Physics 1

Lecture 19

Sahraei

**Physics Department,
Razi University**

http://www.razi.ac.ir/sahraei

Review of Lecture 18

- Potential Energy
  - Work and Potential Energy
  - Conservative & Nonconservative Forces
- Path Independence of Conservative Forces
- Determining Potential Energy Values
  - Gravitational PE
  - Elastic PE
- Conservation of Mechanical Energy

Review of Lecture 18

- Reading a Potential Energy Curve
  - Turning points & Equilibrium points
- Work Done on a System by an External Force
  - With & Without Friction
- Conservation of Energy

Systems of Particles

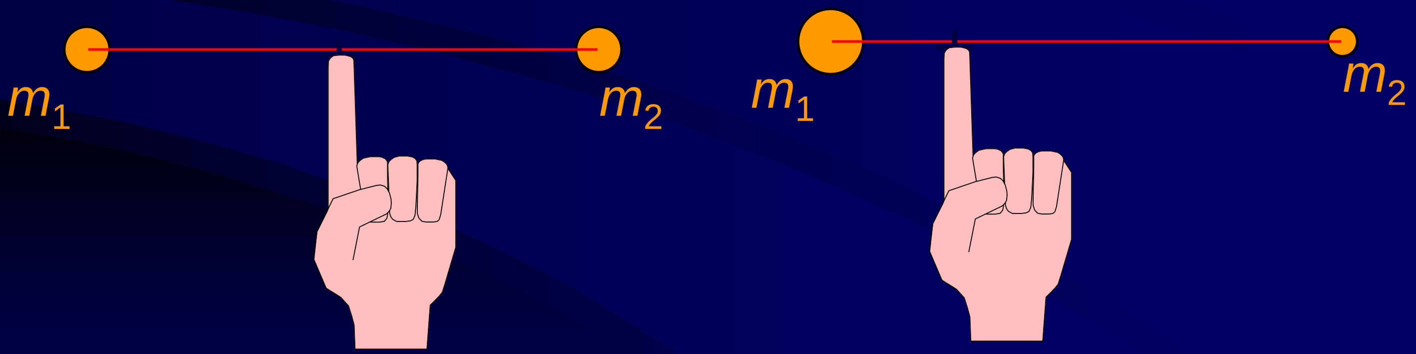
- We have discussed parabolic trajectories using a "particle" as the model for our objects
- But clearly objects are not particles – they are extended and may have complicated shapes & mass distributions
- So if we toss something like a baseball bat into the air (spinning and rotating in a complicated way), what can we really say about it's trajectory?

Center of Mass

- There is one special location in every object that provides us with the basis for our earlier model of a point particle
- That special location is called the *center of mass*
- The center of mass will follow a parabolic trajectory – even if the rest of the bat’s motion is very complicated

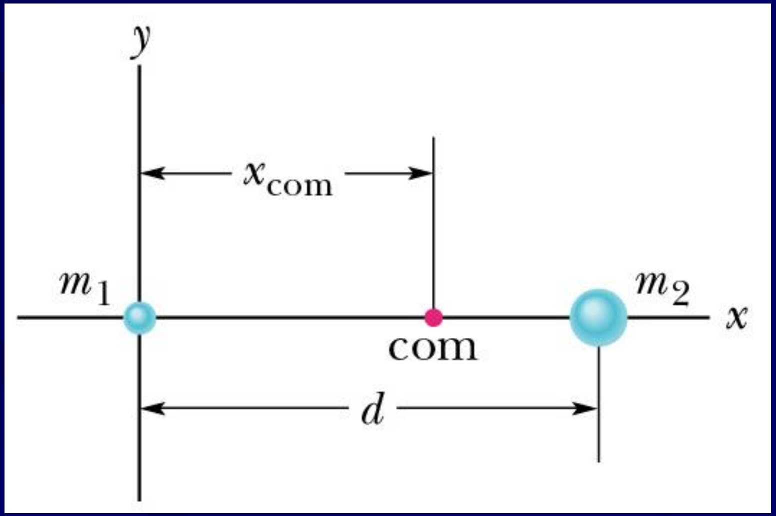
System of Particles: Center of Mass

The center of mass is where the system is balanced!

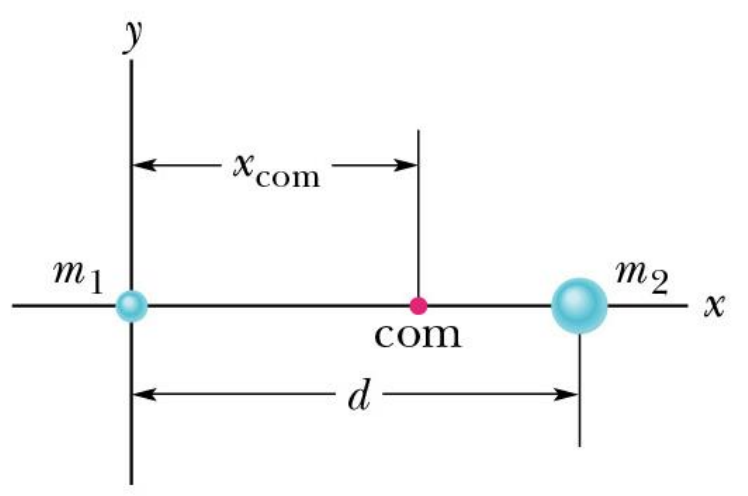


- To start, let's suppose that we have two masses m_1 and m_2, separated by some distance d
- We have also arbitrarily aligned the origin of our coordinate system to be the center of mass m_1
- We define the center of mass for these two particles to be:

$$x_{cm} = \frac{m_2}{m_1 + m_2} d$$



- From this we can see that if $m_2 = 0$, then $x_{cm} = 0$
- Similarly, if $m_1 = 0$, then $x_{cm} = d$

