Physics 1

Lecture 18

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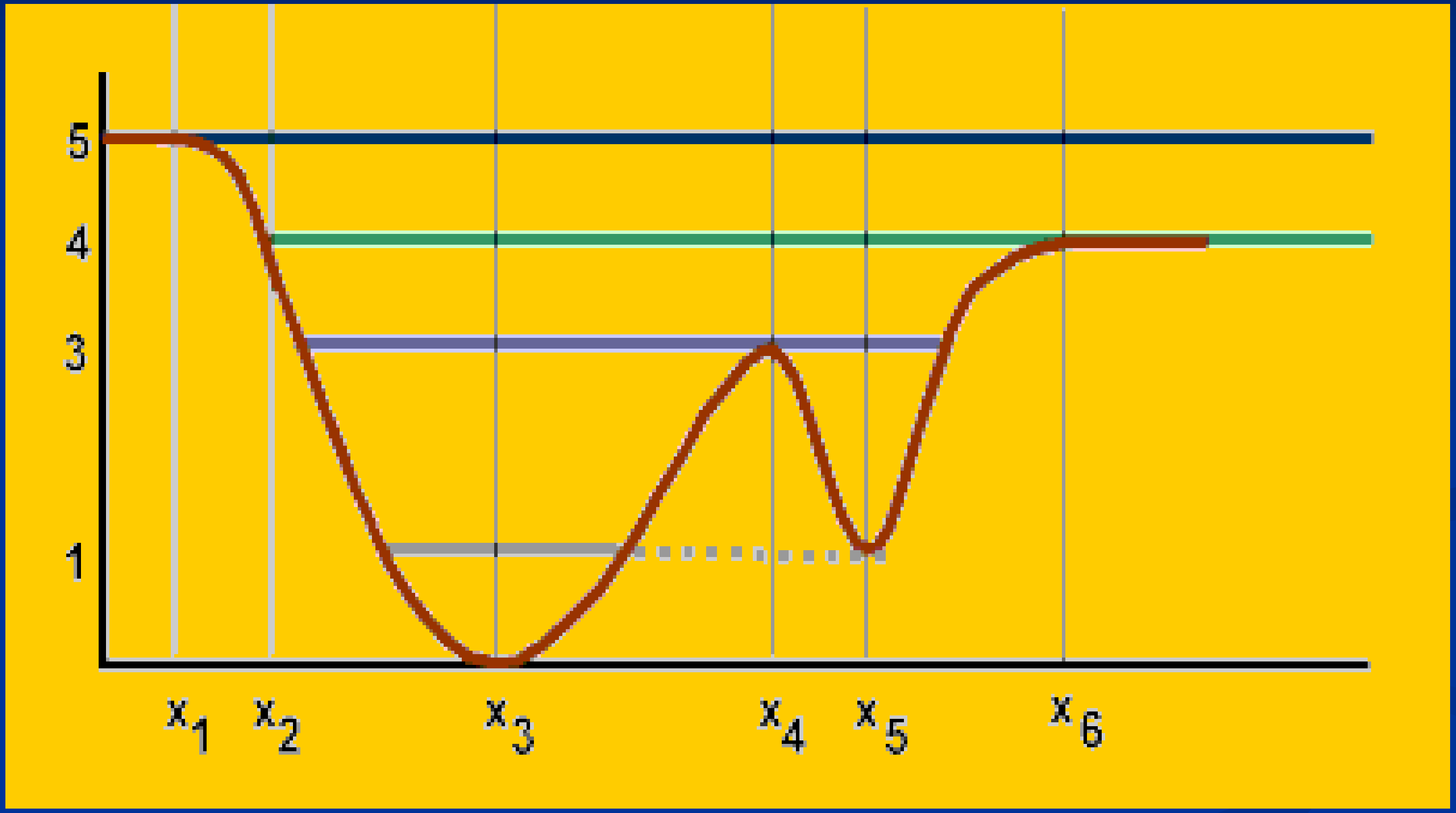
One dimension Conservative systems

$$\frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv_0{}^2 + U(x_0) = E$$

$$U(x) + \frac{1}{2}mv^2 = E$$

$$v = \frac{dx}{dt} = \pm\sqrt{\frac{2}{m}\left[E - U(x)\right]}$$

potential energy function. Remember that in the absence of dissipative or non-conservative forces, the total energy is conserved. This can be expressed as U(x) + K(x) = Etotal



