Physics 1

Lecture 18

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Review of Previous Lecture

- **Energy**
- **Work**
- **Work & Kinetic Energy**
- **Work Done by a Gravitational Force**
- **Work Done by a Spring Force**
- **Work – Energy Theorem**
- **Power**

Types of Forces

There are two general kinds of forces

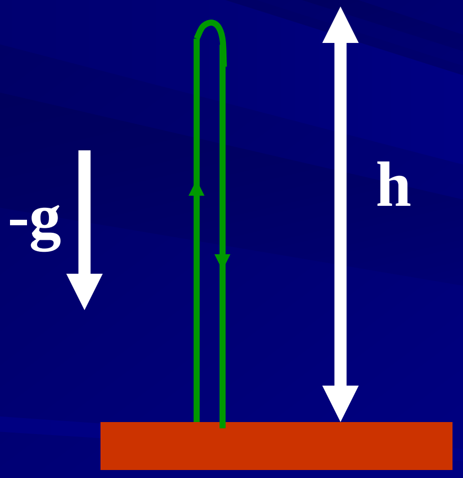
Conservative

Nonconservative

Conservative Forces

A force is conservative if the work it does on a particle that moves through a round trip is zero; otherwise the force is non-conservative

Consider throwing a mass up a height h



work done by gravity for round trip:

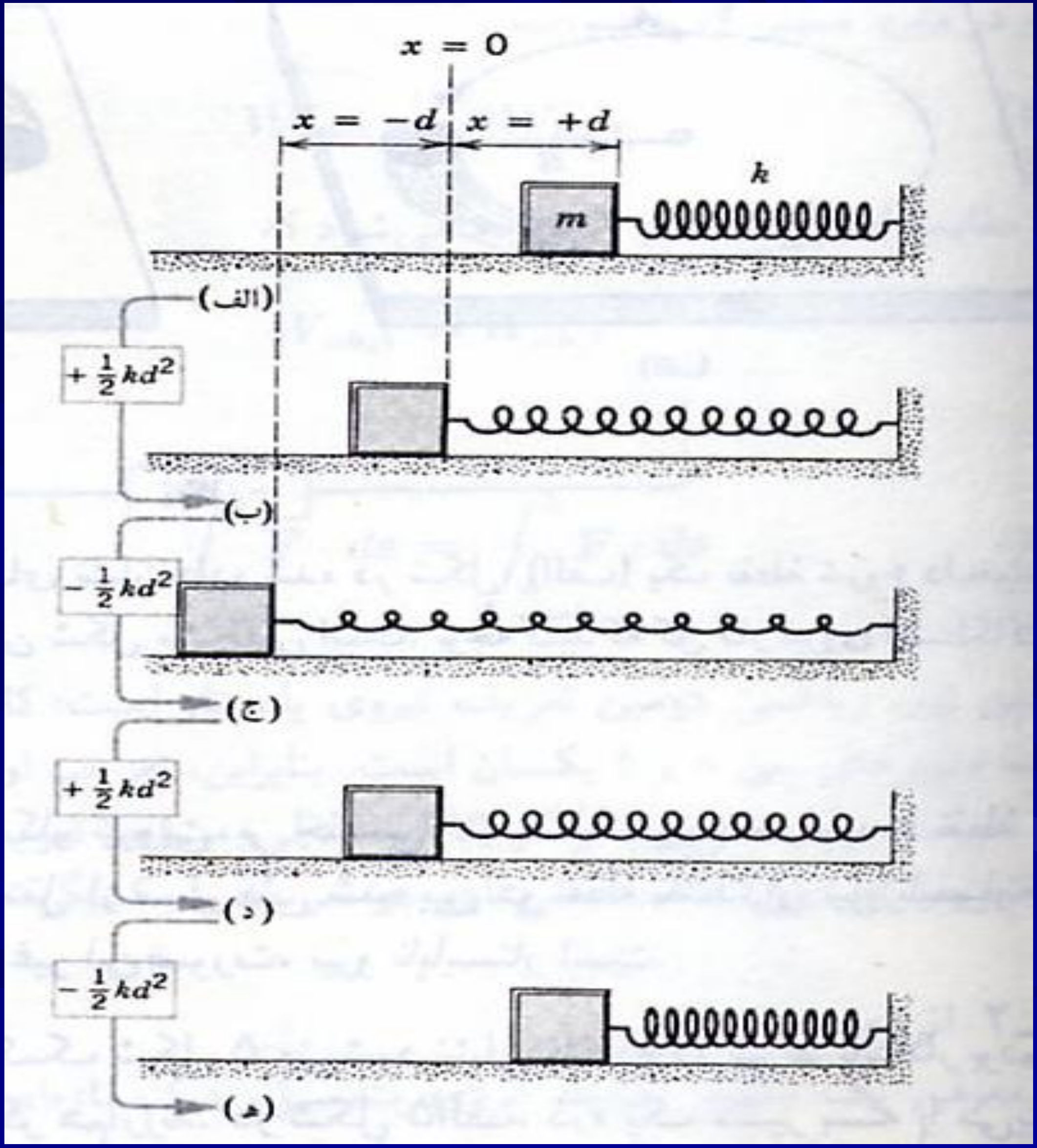
On way up: work done by gravity = -mgh

On way down: work done by gravity = mgh

Total work done = 0

Sometimes written as $\oint \mathbf{F}.d\mathbf{s} = 0$

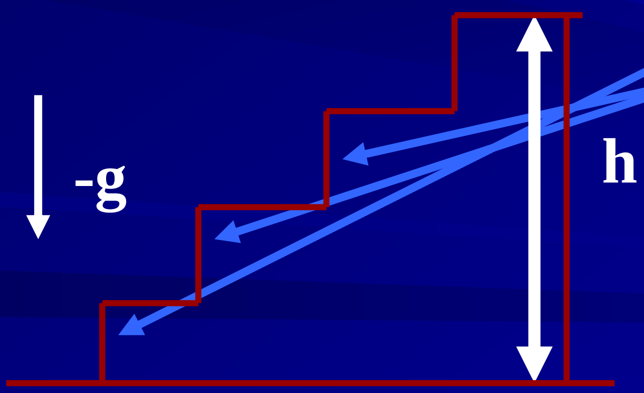
Conservative Forces



Conservative Forces

A force is conservative if the work done by it on a particle that moves between two points is the same for all paths connecting these points: otherwise the force is non-conservative.

Each step height=Δh



Work done by gravity

$$w = -mg\Delta h_1 + -mg\Delta h_2 + -mg\Delta h_3 + \ldots$$

$$= -mg(\Delta h_1 + \Delta h_2 + \Delta h_3 + \ldots\ldots)$$

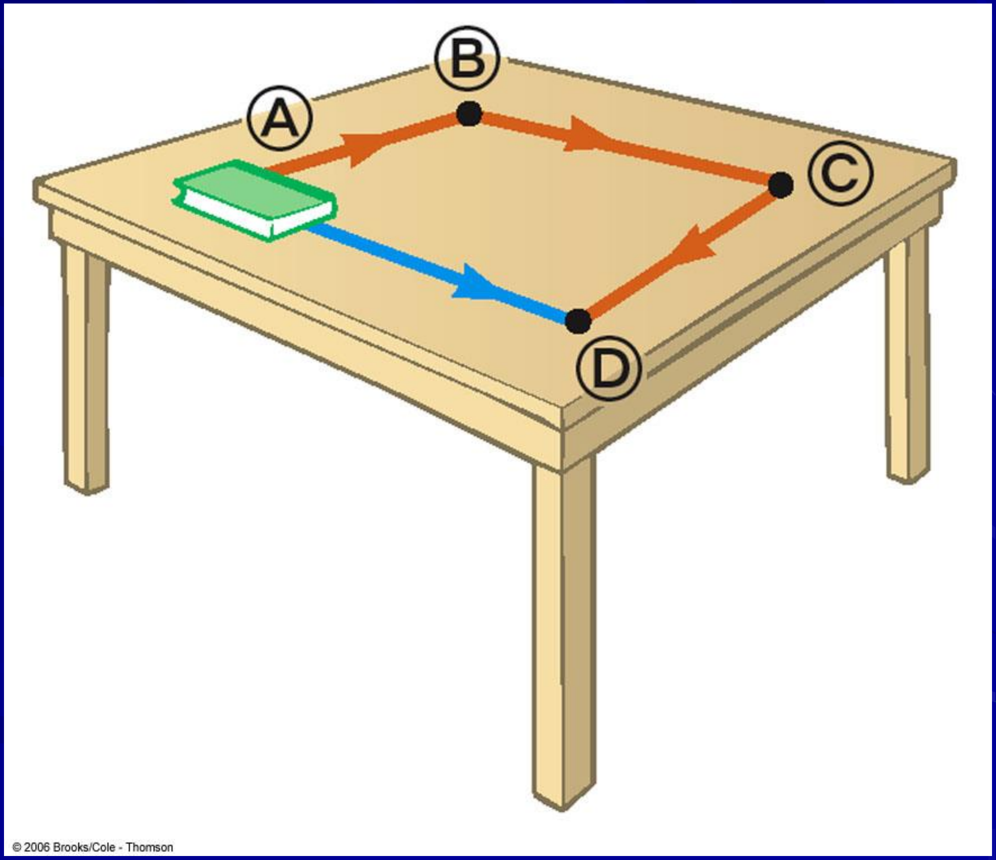
$$= -mgh$$

Same as direct path (-mgh)

$$W_1 = W_2$$

Friction Depends on the Path

- The blue path is shorter than the red path
- The work required is less on the blue path than on the red path
- Friction depends on the path and so is a non-conservative force



Conservative and Nonconservative Forces

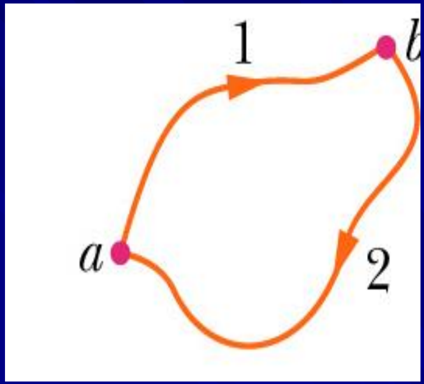
- **We know that as we slide a block along the floor, the floor and the block have friction**
- **The kinetic frictional force does negative work on the block (slowing it down) - this negative work is transferred into heat (thermal energy)**
- **We also know that this energy transfer can't be reserved (e.g., warming up the floor-block interface won't cause the block to start moving)**
- **From this we can conclude that thermal energy (heat) is not a potential energy**