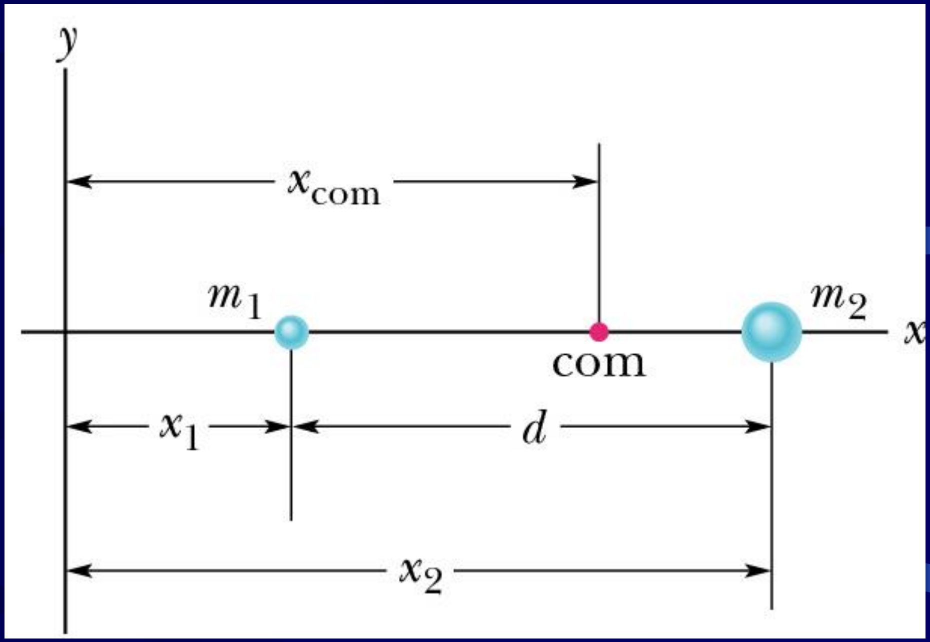


$$x_{cm} = \frac{m_2}{m_1 + m_2} d$$

Finally, if $m_1 = m_2$, then $x_{cm} = ½d$

So we can see that the center of mass in this case is constrained to be somewhere between $x = 0$ and $x = d$

- Now lets shift the origin of the coordinate system a little
- We now need a more general definition of the center of mass



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

- **Note that if x_1 = 0 we are back to the previous equation**

- Now let's suppose that we have lots of particles – all lined up nicely for us on the x axis
- The equation would now be:

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_n x_n}{M}$$

where $M = m_1 + m_2 + \ldots + m_{\text{n}}$

The collection of terms in the numerator can be rewritten as a sum resulting in:

$$x_{\text{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

This result is only for one dimension however, so the more generalized result for 3 dimensions is shown here:

$$x_{\text{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

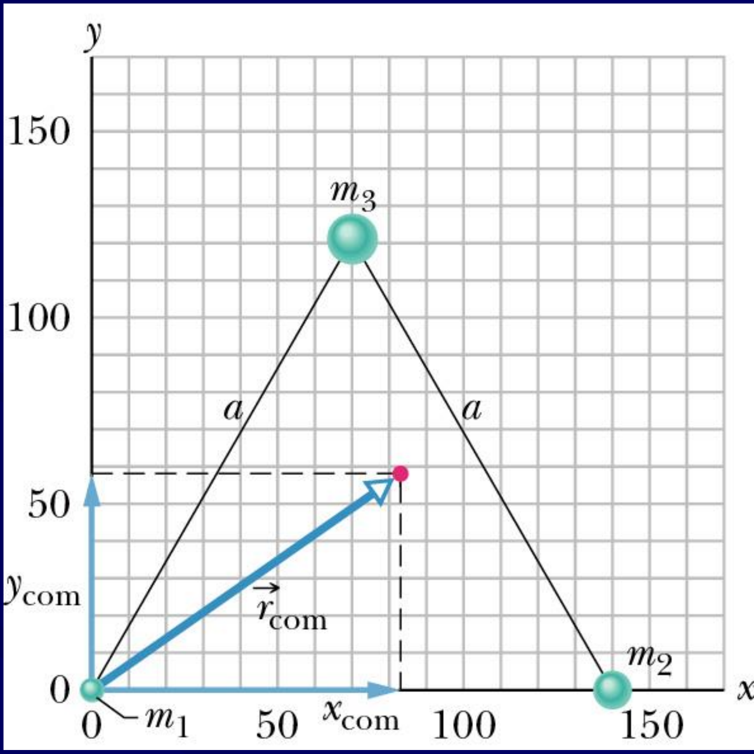
$$y_{\text{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$

$$z_{\text{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

مثال: سه ذره به جرمهای m_1=1kg و m_2=2kg و m_3=3kg در رئوس یک مثلث قرار دارند. مرکز جرم این مجموعه را بدست آورید.

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} =$$

$$\frac{(1kg)(0) + (2kg)(140cm) + (3kg)(70cm)}{(1+2+3)kg} = 81.7cm$$



$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$$

$$= \frac{(1kg)(0) + (2kg)(0) + (3kg)(120)}{(1+2+3)kg} = 60cm$$

- Noticing that x_{cm} and x_i, etc. are distances along the main axis of our coordinate system, we could just as easily switch to vector notation
- First recall that the position of mass m_i using vector notion would be:

$$\vec{r}_i = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$$

- Our center of mass equation using vector notation would therefore be:

$$\vec{r}_{\text{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

$$\vec{r}_{\mathrm{cm}} = \frac{1}{M}(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_n\vec{r}_n) = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

$$v_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{M}(m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + \ldots + m_n\frac{d\vec{r}_n}{dt})$$

$$\vec{v}_{cm} = \frac{1}{M}(m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n) = \frac{1}{M}\sum m_i\vec{v}_i$$

$$a_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{1}{M}(m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots + m_n\vec{a}_n) = \frac{1}{M}\sum m_i\vec{a}_i$$

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots + m_n\vec{a}_n$$

$$M\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n$$

$$\sum \vec{F}_{ext} = M\vec{a}_{cm}$$

$$\begin{cases} \sum F_{ext,x} = Ma_{cm.x} \\ \sum F_{ext,y} = Ma_{cm,y} \\ \sum F_{ext,z} = Ma_{cm,z} \end{cases}$$

- Remember that the CM is a point that acts as though all of the mass in the system were located there
- So even though we may have a large number of particles – possibly of different masses, we can treat the assembly as having all of it's mass at the point of it's CM
- So we can assign that point a position, a velocity and an acceleration

- $\sum F_{net}$ is the net of all of the *external* forces acting upon our system of particles; internal forces are not included (as they generally have no net effect)