In the region between 0 and ⟶ **5 J of potential energy**

At x_3 the mass would have converted all of its energy into kinetic energy

Now consider the green line representing E total equal to only 4 J

We call x_2 a turning point. From x_6 onward, the mass possesses exactly 4 J of potential energy and therefore has no kinetic energy Hence no forces would be acting on it to cause any acceleration. We say that the mass is in a state of neutral equilibrium.

Next consider the blue line representing Etotal equal to only 3 J

there are now two turning points: (1) one is just to the right of x_2, and (2) the other is between x_5 and x_6, showing us that the mass can no longer reach x_6.

At the point x_4 the mass possesses exactly 3 J of potential energy and therefore has no instantaneous kinetic energy. This represents a state of unstable equilbrium

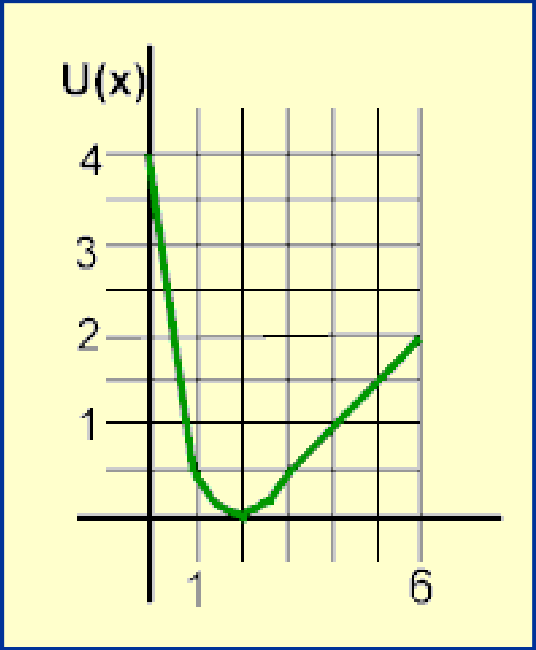
since the negative derivative of the potential energy function tells us that a positive non-zero force acts on the mass to either side of this position. Any slight adjustment to the left or right will cause the particle accelerate and gain KE.

Finally consider the light grey line representing Etotal equal to only 1 J.

The point x_5 represents a position of stable equilibrium. Since its potential energy equals the total energy available, the mass while at x_5 is trapped - it cannot move, climb, to either side where $U(x) > 1$ because that would represent acquiring unavailable negative kinetic energy

That is, an expression for a conservative, one-dimensional force with respect to position, x, can be calculated as the negative derivation of the system's potential energy function with respect to x.