Non-conservative forces

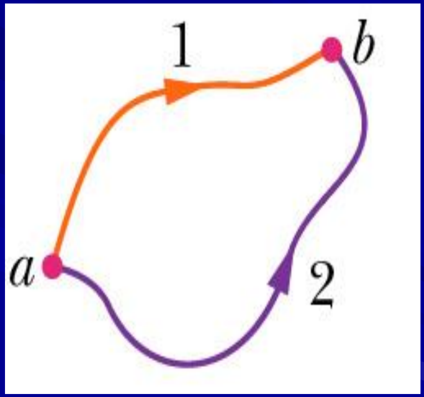
- When moving a mass in a gravitational field, the amount of work done by gravity is independent of the path taken.
- The same is not true of friction as it always opposes the direction of motion.
- Whereas gravity can do positive and negative work on an object, friction only does negative.

Path Independence of Conservative Forces

If the force is conservative, then
$W_{ab,1} + W_{ba,2} = 0$ and thus $W_{ab,1} = -W_{ba,2}$



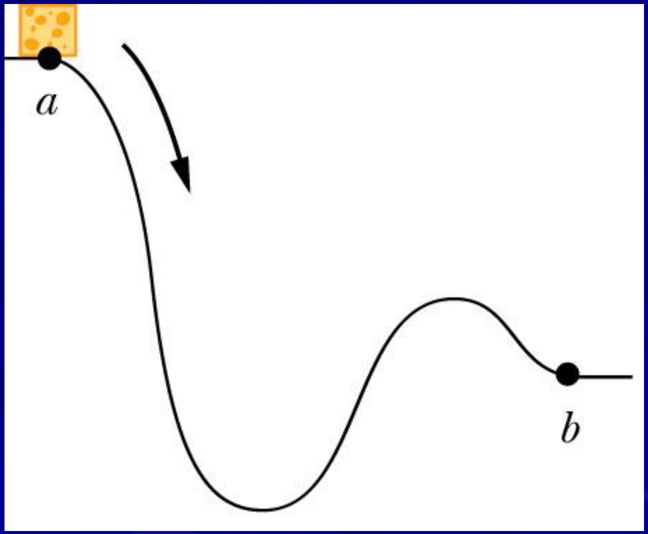
- But we also know that – if the force is conservative – the work done in getting from *a* to *b* along path 2 must be the negative of the work done in getting from *b* to *a* along path 2; thus $W_{ab,2} = -W_{ba,2}$



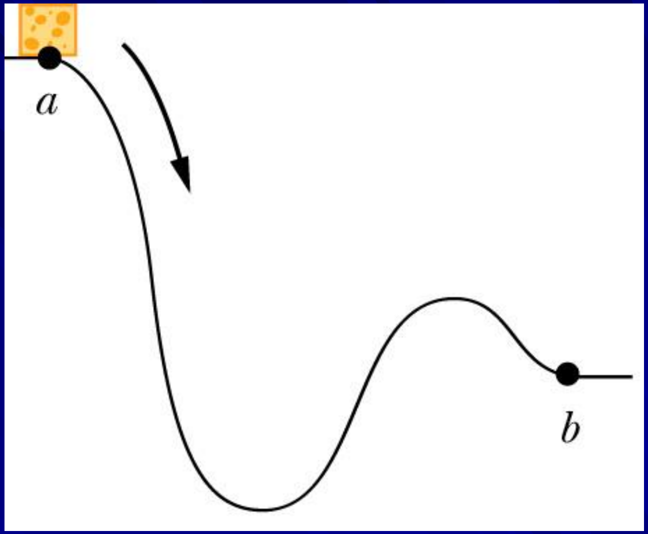
we then get: $W_{ab,1} = W_{ab,2}$

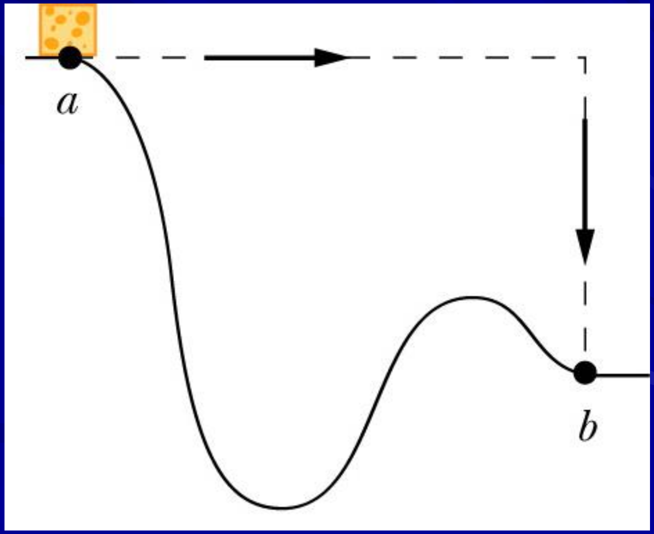
Sample Problem 8-1

- A 2.0 kg block of slippery cheese slides along a frictionless track from point *a* to point *b*
- The total distance traveled along the track is 2.0 m and the net vertical drop is 0.80 m
- How much work is done on the cheese by the gravitational force during the trip?