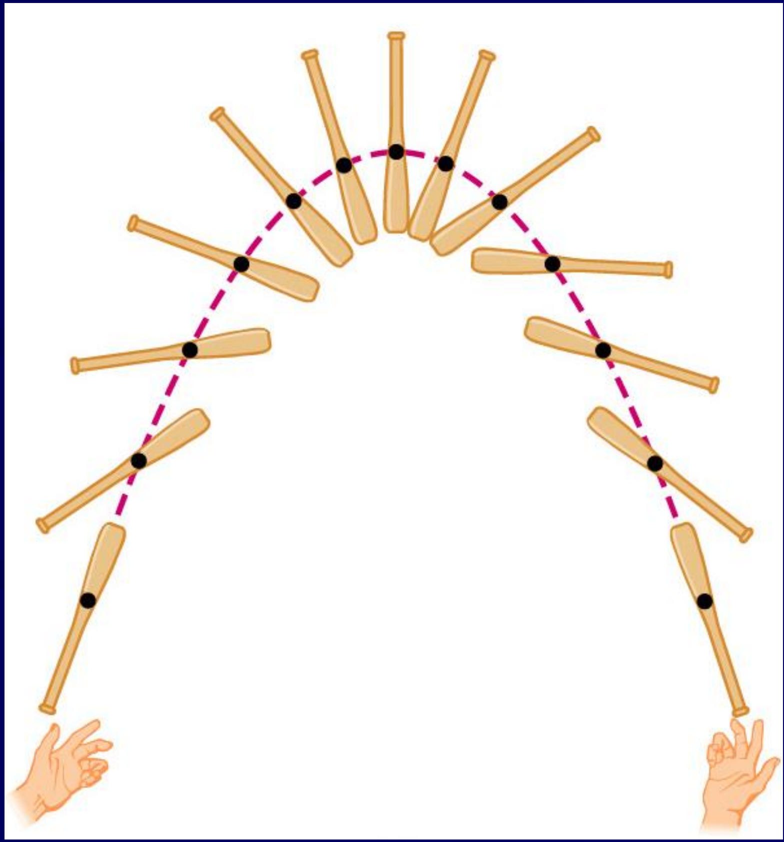
- M is the total mass of the system (and is assumed to be constant)

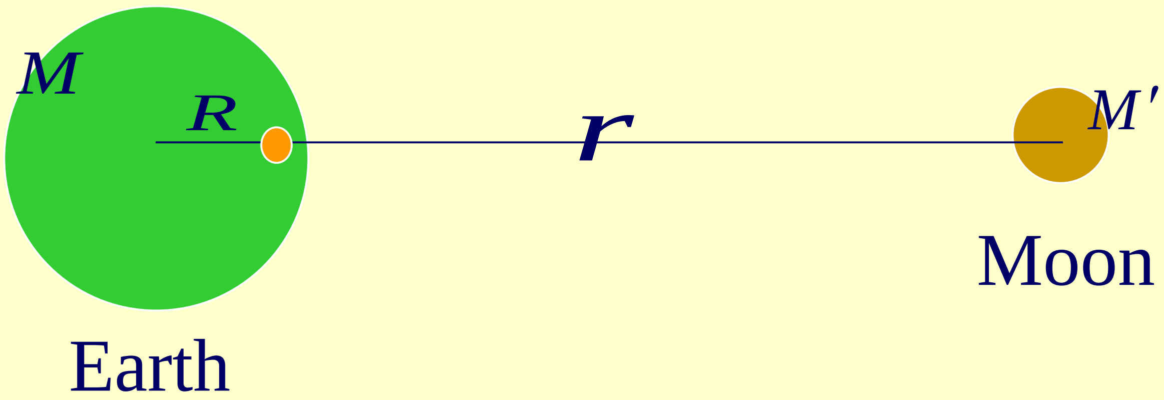
- a_{cm} is the acceleration of the center of mass; we can't say anything about what the acceleration might be of any other part of the system

- **Note however that the center of mass doesn't necessarily have to lie within the object or have any mass at that point:**
  - **The CM of a horseshoe is somewhere in the middle along the axis of symmetry.**
  - **The CM of a doughnut is at it's 'geographic' center, but there is no mass there either**

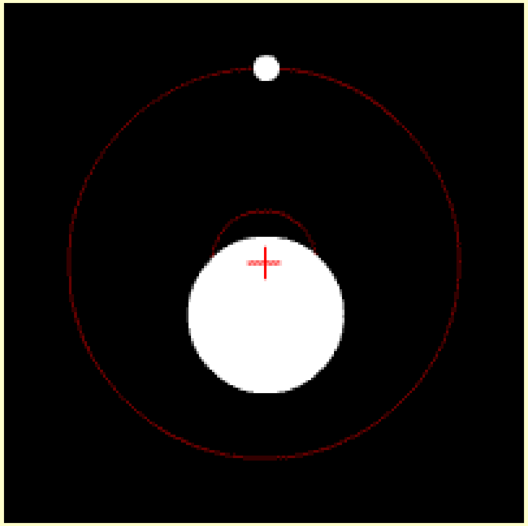
Center of Mass

- Going back to our baseball bat, the CM will lie along the central axis (the axis of symmetry)
- And it is the CM that faithfully follows the line of a parabola