$$F(x) = -d/dx\,[U(x)]$$



segment | **slope** | **force**

$0 \le x < 1$ | -3.5 | $+3.5$ N

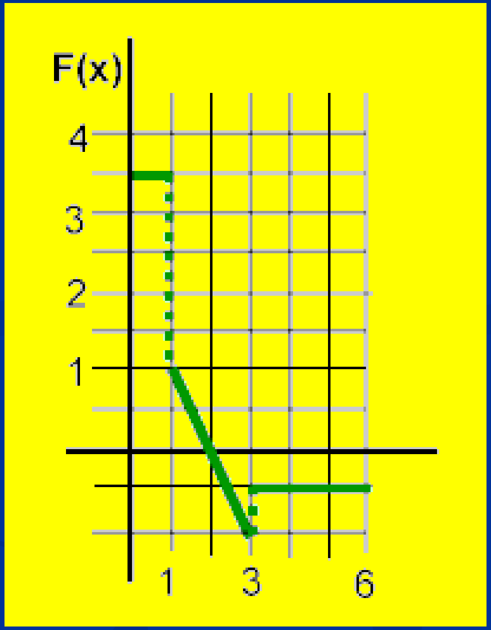
$$U(x) = 4 - 3.5x$$

$1 < x < 3$ | $2-x$ | $+1 > F > -1$

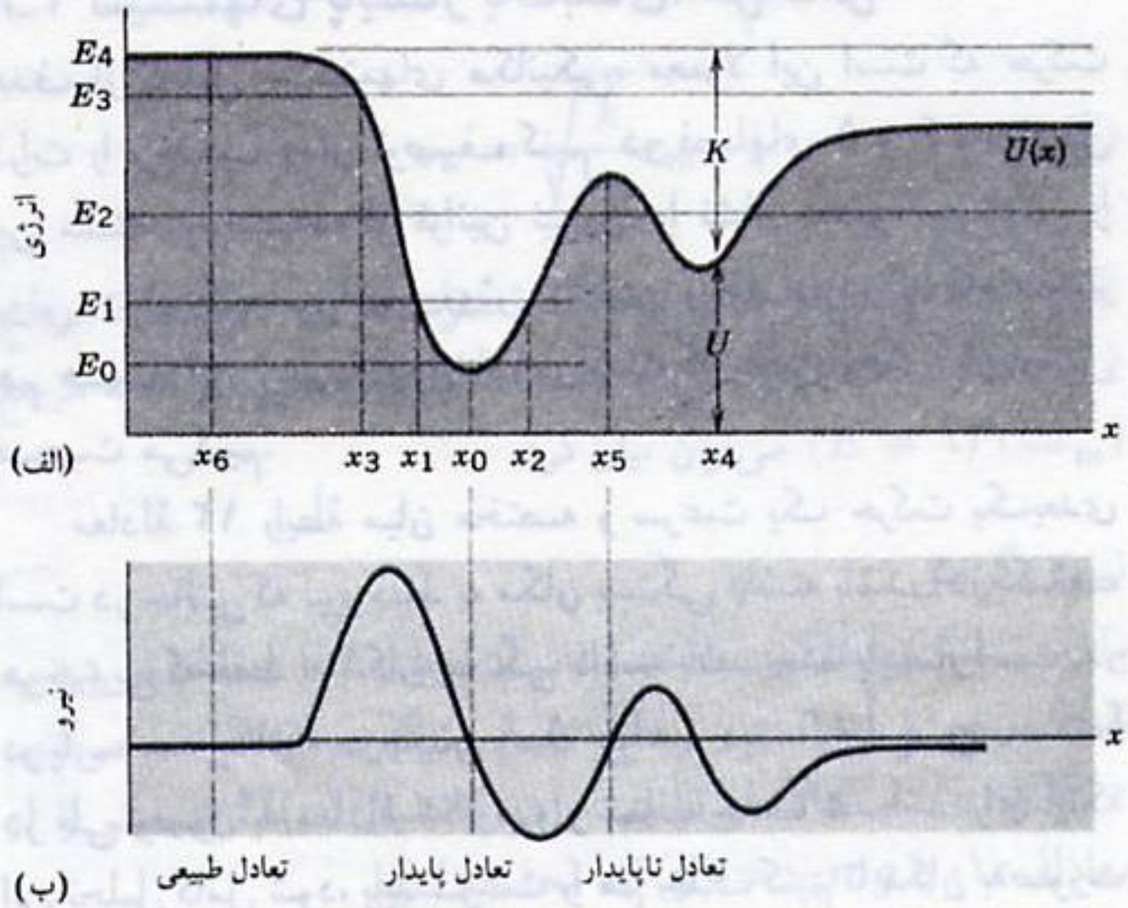
$$U(x) = \tfrac{1}{2}(x - 2)^2$$

$3 < x \le 6$ | 0.5 | -0.5 N

$$U(x) = \tfrac{1}{2}x - 1$$



Potential Energy Curve



شکل۹. (الف) یک تابع انرژی پتانسیل $U(x)$. (ب) نیروی متناظر با این انرژی پتانسیل.