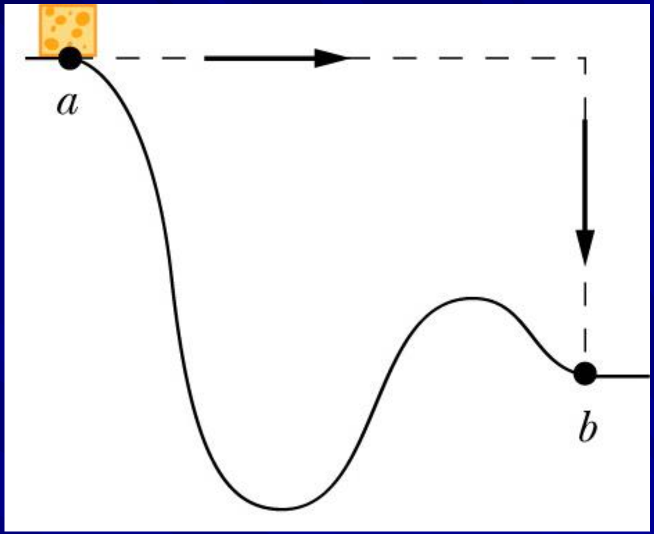
- We know that the total work done is the same regardless of the path – so let's pick an alternative path that allows us an easy solution to the problem
- We can do this because the only force we are dealing with here is the force of gravity – and we know that the gravitational force is conservative





First look at the horizontal segment of the path

The work done is:



$$W_h = mgd\cos 90^\circ = 0$$

Now let's look at the vertical segment of the path

The work done is:

$$W_v = mgd\cos 0^\circ$$

$$W_v = (2.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0.80\,\mathrm{m})(1)$$

$$W_v = 16\,\mathrm{J}$$

$$W_T = W_h + W_v$$

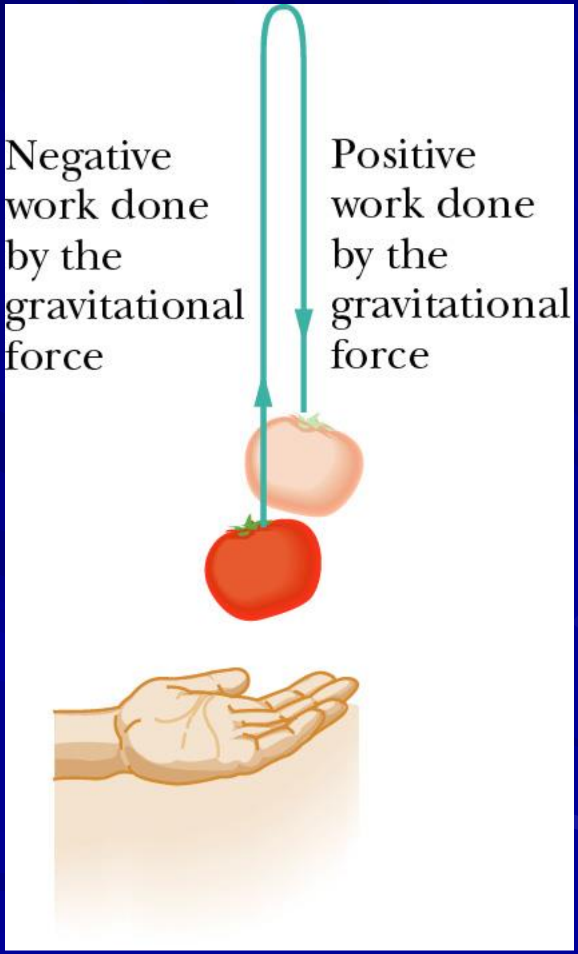
$$W_T = 0 + 16\,\mathrm{J} = 16\,\mathrm{J}$$

Potential Energy

- Last week we defined something called kinetic energy – the energy associated with the motion of an object
- We will now define a 2nd kind of energy called *potential energy*
- This is the energy associated with a *change in the configuration of a system*
- OK – but what the heck does that mean???

Work and Potential Energy

- when we threw the tomato up we noted that negative work was being done on the tomato which caused it to slow down during it's ascent
- As a result, the kinetic energy of the tomato was reduced – eventually to zero



Where it went was into an <u>increase</u> in the *gravitational potential energy* of the tomato

Work and Potential Energy

- From this we can see that for either the rise or fall of the tomato, the change ΔU in the gravitational potential energy is the negative of the work done on the tomato by the gravitational force
- In equation form we get:

$$\Delta U = -W$$

Determining Potential Energy Values

- In the general case, we can relate the work done on an object as:

$$W = \int_{x_1}^{x_2} F(x)dx$$

- Substituting in our earlier relationship for work and potential energy we get:

$$\Delta U = -\int_{x_1}^{x_2} F(x)dx$$

Gravitational Potential Energy

- **Let's imagine a particle moving along the *y* axis (positive upwards) from point y_i to point y_f**
- **As the particle moves, the gravitational force *F* does work on it; we therefore get:**

$$\Delta U = -\int_{y_i}^{y_f} (-mg)\,dy$$

- **Carrying the integral on we get:**

$$\Delta U = mg\int_{y_\mathrm{i}}^{y_\mathrm{f}} dy = mg\left[y\right]_{y_\mathrm{i}}^{y_\mathrm{f}}$$

which yields:

$$\Delta U = mg\left(y_\mathrm{f} - y_\mathrm{i}\right) = mg\Delta y$$

Gravitational Potential Energy

If **we let the initial value of y = 0, then we finally get:**

$$\Delta U = mgy$$

This equation tells us that:

The gravitational energy associated with a particle-earth system depends only on the vertical position y (or height) of the particle relative to the reference position (y = 0)

Elastic Potential Energy

- Now let's do the same analysis for a spring-block system (where the spring has a spring constant k)
- As the block moves from point x_i to point x_f, the spring force $F = -kx$ does work on the block

Elastic Potential Energy

- Substituting in $-kx$ for the force in our earlier equation we get:

$$\Delta U = -\int_{x_\mathrm{i}}^{x_\mathrm{f}} (-kx)\,dx$$

$$\Delta U = k\int_{x_\mathrm{i}}^{x_\mathrm{f}} x\,dx = \frac{1}{2}k\left[x^2\right]_{x_\mathrm{i}}^{x_\mathrm{f}}$$

Elastic Potential Energy

- Which finally results in:

$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

- Again, if we let $x_i = 0$, we get:

$$\Delta U = \frac{1}{2}kx^2$$

Conservative and Non-conservative Forces

Conservative force is one for which the work done does *not* depend on the path.

Gravitational force: mechanical work against it depends just on difference in elevation not how an object is lifted.

Elastic force: work against spring only depends on length change.

Conservation of Mechanical Energy

- **The mechanical energy of a system is simply the sum of its potential energy and the kinetic energy of the objects within it:**

$$E_{\mathrm{mch}} = K + U$$

- **For the moment, we will assume that all of the forces acting on the system are conservative – in other words, there are no frictional or drag forces present**

Conservation of Mechanical Energy

- We will also assume that the system is *isolated* – meaning that there are no external forces acting on it.
- We know that when conservative forces do work they act to transfer energy between the kinetic energy of objects in the system and the potential energy of the system

Conservation of Mechanical Energy

- We know that the change in kinetic energy is:

$$\Delta K = W$$

- We also know that the change in potential energy is:

$$\Delta U = -W$$

Conservation of Mechanical Energy

- We can therefore combine these two equations to get:

$$\Delta K = -\Delta U$$

which tells us that, *in an isolated system with conservative forces, the kinetic energy increases exactly as much as the potential energy decreases*

- **Written a little differently, we have:**

$$K_2 - K_1 = -(U_2 - U_1)$$

which can be rearranged to be:

$$K_2 + U_2 = K_1 + U_1$$

where the subscripts indicate two different states of the system

$$\frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv_0{}^2 + U(x_0) = E$$

- **But we said from the outset that the sum of the kinetic and potential energies was defined to be the mechanical energy of the system:**

$$E = K + U$$

- **So we can now see that, for an isolated system with only conservative forces, the mechanical energy of the system cannot change**

- Or said slightly differently – *In an isolated system where only conservative forces cause change, the kinetic and potential energies can change, but their sum, the mechanical energy of the system, cannot change*