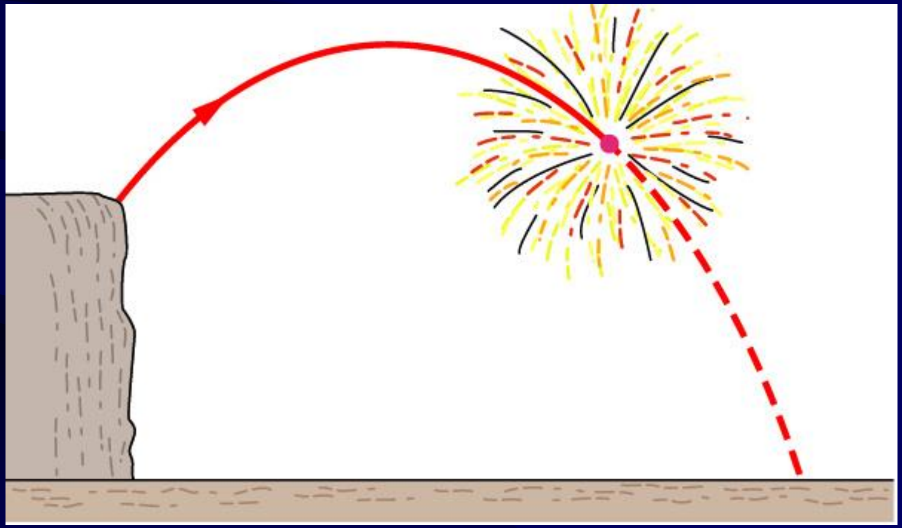




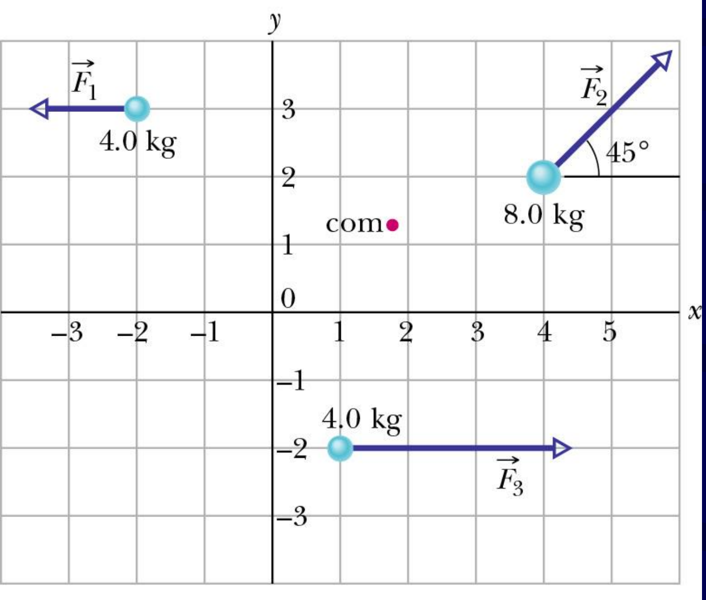
the center of mass of the Earth-moon system is about 1600 km below the surface of the Earth.



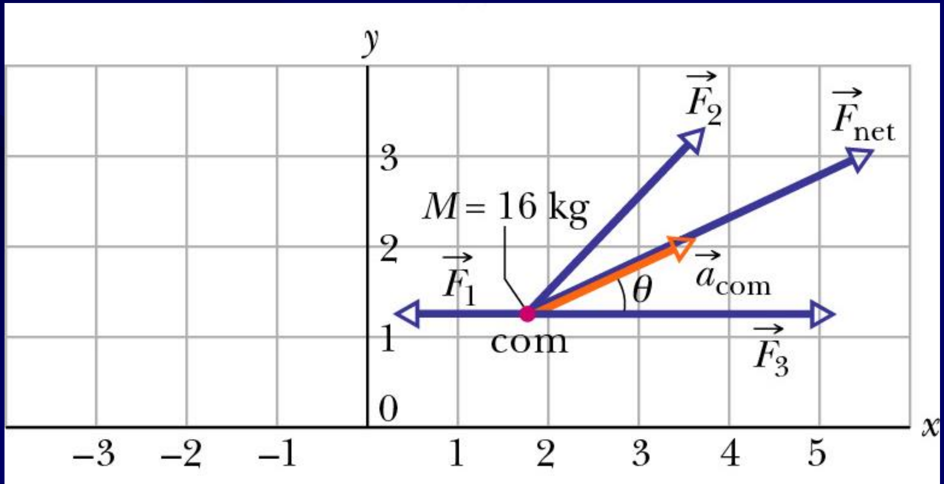
- When a fireworks rocket explodes, the CM of the system does not change; while the fragments all fan out, their CM continues to move along the original path of the rocket



- What is the acceleration of the CM and in what direction does it move?



- Solve for the net force; then use $\sum F_{net} = Ma_{com}$
- Assume that all of the mass is concentrated at the CM; e.g., $M = m_1 + m_2 + m_3$

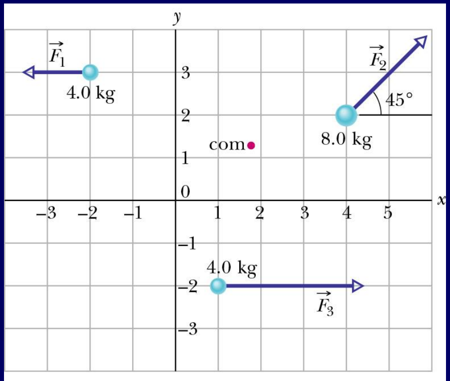


$$x_{cm} = \frac{1}{M}(m_1x_1 + m_2x_2 + m_3x_3)$$

$$= \frac{1}{16kg}\left[(4kg)(-2cm) + (8kg)(4cm) + (4kg)(1cm)\right] = 1.75cm$$

$$y_{cm} = \frac{1}{M}(m_1y_1 + m_2y_2 + m_3y_3)$$

$$= \frac{1}{16kg}\left[(4kg)(3cm) + (8kg)(2cm) + (4kg)(-2cm)\right] = 1.25cm$$



$$F_{ext,x} = F_{1x} + F_{2x} + F_{3x}$$

$$= -6N + (12N)(\cos 45^0) + 14N = 16.5N$$

$$F_{ext,y} = F_{1y} + F_{2y} + F_{3y}$$

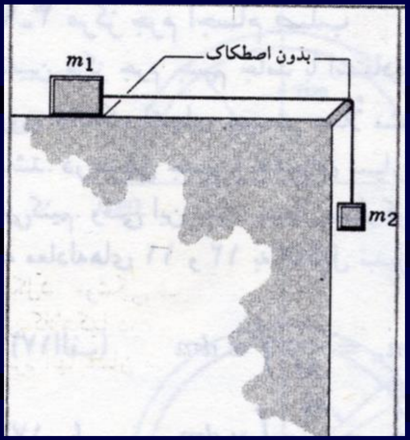
$$= 0 + (12N)(\sin 45^0) + 0 = 8.5N$$

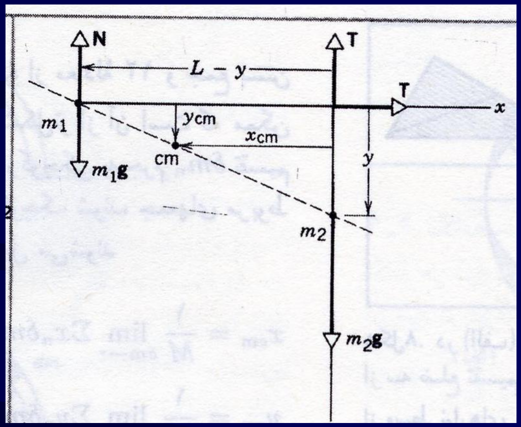
$$F_{net} = \sqrt{(F_{ext,x})^2 + (F_{ext.y})^2} = \sqrt{(16.5N)^2 + (8.5N)^2} = 18.6N$$

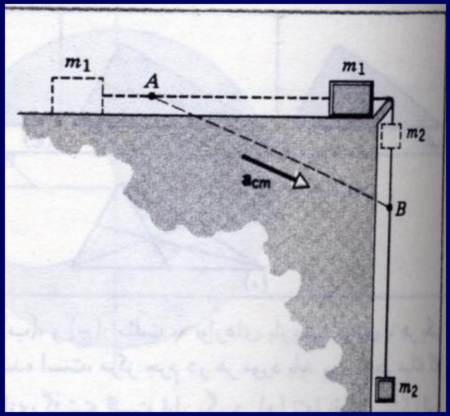
$$\phi = \tan^{-1}\frac{F_{ext,y}}{F_{ext,x}} = \tan^{-1}\frac{8.5N}{16.5N} = 27^0$$

$$a_{cm} = \frac{F_{ext}}{M} = \frac{18.6N}{16.4kg} = 1.1m/s^2$$

مثال:با در نظر گرفتن مرکز جرم سیستم دو ذره ای شتاب مشترک آنها را بدست آورید.