$$v = \pm\sqrt{\frac{2}{m}\left[E - U(x)\right]}$$

$$F(x) = -\frac{dU(x)}{dx}$$

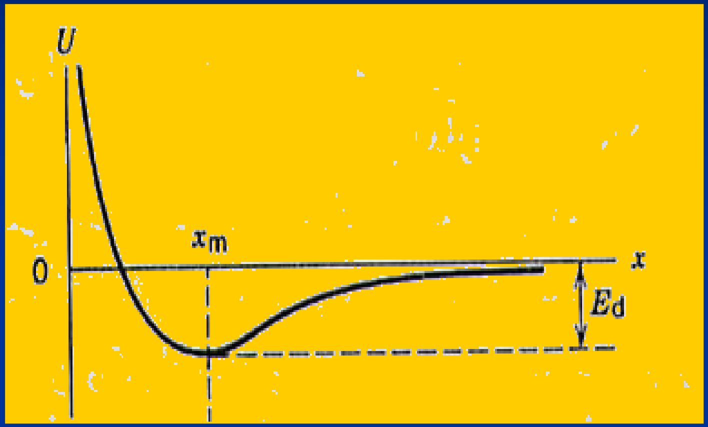
A minimum in potential energy is a *stable* equilibrium

The turning points are where potential energy equals mechanical energy.

A maxiumm in potential energy is an *unstable* equilibrium

مثال: تابع انرژی پتانسیل نیروی بین اتمهای یک ملکول دو اتمی را چنین می توان نوشت.

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^{6}}$$



a و b دو ثابت مثبت اند و x فاصله بین دو اتم است.

(الف) فاصله دو اتم در حالت تعادل چقدر است؟

تعادل در نقطه x_m حاصل می شود که در آن U(x) کمینه می شود. این نقطه از رابطه زیر بدست می آید.

$$\left(\frac{dU}{dx}\right)_{x=x_m} = 0$$

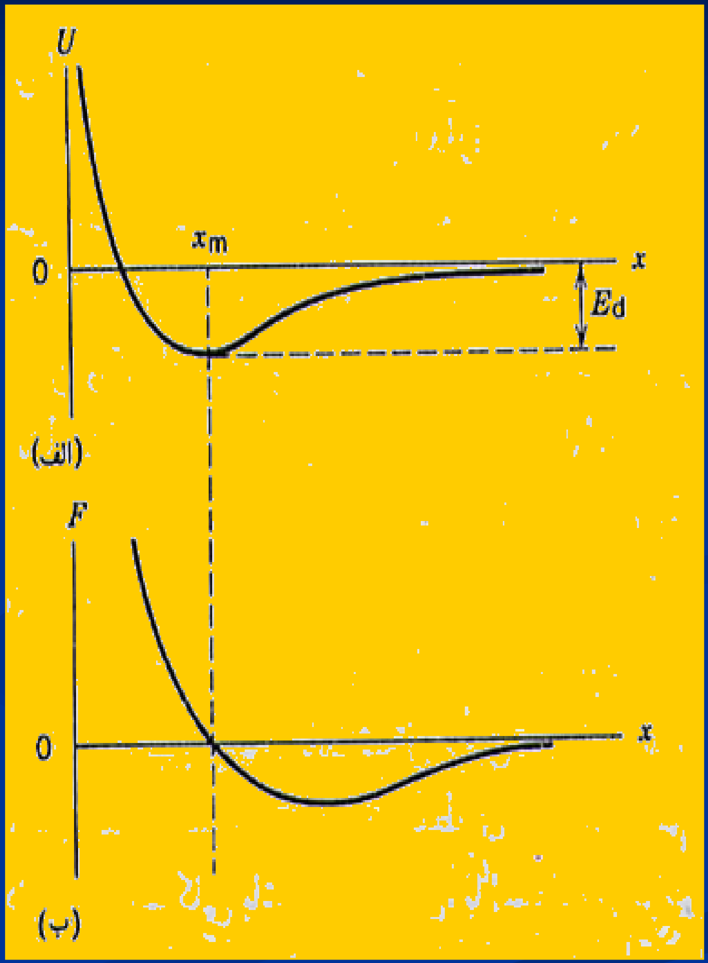
$$x_m = \left(\frac{2a}{b}\right)^{1/6}$$