$$\Delta E = \Delta K + \Delta U = 0$$

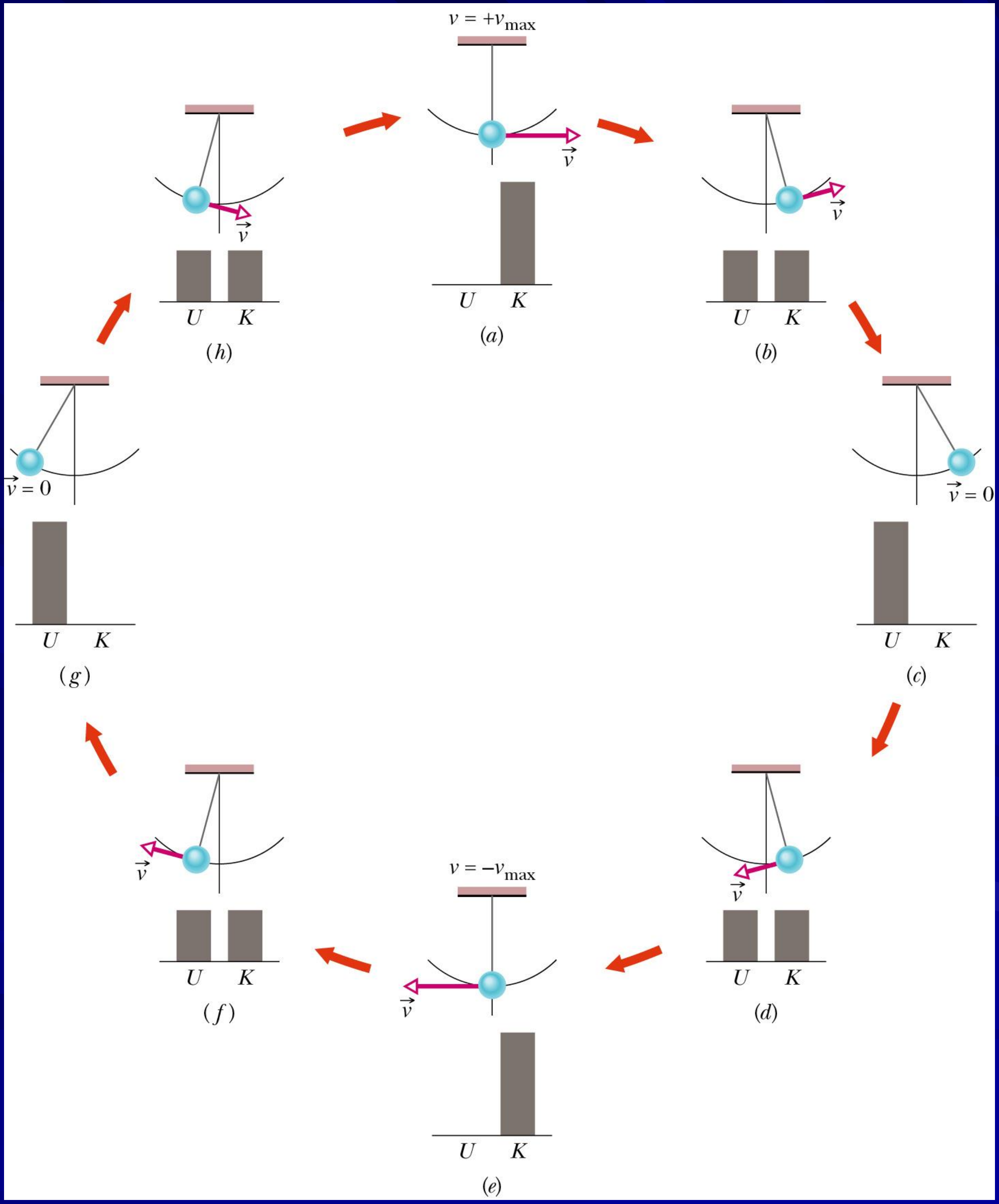
- This result is called the ***principle of conservation of mechanical energy***

$$\Delta E = \Delta K + \Delta U = 0$$

- **It allows us to examine complicated systems without having to consider what happens at all times (e.g., all of the intermediate states) and without having to consider the work done by the force(s) involved**

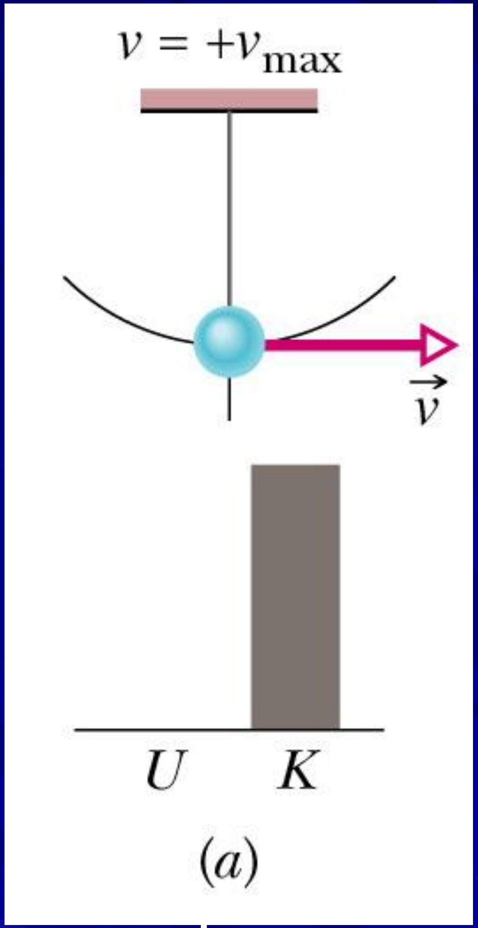
Conservation of Mechanical Energy

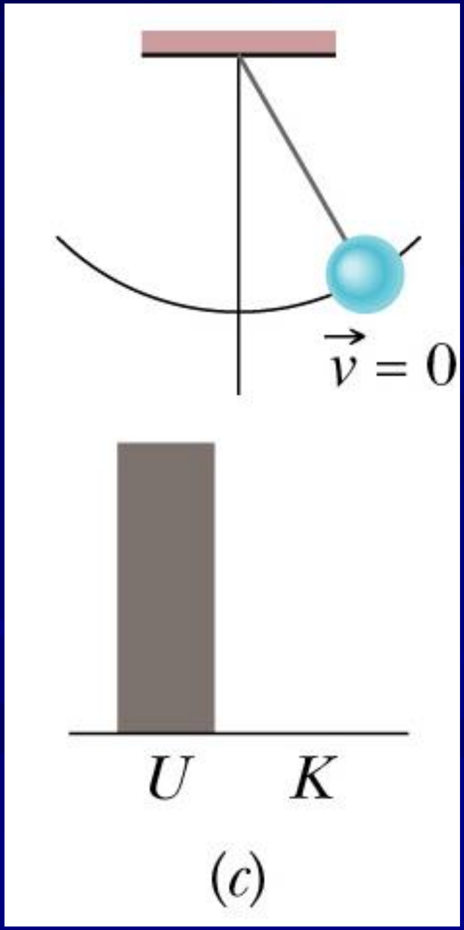
- A great illustration of the principle of conservation of mechanical energy is the pendulum.