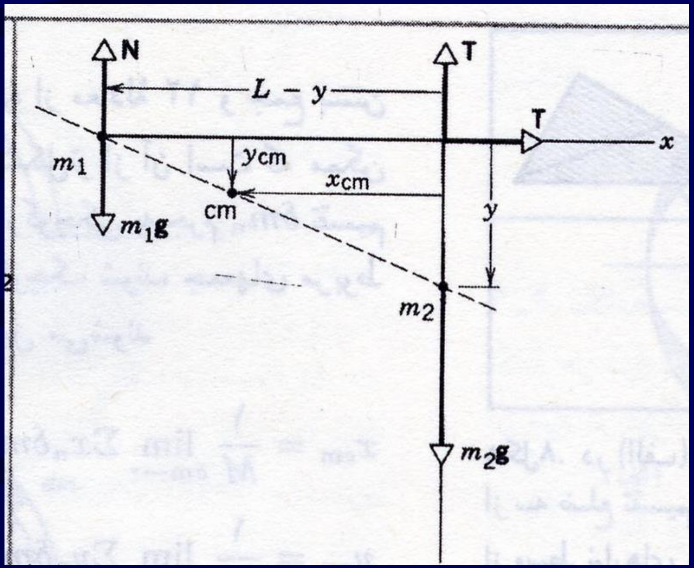




$$x_{cm} = -\frac{m_1}{M}(L-y) \quad , \quad y_{cm} = \frac{m_2}{M} y$$

$$v_{cm,x} = \frac{m_1}{M} v \quad , \quad v_{cm,y} = \frac{m_2}{M} v$$

$$a_{cm,x} = \frac{m_1}{M} a \quad , \quad a_{cm,y} = \frac{m_2}{M} a$$



x component $\quad T = Ma_{cm,x}$

y component $: m_1 g - N + m_2 g - T = Ma_{cm,y}$

$$m_1 g = N$$

$$a = \frac{m_2}{M} g$$

مرکز جرم اجسام صلب

- As the number of particles gets to be large (as it would be for everyday objects like a baseball bat or a fighter jet), the easiest thing to do is to treat the object as a continuous distribution of matter
- The ‘particles’ then become differential mass elements and the sums become integrals

This result is only for one dimension however, so the more generalized result for 3 dimensions is shown here:

$$x_{\mathrm{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

$$y_{\mathrm{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$

$$z_{\mathrm{cm}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

مرکز جرم اجسام صلب

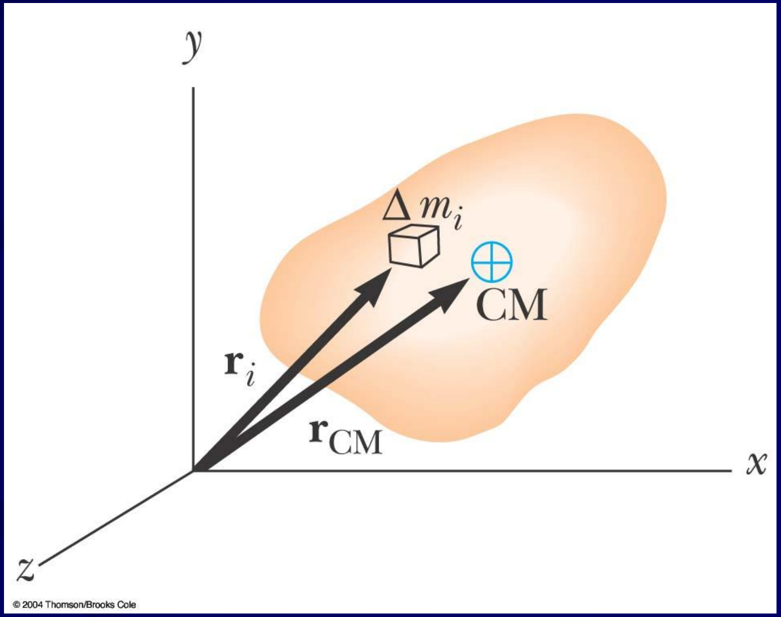
Given continuous matter, the location of the center of mass becomes:

$$x_{\text{cm}} = \frac{1}{M}\lim_{\delta m\to 0}\sum x_n\,\delta m_n = \frac{1}{M}\int x\,dm$$

$$y_{\text{cm}} = \frac{1}{M}\int y\,dm$$

$$z_{\text{cm}} = \frac{1}{M}\int z\,dm$$

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$$



Moving over to the integral form is nice, but now the problem is one of dealing with the non-uniformity of mass distribution in our everyday objects

For this course, we will assume *uniform* objects – objects with a uniform density