$$\frac{-12a}{x_m^{\ 13}} + \frac{6b}{x_m^{\ 7}} = 0$$

ب) نیروی بین دو اتم چقدر است؟

$$F(x) = -\frac{dU(x)}{dx}$$

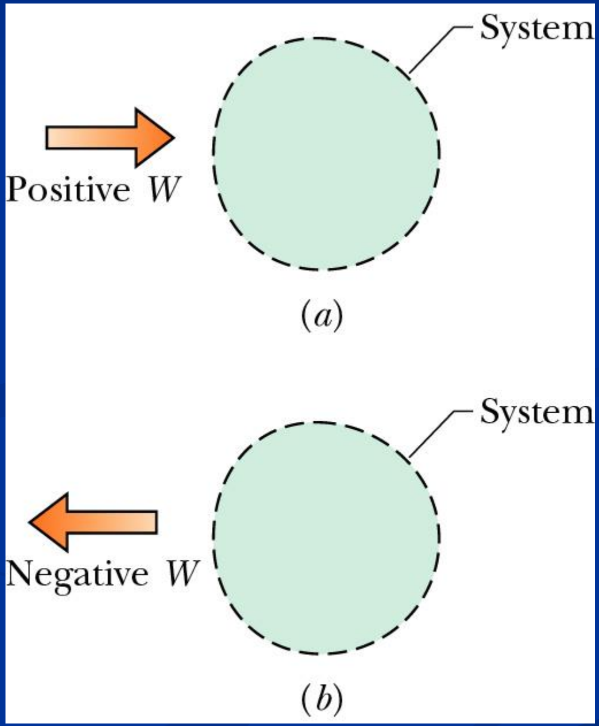
$$F(x) = \frac{12a}{x^{13}} - \frac{6b}{x^{7}}$$



Work Done on a System By An External Force

- So far we have defined work as the energy transferred to or from an object by means of a force acting on that object.
- We will now extend the definition to a system of particles.
- *Work is energy transferred to or from a system by an external force acting on that system*

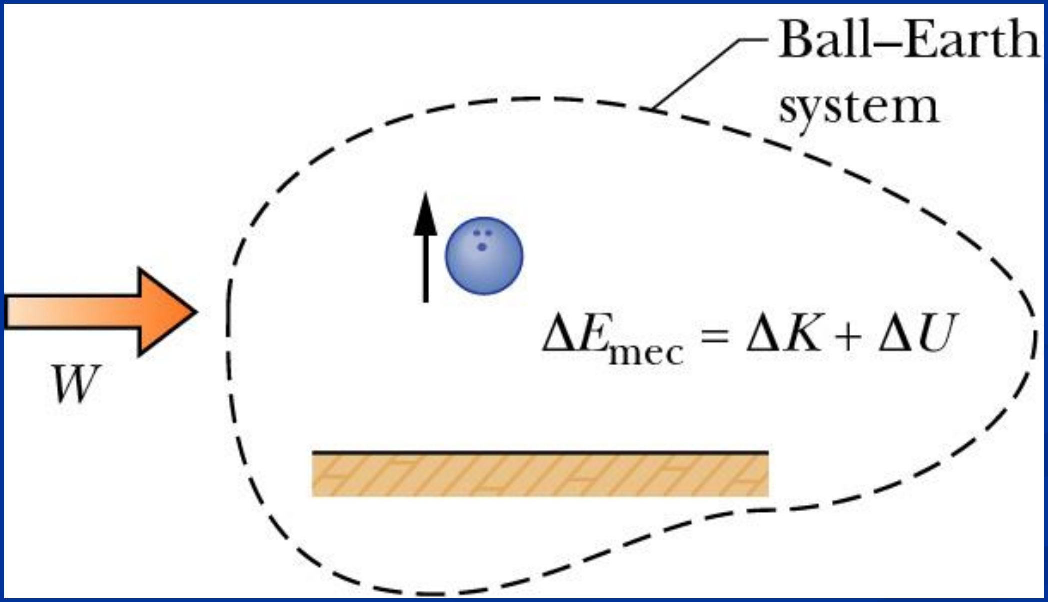
- In (a) energy is transferred to the system – thus the work done is positive
- In (b) energy is transferred from the system – thus the work done is negative



