

General Meteorology

Lecture 5

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Thermodynamics and Statics

Thermodynamics: The study of equilibrium states of a **system**

System: Any specific sample of matter
(Examples: an "air parcel" or the air within a balloon)

Open system: Exchanges matter and energy with its surroundings. (Example: a balloon with a lot of holes)

Closed system: Does not exchange matter with its surroundings, but can exchange energy. (Example: a balloon without insulation)

Isolated System: Does not exchange matter or energy with its surroundings. (Example: a balloon with insulation)

Note: In meteorology we assume most systems are closed

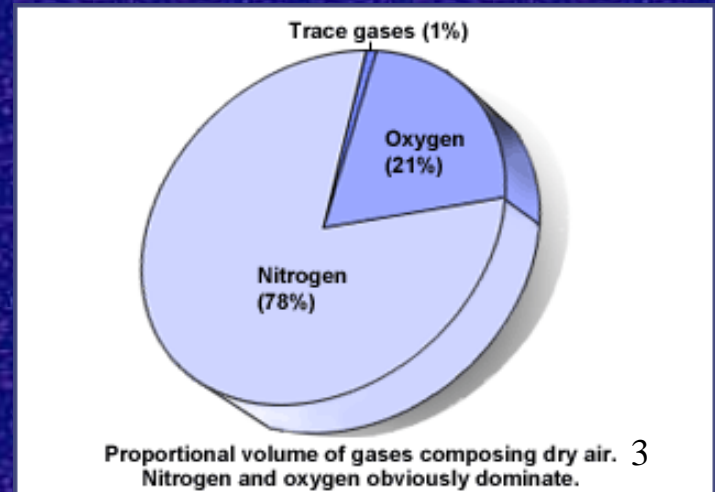
Thermodynamics: The study of equilibrium **states** of a system

State: Defines the total volume (V), pressure (p), and temperature (T) of each constituent composing a system at any given time

If the system is homogeneous (contains only one component), then only the V , p , T values are needed.

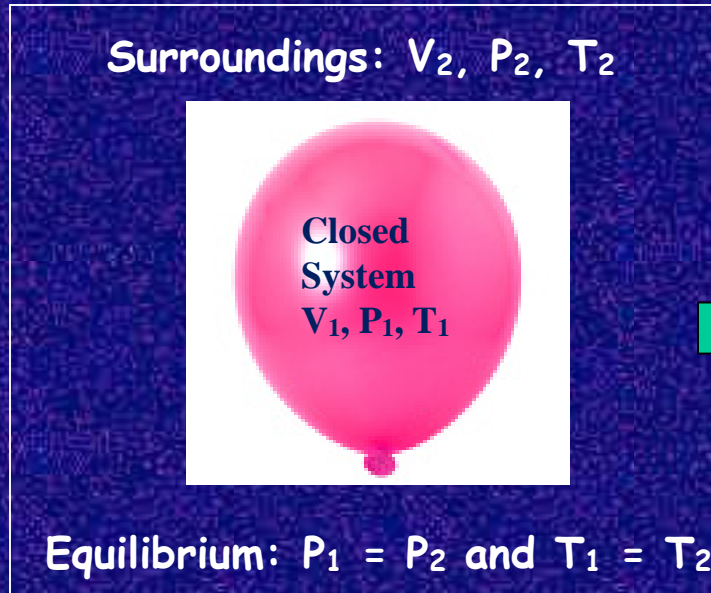
If the system is heterogeneous (contains multiple components), then the concentrations of the different components as well as the V , p , T are needed.

Note: Our atmosphere is a heterogeneous system:



Thermodynamics: The study of **equilibrium** states of a system

Equilibrium: Defines as a state in which a system's properties do not change as long as the properties of the surroundings remain unchanged



What would happen if the air pressure in the surroundings decreases?

What would happen if the air temperature in the surroundings decreases?

Transformation: When a system changes from an initial state to a new final state.

Symbolically: $i \rightarrow f$

Reversible transformation: Any transformation that can be reversed and arrive back at the initial state ($i \rightarrow f \rightarrow i$)

Occur when the external surroundings change very slowly so the system has time to adjust to new equilibrium states along the way.

Irreversible transformation: Any transformation that can not be reversed and arrive back at its original state.

Occur when surroundings or the system experiences a rapid change.