$$\rho = \frac{dm}{dV} = \frac{M}{V}$$

$$dm = \frac{M}{V} dV$$

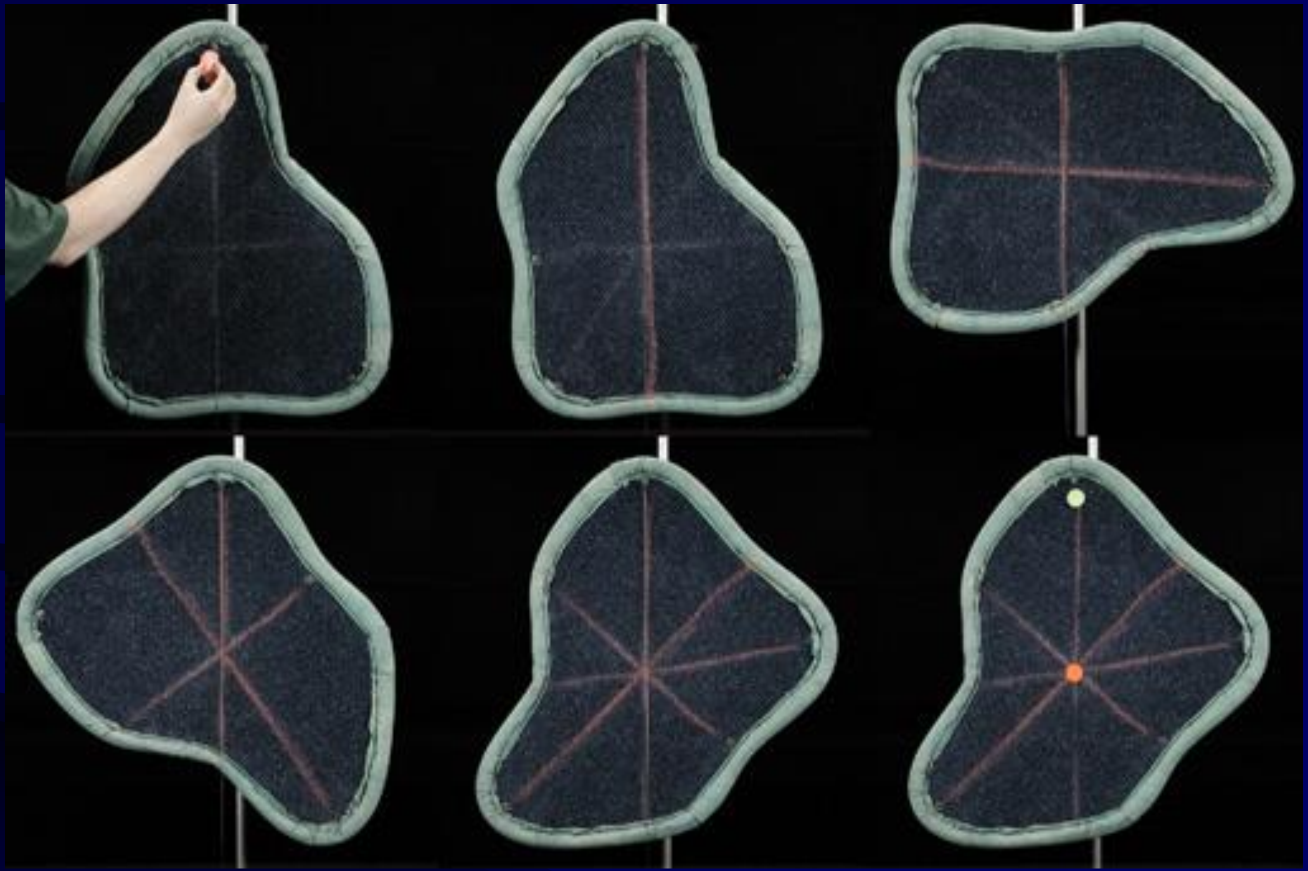
Center of Mass

- If we now substitute for *dm* in the previous integrals, we get:
- Now we are simply integrating over the volume of the object

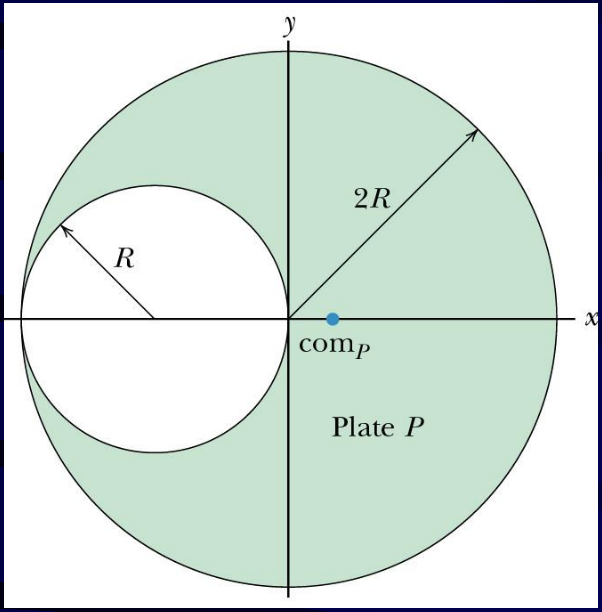
$$x_{\text{cm}} = \frac{1}{V}\int x\,dV$$

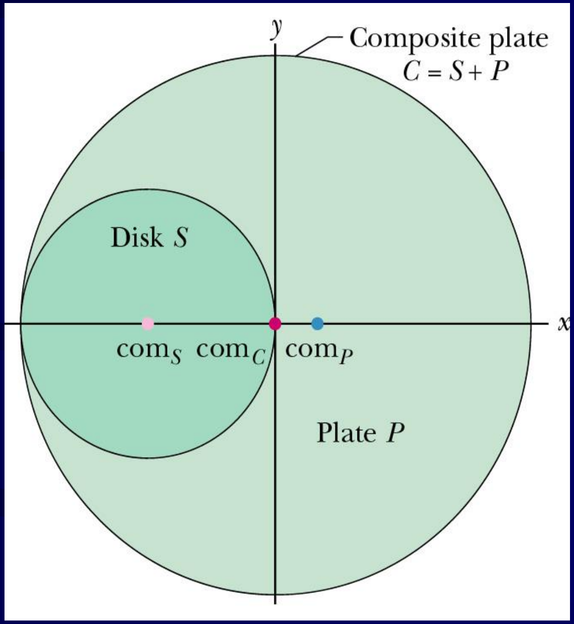
$$y_{\text{cm}} = \frac{1}{V}\int y\,dV$$

$$z_{\text{cm}} = \frac{1}{V}\int z\,dV$$

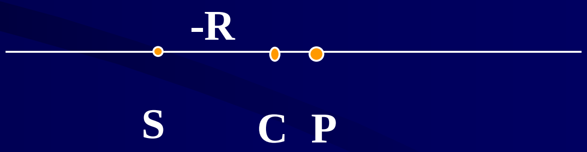


مثال: یک ورقه فلزی دایره ای به شعاع 2R که از آن قرصی به شعاع R درآورده شده است.محل دقیق مرکز جرم را پیدا کنید.