- If you toss a bowling ball up in the air (ignoring air friction) you have obviously done some work – but what is the “system” that you did the work on?
- Ask yourself – where did the energy change?
- You clearly changed the KE of the ball
- And because the separation of the ball and the earth increased, you also changed the PE of the ball-earth system
- So the “system” must include both the ball and the earth

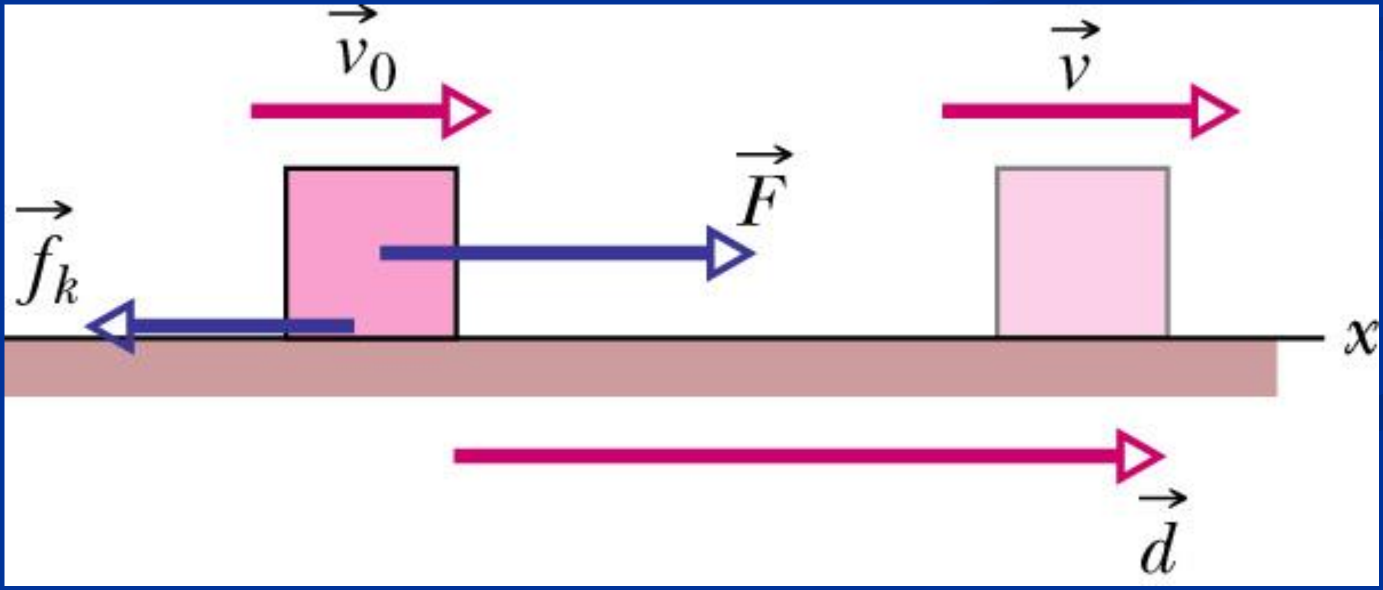
- The work must include both changes in energy, thus:

$$W = \Delta K + \Delta U \quad \text{or} \quad \mathrm{W} = \Delta E_{\mathrm{mch}}$$



Work Done By An External Force (Friction Involved)

- **We have a box being propelled by a constant force *F*, with a retarding frictional force f_k**
- **The box starts with an initial velocity v_0, and travels distance *d* ending with a final velocity *v***



- We begin by applying Newton's 2nd law to get:

$$F - f_\mathrm{k} = ma$$

- Because the forces are all constant, so is the acceleration *a*; as a result we can use the equation:

$$v^2 = v_0^2 + 2ad$$

$$Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + f_\mathrm{k}d$$

$$Fd = \Delta K + f_\mathrm{k}d$$

- As posed, the problem is 1-dimensional
- But suppose the block were being pulled up a ramp – in that case there would also be a ΔU to contend with
- So to make the solution a little more general we will change the ΔK term to ΔE_{mec}

$$Fd = \Delta E_{\text{mec}} + f_{\text{k}} d$$

By experimentation, we find that that as the block slides along, the block and the floor get warmer due to the friction
As you will find out next term, the temperature of the block is related to an object's thermal energy