Transformation: When a system changes from an initial state to a new final state

Isothermal transformation: Any transformation that occurs without a change in temperature

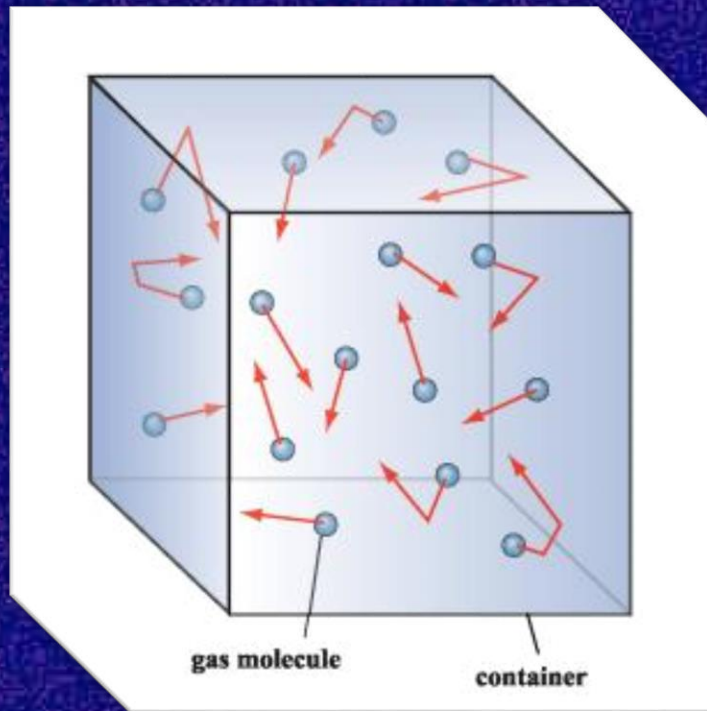
Isochoric transformation: Any transformation that occurs without a change in volume

Isobaric transformation: Any transformation that occurs without a change in pressure

Adiabatic transformation: Any transformation that occurs without exchanging heat with the environment

Note: Adiabatic transformations are not isothermal

Energy: The total internal energy of a system (U) is the sum of the kinetic energy and potential energy of all the particles that compose the system. (of the molecules in the system)



Each particle is moving in some random direction (kinetic energy)

Each particle is being pulled down by gravity (potential energy)

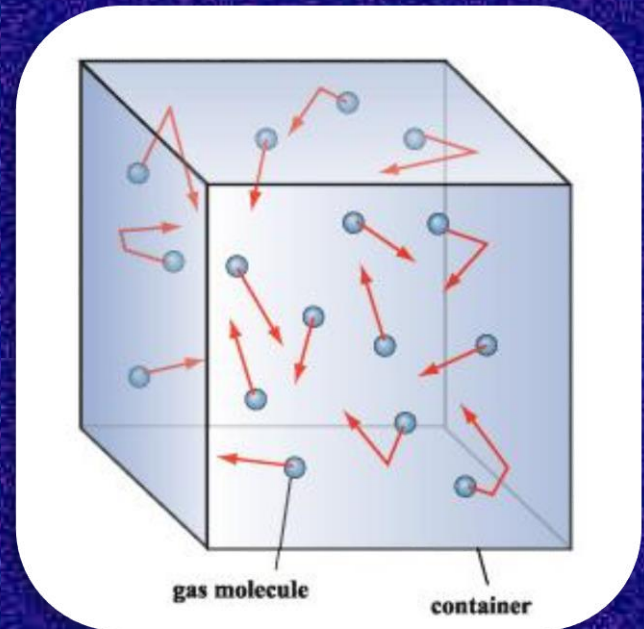
Kinetic Theory of Gases

Basic Idea: We have a closed system containing N molecules moving in random directions and random speeds:

The mean kinetic energy of all molecules is the system temperature (T)

The mean force exerted on the system's edges due to particle collisions with the edge is the system pressure (p).

The three-dimensional space occupied by the moving molecules is the system volume (V)



Kinetic Theory of Gases

Basic Idea: By applying some simple laws of physics a simple equation that combines the three state variables into a single equation can be obtained:

$$pV = NkT$$

where: p = pressure

V = volume

N = Number of molecules

k = Boltzmann's constant

T = temperature

Note: This is one form of the ideal gas law

This equation was obtained from the results of numerous laboratory experiments during the 18th and 19th centuries. Let's examine some of those early results...