$$x_C = \frac{m_S x_S + m_P x_P}{m_S + m_P}$$

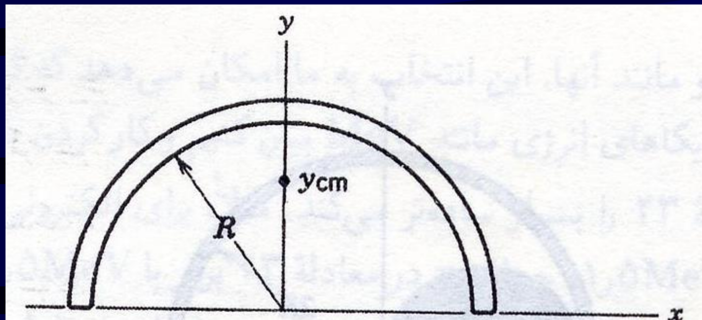


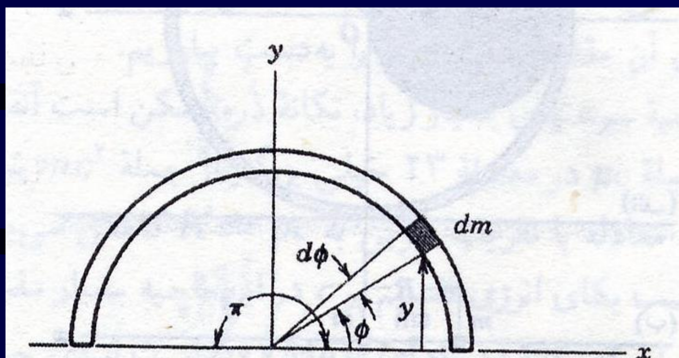
$$x_C = 0 \leftarrow x_P = -\frac{x_S m_S}{m_P}$$

$$x_S = -R \rightarrow x_P = \frac{1}{3}R$$

$$\frac{m_S}{m_P} = \frac{S\ \ area}{P\ \ area} = \frac{S\ \ area}{C\ \ area - S\ \ area} = \frac{\pi R^2}{\pi (2R)^2 - \pi R^2} = \frac{1}{3}$$

مثال: نوار نازکی را به صورت نیمدایره ای به شعاع R درآورده ایم مرکز جرم این جسم را بدست آورید.





$$\frac{dm}{M} = \frac{d\phi}{\pi} \rightarrow dm = (M/\pi)d\phi$$

$$y = R\sin\phi$$

$$y_{cm} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^{\pi}(R\sin\phi)\frac{M}{\pi}d\phi$$

$$= \frac{R}{\pi}\int_0^{\pi}\sin\phi\, d\phi = \frac{2R}{\pi} = 0.637R$$

Linear Momentum

The *linear momentum* of a particle is a vector defined as:

$$\vec{p} = m\vec{v}$$

- **Newton actually expressed his 2nd law of motion in terms of momentum:**
  - *The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force*

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Linear Momentum

- Because acceleration is the time derivative of velocity, we can also state the law as:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m\vec{v}\right) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

- These two ways of expressing the force on a particle are entirely equivalent

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_{net} = m\vec{a}$$

$$K = \frac{1}{2}mv^2 \quad , \quad P = mv$$

$$K = \frac{P^2}{2m}$$

Linear Momentum of a System of Particles

- Now suppose that we have a system of *n* particles, each with their own mass and velocity – and therefore momentum.
- The particles may interact with each other and there may be external forces acting on the system (e.g., the collection of particles)

- The system as a whole has a total linear momentum which is defined to be the vector sum of the momenta of the individual particles, thus:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_{\mathrm{n}}$$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_{\mathrm{n}}$$

$$\vec{v}_{cm} = \frac{1}{M}(m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n) = \frac{1}{M}\sum m_i\vec{v}_i$$

$$\vec{P} = M\vec{v}_{\mathrm{cm}}$$

$$\vec{P} = M\vec{v}_{\text{cm}}$$

- where M = the total mass and v_{com} is the velocity of the center of mass.

- *The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.*

If we take the time derivative of the previous equation, we get:

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{\mathrm{cm}}}{dt} = M\vec{a}_{\mathrm{cm}}$$

which of course leads us straight to:

$$\sum \vec{F}_{\mathrm{net}} = \frac{d\vec{P}}{dt}$$

Conservation of Linear Momentum

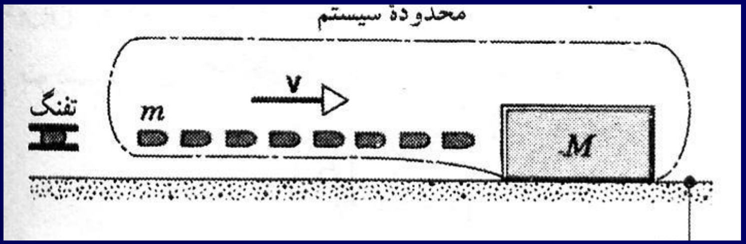
- Suppose we have a system that has no external forces acting upon it (the system is isolated) and no particles enter or leave the system (the system is closed)
- We then know that $\sum F_{net} = 0$ which in turn means that $dP/dt = 0$, or that:

P = constant (in a closed, isolated system)

- In other words:
  - *If no net force acts upon a system of particles, the total linear momentum $\sum P$ of the system cannot change*
- This very important result is called the *law of conservation of linear momentum*
- It can also be written as: $\sum P_{\mathrm{i}} = \sum P_{\mathrm{f}}$

- Because these are vector equations, we can derive a little more insight if we further examine what they mean along each dimension:
  - If the component of a net *external* force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change

مثال: رگباری از گلوله هایی به جرم m=3.8g به طور افقی با سرعت 1100m/s به قطعه چوب بزرگی به جرم M=12kg که در ابتدا روی سطح میزی افقی ساکن است، شلیک می شود. اگر قطعه چوب بتواند بدون اصطکاک روی سطح میز بلغزد سرعت آن پس از دریافت 8 گلوله چقدر است؟



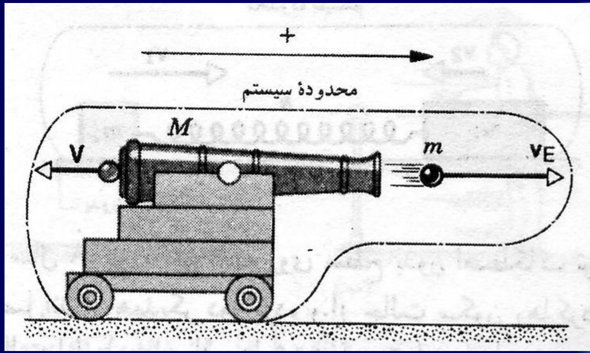
$$P_i = N(mv)$$

$$P_f = (M + Nm)V$$

$$P_i = P_f \rightarrow N(mv) = (M + Nm)V$$

$$V = \frac{Nm}{M + Nm} v = 2.8 m/s$$

مثال:توپی به جرم M گلوله ای 72 کیلوگرمی را در راستای افقی با سرعت $v=55m/s$ شلیک می کند. توپ می تواند آزادانه پس بزند.(الف) سرعت پس زنی توپ (V) را نسبت به زمین پیدا کنید.
(ب) سرعت اولیه گلوله توپ(v_E) را نسبت به زمین پیدا کنید.