Again by experimentation, we have found that the thermal energy ΔE_{th} is:

$$\Delta E_{\text{th}} = f_{\text{k}} d$$

So we can rewrite our earlier equation as:

$$Fd = \Delta E_{\text{mec}} + \Delta E_{\text{th}}$$

Recall that $W = Fd$, so we finally end up with:

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}$$

This is the energy statement for the work done on a system by an external force when friction is involved…

Conservation of Energy

- As we have progressed through this lecture, we have assumed (correctly) that energy cannot magically appear or disappear
- More formally, we have assumed that energy obeys a law called *the law of conservation of energy* which is concerned with the **total energy** E of a system

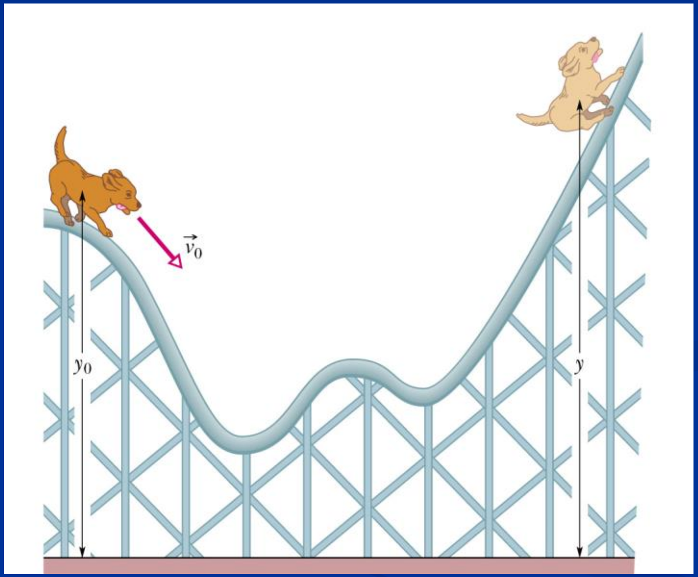
Conservation of Energy

- This total includes:
  - Mechanical energy (kinetic & potential),
  - Thermal energy,
  - and any other form of internal energy in addition to thermal energy
- The law states that:
  - *The total energy E of a system can change only by the amounts of energy that are transferred to or from the system*

Conservation of Energy

- If you have an isolated system, then the law of conservation of energy states that:
  - *The total energy of an isolated system cannot change*

- **A circus beagle of mass *m* = 6.0 kg runs onto a curved ramp with a speed of v_0 = 7.8 m/s at a height of y_0 = 8.8 m above the floor**
- **It slides along the ramp to the right and eventually (momentarily) comes to a stop 11.1 m above the floor**
- **The ramp is not frictionless**
- **What is the increase in thermal energy E_{th} in the beagle-ramp system because of the sliding?**



- **Let's start by looking at the forces involved;**
  - **The normal force on the beagle from the ramp does no work as it is always perpendicular to the beagle's displacement**
  - **The gravitational force is clearly doing work as the height of the beagle changes as it slides along the ramp**
  - **And finally, because the ramp is not frictionless there is an increase in thermal energy in both the beagle and the ramp**

- The system includes the ramp, the beagle and the earth
- We can take this system to be isolated (meaning that there are no other forces involved)
- Therefore we know that the total energy of the system cannot change

$$\Delta E = \Delta E_{mch} + \Delta E_{th} = 0$$

- We also know that the change in mechanical energy of the system (ΔE_{mch}) is the sum of the change in the kinetic energy and the change in the potential energy:

$$\Delta K = 0 - \frac{1}{2} m v_0^2$$

$$\Delta U = mgy - mgy_0$$

$$\Delta E_{th} = \frac{1}{2} m v_0^2 - mg(y - y_0)$$

$$\Delta E_{th} \approx 30 \text{ J}$$