Early Gas Laws

Gay-Lussac Law 1:

When pressure (p) is constant:

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Gay-Lussac Law 2:

When volume (V) is held constant:

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

Boyles Law:

When temperature (T) is held constant:

$$p_1 V_1 = p_2 V_2$$

Note: The subscripts refer to two different system states

Avogadro's Hypothesis (Law)

Avogadro's Number: The number of atoms in 12 g of carbon Called a *mole* → 1 mole = 6.022×10^{23}

Avogadro's Hypothesis: One mole of any gas at constant temperature and pressure occupies the same volume

If we increase the number of molecules (or mass) of the gas at a constant temperature and pressure, then the volume of the system increases.

Ideal Gas Law

If we combine Boyles Law and Gay-Lussac's second law we obtain:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

which implies that if we change either p , V , or T , the value of pV/T remains constant. Thus we can re-write the equation above as:

$$pV = AT$$

where A is a constant that depends on the type and amount of the gas.

Note: The dependence on the amount (or mass) of the gas is a result of Avogadro's Hypothesis.

Ideal Gas Law

- Since A is a function of both gas type and its mass, we can define A as:

$$A = mR$$

where: m = total mass of the gas

R = constant independent of mass, but dependent on gas type
(will change for different gas types: oxygen vs. hydrogen)

and re-write our equation as:

$$pV = mRT$$

Ideal Gas Law

$$\rho = \frac{1}{\alpha} = \frac{m}{V}$$

$$p = \rho RT$$

$$p \alpha = RT$$

Ideal Gas Law

$$n = \frac{m}{M}$$

where: n = number of moles
 m = total mass of the gas
 M = molecular weight of the gas

$$pV = nMRT$$

$$R^* = MR$$

$$pV = nR^*T$$

where: $R^* = 8.3143 \text{ J K}^{-1} \text{ mol}^{-1}$

Both M and R are constants and each are dependent on gas type

However, their product results in a constant that is independent of gas type, and is thus called the universal gas constant (R^*):

Ideal Gas Law: Earth's Atmosphere

Ideal Gas Law for Earth's Dry Atmosphere:

We need to find the gas constant (R) valid for our atmosphere, which contains multiple gases in various percentages:

$$R = \frac{R^*}{M} \rightarrow R_d = \frac{R^*}{M}$$

We will neglect water vapor, and find the gas constant for our dry atmosphere

<u>Gas</u>	<u>Molecular Weight</u>	<u>Percentage by mass</u>
Nitrogen (N ₂)	28.016	75.51%
Oxygen (O ₂)	32.000	23.14%
Argon (Ar)	39.940	1.30%
Carbon Dioxide (CO ₂)	44.010	0.05%

Ideal Gas Law: Earth's Atmosphere

Ideal Gas Law for Earth's Dry Atmosphere:

Thus, we need to find the mean molecular weight of our atmosphere:

Dalton's Law:

$$\overline{M} = \frac{\sum_{i=1}^K m_i}{\sum_{i=1}^K \frac{m_i}{M_i}}$$

$$\overline{M} = 28.97 \text{ g / mol}$$

$$R_d = 287 \text{ J / kgK}$$

$$p = \rho R_d T$$

$$dp = \rho R_d dT + R_d T d\rho$$

The ideal gas law relates three variables (p, ρ or V, and T)

A change in pressure (dp) can lead to either a change in temperature (dT) or a change in density (dp), or both.

The Concept of Work

Work is a Mechanical form of Energy:

$$dW = F \times \Delta x$$

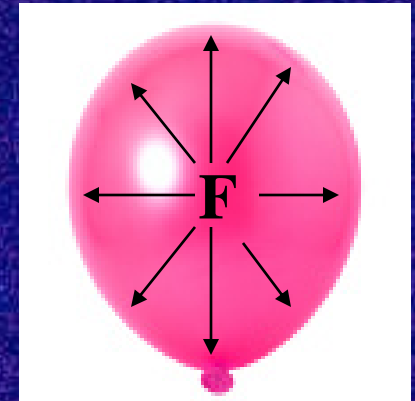
$$p = \frac{\text{Force}}{\text{Area}} = \frac{F}{(\Delta x)^2}$$

