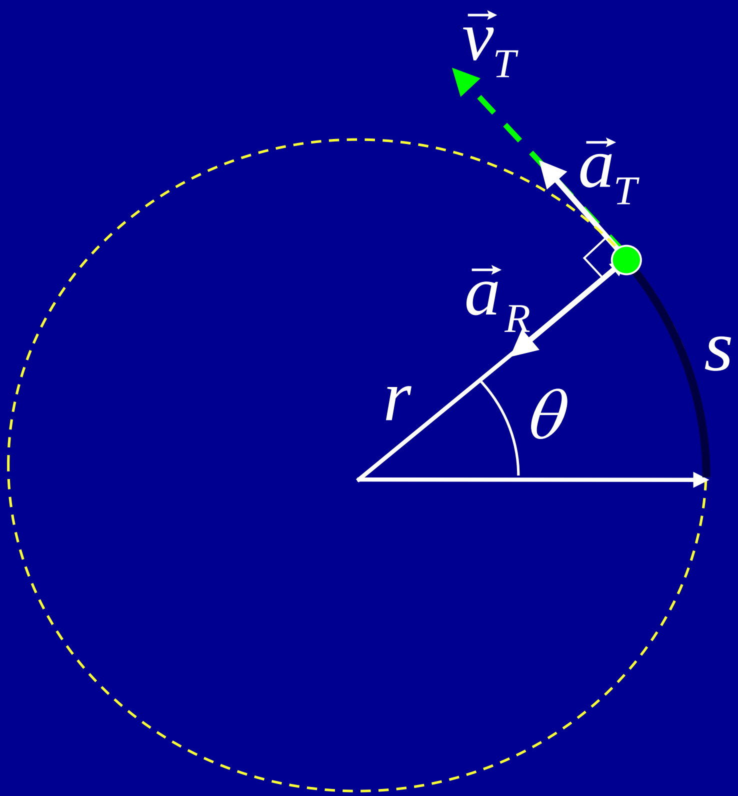
$$\theta = \theta_0 + \frac{\omega + \omega_0}{2} t = 0 + \frac{8.6 rad/s + 0}{2}(192 s)$$

$$= 826 rad = 131 rev$$

Angular, tangential and radial variables

A point on an object undergoing rigid rotation is described by its radius r and the following angular variables:

- θ – angle
- ω – angular velocity
- α – angular acceleration



tangential components:

v_T - tangential velocity

a_T- tangential acceleration

radial components:

a_R - radial acceleration

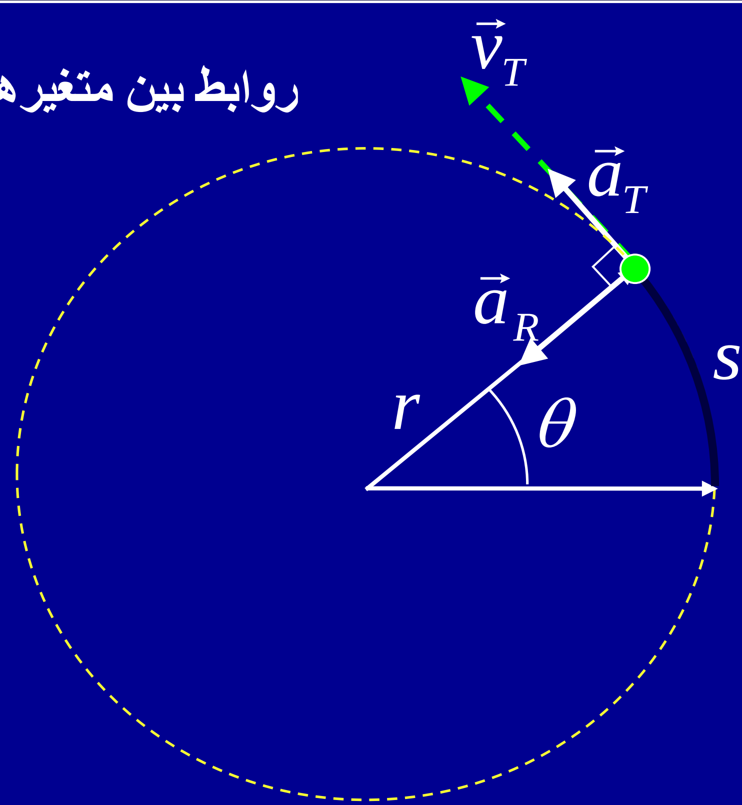
the radial velocity is 0!

- **Notes:**

  the uniform circular motion we studied in chapter 5 is a special case of rigid rotation, where the angular acceleration is zero.

  if the angular acceleration is non-zero, then the motion traced out by a point on the circle is called *nonuniform circular motion*

روابط بین متغیرهای خطی و زاویه ای



$$s = r\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$v_T = r\omega$$

$$v = \sqrt{{v_T}^2 + {v_R}^2} = v_T$$

$$\frac{dv_T}{dt} = r\frac{d\omega}{dt}$$

$$a_T = r\alpha \qquad a_R = \frac{v^2}{r} = r\omega^2$$

$$a = \sqrt{{a_T}^2 + {a_R}^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$