v = +v max
v
U K
(a)
v
U K
(b)
v = 0
U K
(c)
v
U K
(d)
v = −v max
v
U K
(e)
v
U K
(f)
v = 0
U K
(g)
v
U K
(h)

- Suppose we knew that the kinetic energy at the bottom of the arc (point *a* in Fig was 20 J





- Then without any further work we would also know that the potential energy at the top of the arc (point *c* in Fig) is also 20 J

Potential-energy diagrams

$$\Delta U = -W = -\int_{x_0}^{x} F(x)dx$$

$$F(x) = -\frac{dU(x)}{dx}$$

The force is the negative gradient

of the PE curve

If we know how the PE varies with position, we can find the conservative force as a function of position

Spring Force

$$\Delta U = -W = -\int_{x_0}^{x} F(x)dx$$

$$U(x) - U(x_0) = -\int_{x_0}^{x} F(x)dx$$

$$F(x) = -kx$$

$$U(x) = \frac{1}{2}kx^2$$

$$-\frac{dU}{dx} = -\frac{d}{dx}(\frac{1}{2}kx^2) = -kx = F$$

$$\frac{1}{2}kx_m{}^2$$ P.E.

$$\frac{1}{2}kx_m{}^2$$ system Mec. E.

$$\frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv_0{}^2 + U(x_0) = E$$

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E = \frac{1}{2}kx_m{}^2$$

$$v = \sqrt{\frac{k}{m}(x_m{}^2 - x^2)}$$

$$v = \sqrt{\frac{k}{m}(x_m{}^2 - x^2)} \qquad x = \pm x_m \rightarrow v = 0$$

$$x = x_0 = 0 \rightarrow v_0 = \sqrt{\frac{k}{m}} m_m$$

$$E = \frac{1}{2} m v_0{}^2$$

$$E = \frac{1}{2} k x_m{}^2$$

PE of a spring

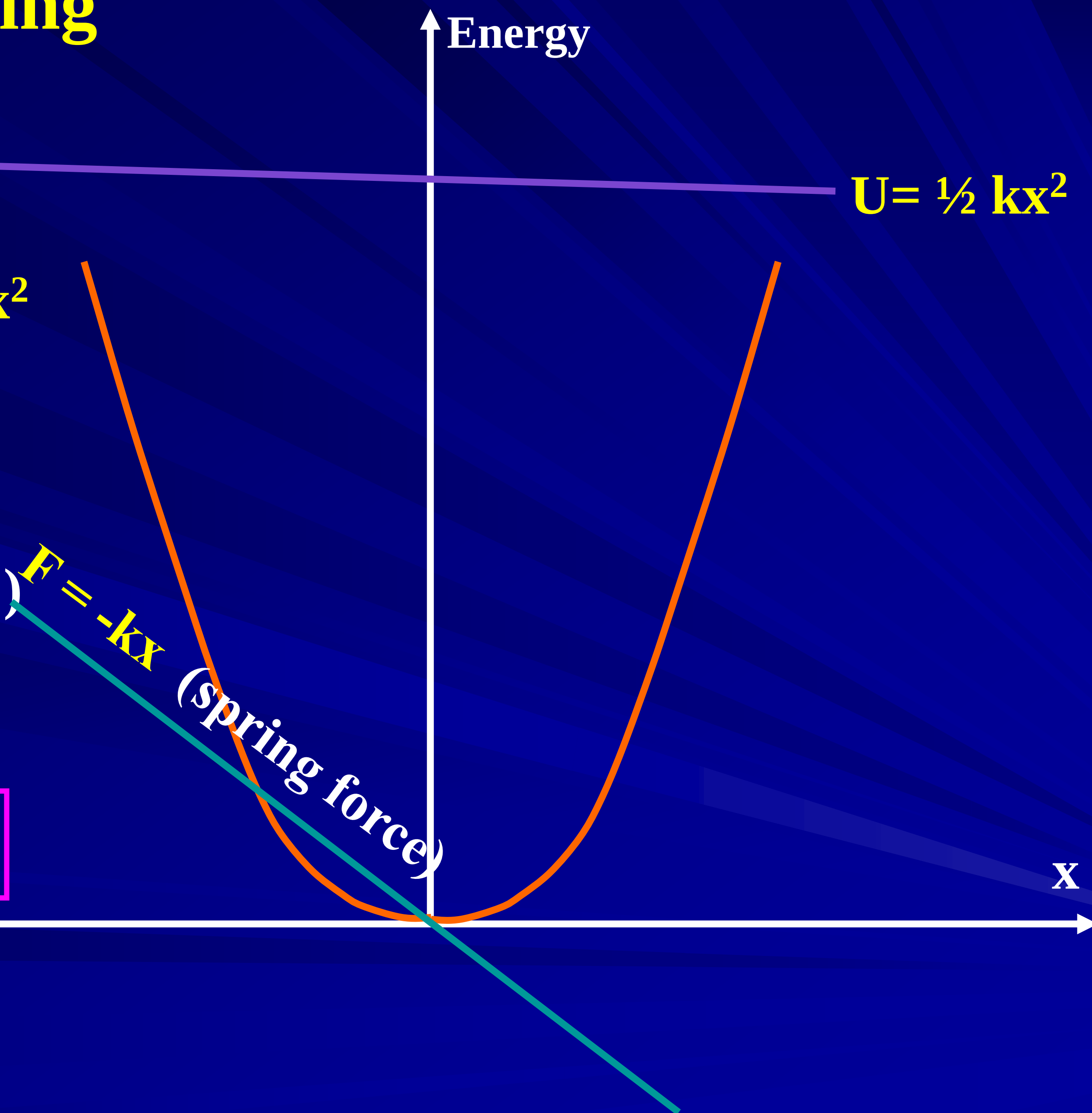
$$F = -\frac{dU}{dx}$$

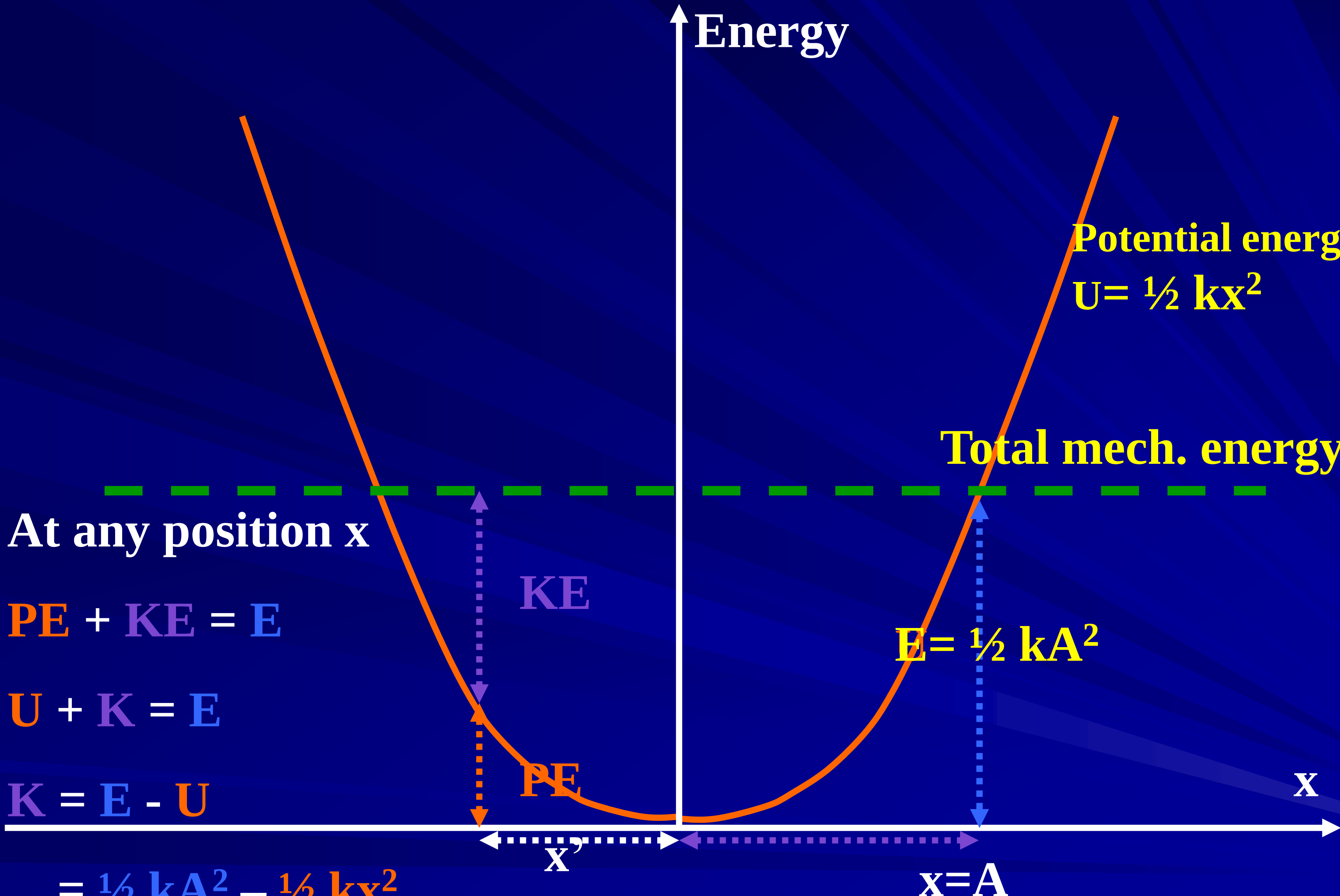
here $U = \frac{1}{2} kx^2$

$$\text{so } F = -\frac{dU}{dx}$$

$$= -\frac{d}{dx}\left(\frac{1}{2}kx^2\right)$$

$$\therefore F = -kx$$





At any position x

PE + KE = E

U + K = E

K = E - U

$= \frac{1}{2} kA^2 – \frac{1}{2} kx^2$

$= \frac{1}{2} k(A^2 - x^2)$