$$P_i = 0$$

$$P_f = MV + m(v+V)$$

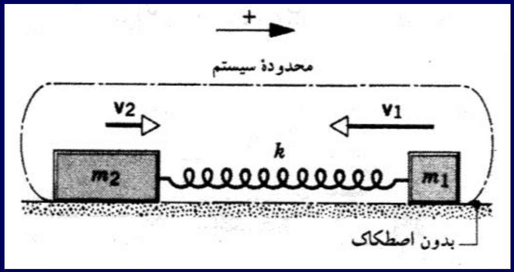
$$P_i = P_f$$

$$0 = MV + m(v+V)$$

$$V = -\frac{mv}{M+m} = -\frac{(72kg)(55m/s)}{1300kg + 72kg} = -2.9m/s$$

$$v_E = v + V = 55m/s + (-2.9m/s) = 52m/s$$

مثال: دو جسم را که توسط فنری به هم متصل شده اند می توانند آزادانه روی سطح افقی بدون اصطکاکی بلغزند.دو جسم را که جرم آنها برابر m_1 و m_2 است از هم دیگردور می کنیم و سپس از حال سکون رها می کنیم. در زمانهای بعدی هر کدام از دو جسم حامل چه کسری از انرژی جنبسی کل سیستم خواهند بود؟



$$P_i = 0$$

$$P_f = m_1 v_1 + m_2 v_2$$

$$0 = m_1 v_1 + m_2 v_2$$

$$\frac{v_1}{v_2} = -\frac{m_2}{m_1}$$

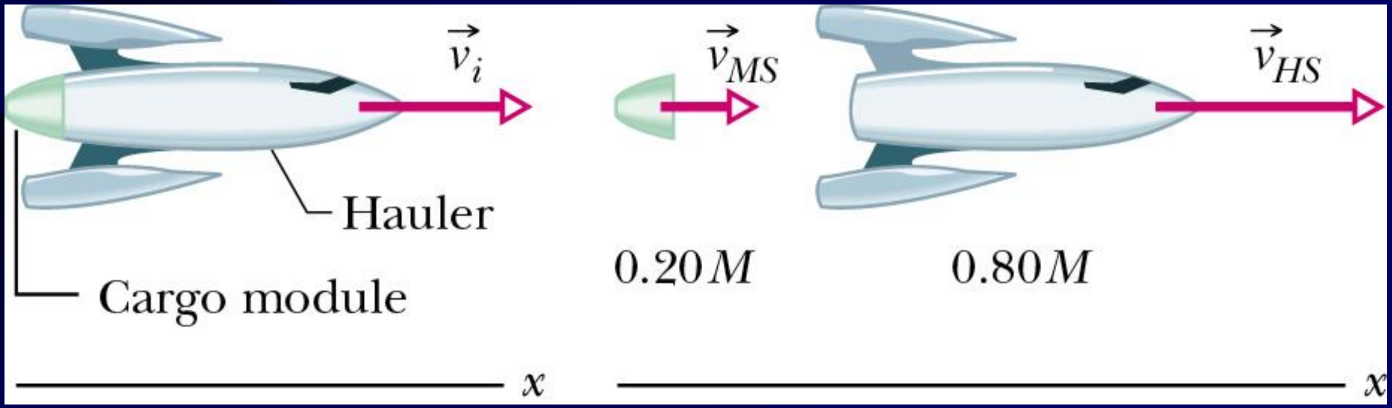
$$K_1 = \frac{1}{2} m_1 v_1{}^2 \quad , \quad K_2 = \frac{1}{2} m_2 v_2{}^2$$

$$f1 = \frac{K_1}{K_1 + K_2} = \frac{\frac{1}{2} m_1 v_1{}^2}{\frac{1}{2} m_1 v_1{}^2 + \frac{1}{2} m_2 v_2{}^2}$$

$$f_1 = \frac{m_2}{m_1 + m_2} \qquad f_2 = \frac{m_1}{m_1 + m_2}$$

$$if \quad m_2 = 10 m_1 \quad , \quad f1 = 0.91 \quad , \quad f2 = 0.09$$

- A space hauler with an initial velocity v_i = 2100 km/hr separates from it's cargo container
- The cargo module has a mass of 0.2*M* where *M* is the initial mass of the hauler plus cargo module
- After separation, the hauler has a velocity of +500 km/hr relative to the cargo module
- What is the velocity of the hauler relative to the sun?

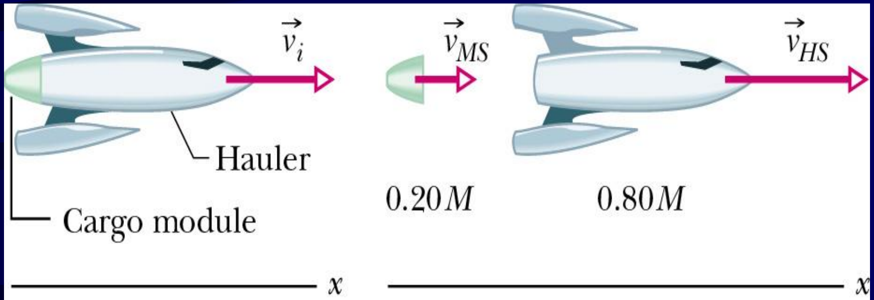


- We will assume that the system consists of the hauler & cargo module – and is closed
- Because it is closed, we know that conservation of momentum will hold and that $\sum P_i = \sum P_f$

$$P_i = Mv_i$$

where *vi* is the velocity of the combined hauler & module relative to the sun

Let v_{MS} and v_{HS} be the velocities of the ejected cargo module and the hauler respectively, relative to the sun



- After the cargo module has been ejected, the total linear momentum of the system is:

$$P_f = (0.20M)v_{MS} + (0.80M)v_{HS}$$

- We don't know what v_{MS} is, but we can relate it to other factors as follows:

$$v_{HS} = v_{rel} + v_{MS}$$

$$v_{MS} = v_{HS} - v_{rel}$$

We can now substitute v_{MS} back into our equation for the final linear momentum:

$$P_f = 0.20M\left(v_{HS} - v_{rel}\right) + 0.80Mv_{HS}$$

Rearranging a little gives us:

$$Mv_i = M\left(v_{HS} - 0.20Mv_{rel}\right)$$

$$v_{HS} = v_i + 0.20v_{rel}$$

Finally, we can substitute in the known values and solve for v_{HS} to get:

$$v_{HS} = 2100 \text{ km/hr} + (0.20)(500 \text{ km/hr})$$

$$v_{HS} = 2200 \text{ km/hr}$$


از توجه
شما
متشکرم