$$dW = pdV$$

$$dW > 0$$

Changes in Volume Cause Work:
Work is performed when air expands

$$dW < 0$$



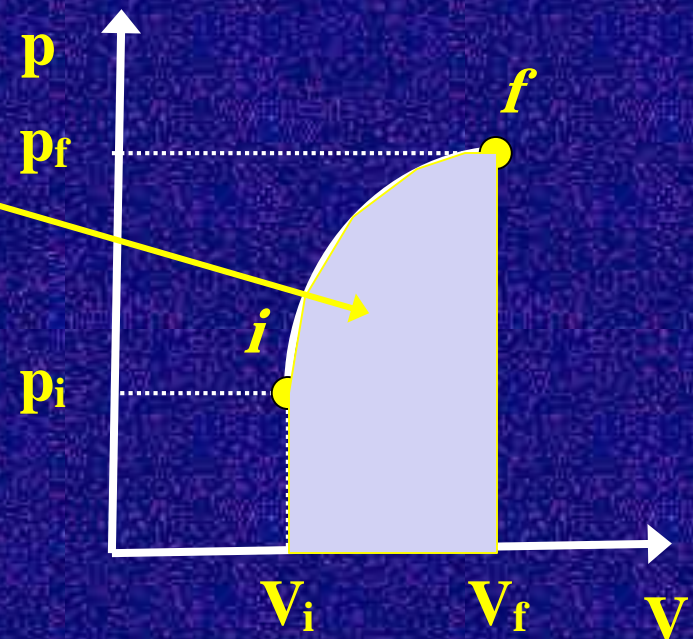
PV Diagrams

Another Way of Depicting Thermodynamic Processes:

Consider the transformation: $i \rightarrow f$

$$W = \int_i^f p dV$$

The work done is the area under the $i \rightarrow f$ curve



Energy Conservation

The **First Law of Thermodynamics** states that total energy is conserved for any thermodynamic system

$$E_{\text{internal}} + E_{\text{kinetic}} + E_{\text{potential}} + E_{\text{mechanical}} + E_{\text{heat}} + E_{\text{chemical}} + E_{\text{electrical}} = \text{constant}$$

Our main concern in meteorology...

First Law of Thermodynamics

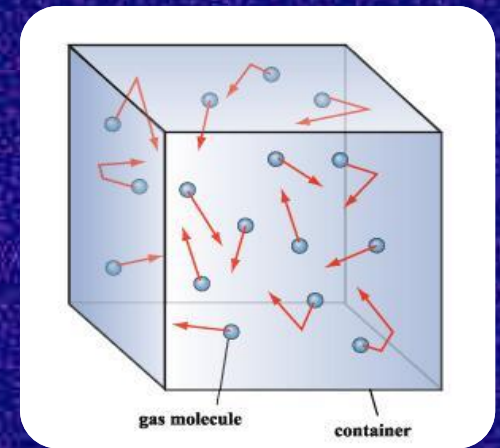
Internal Energy = Kinetic Energy + Potential Energy

Depends only on the current system state (p, V, T)

Does not depend on past states

Does not depend on how state changes occur

Changes are the result of external forcing
on the system (in the form of work or heat)



$$\Delta E_{\text{internal}} = \text{Work}_{\text{environment}} + \text{Heat}_{\text{environment}}$$

$$dU = -dW + dQ$$

$$dU = 0$$

$$dU = -pdV + dQ$$

$$U = U(T)$$

Thermal Capacities (Specific Heats)

$$c_v = \left(\frac{dQ}{dT} \right)_{\text{constant volume}}$$

Parcel experiences
no change in volume

$$c_p = \left(\frac{dQ}{dT} \right)_{\text{constant pressure}}$$

Parcel experiences no
change in pressure

$$dU = -pdV + dQ$$



$$dU = dQ$$

$$dU = c_v dT$$

$$dQ = c_v dT + p dV$$

$$dQ = c_p dT - V dp$$

First Law of Thermodynamics

Statement of Energy Balance / Conservation:

Energy in = Energy out
Heat in = Heat out

$$dq = du + dw$$

$$dq = c_v dT + p d\alpha$$

Heating

Sensible heating

Latent heating

Evaporational cooling

Radiational heating

Radiational cooling

Change in
Internal Energy

Work Done

Expansion

Compression

$$c_p = c_v + R_d$$

$$dq = c_p dT - \alpha dp$$

Types of Processes

Isothermal Processes:

Transformations at constant temperature ($dT = 0$)

Isochoric Processes:

Transformations at constant volume ($dV = 0$ or $da = 0$)

Isobaric Processes:

Transformations at constant pressure ($dp = 0$)

Adiabatic processes:

Transformations without the exchange of heat between the environment and the system ($dQ = 0$ or $dq = 0$)

Adiabatic Processes

Basic Idea:

No heat is added to or taken from the system which we assume to be an air parcel

$$\cancel{dq} = c_v dT + p d\alpha = 0$$

$$\cancel{dq} = c_p dT - \alpha dp = 0$$



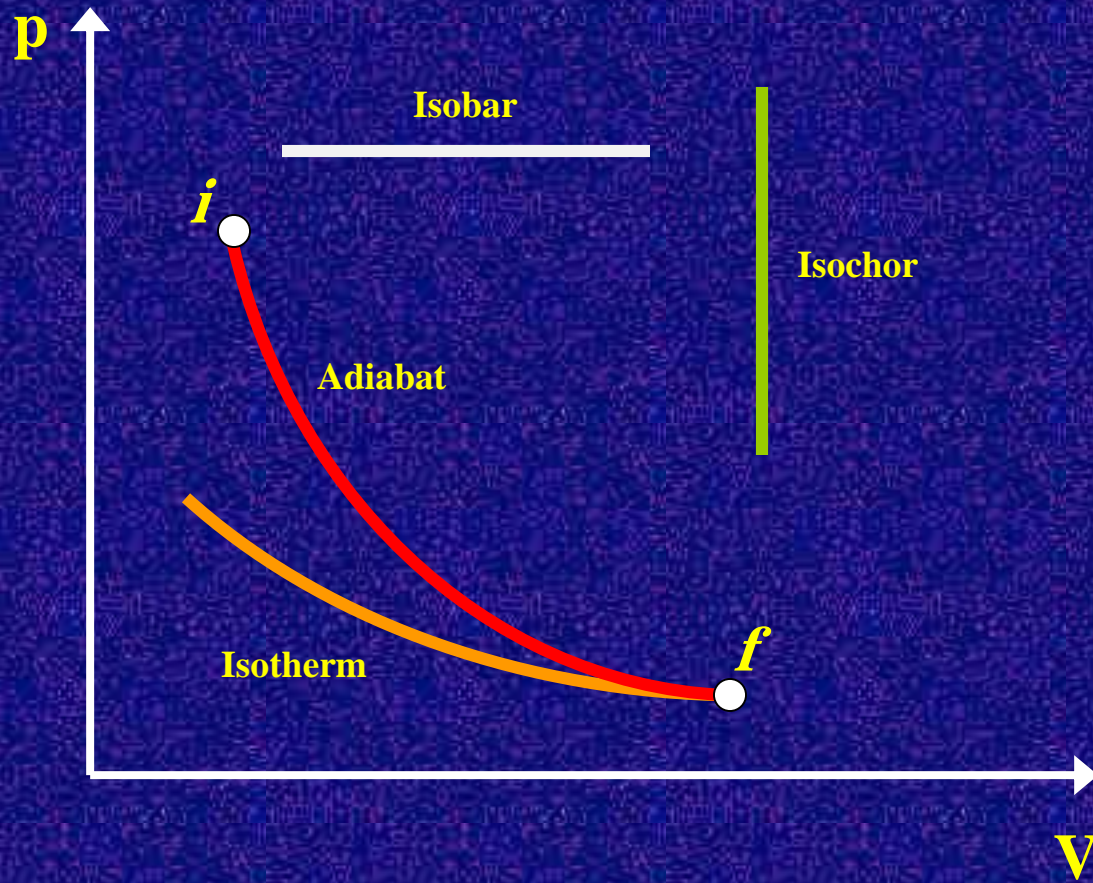
Changes in temperature result from either expansion or contraction

Many atmospheric processes are “dry adiabatic”
We shall see that dry adiabatic process play a large role in deep convective processes

Vertical motions
Thermals

Adiabatic Processes

P-V Diagrams:



Poisson's* Relation

A Relationship between Temperature and Pressure:

$$c_p dT = \alpha dp$$

$$p \alpha = R_d T \rightarrow \alpha$$

$$\frac{dT}{T} = \frac{R_d}{c_p} \frac{dp}{p}$$

$$\int_{T_{\text{initial}}}^{T_{\text{final}}} \frac{dT}{T} = \frac{R_d}{c_p} \int_{p_{\text{initial}}}^{p_{\text{final}}} \frac{dp}{p}$$

$$\ln \frac{T_{\text{final}}}{T_{\text{initial}}} = \frac{R_d}{c_p} \ln \frac{p_{\text{final}}}{p_{\text{initial}}}$$

$$T_{\text{final}} = T_{\text{initial}} \left(\frac{p_{\text{final}}}{p_{\text{initial}}} \right)^{R_d / c_p}$$

Applications of Poisson's Relation

Comparing Temperatures at different Altitudes:

Are they relatively warmer or cooler?

$$T_{\text{final}} = T_{\text{initial}} \left(\frac{p_{\text{final}}}{p_{\text{initial}}} \right)^{\frac{R_d}{c_p}}$$

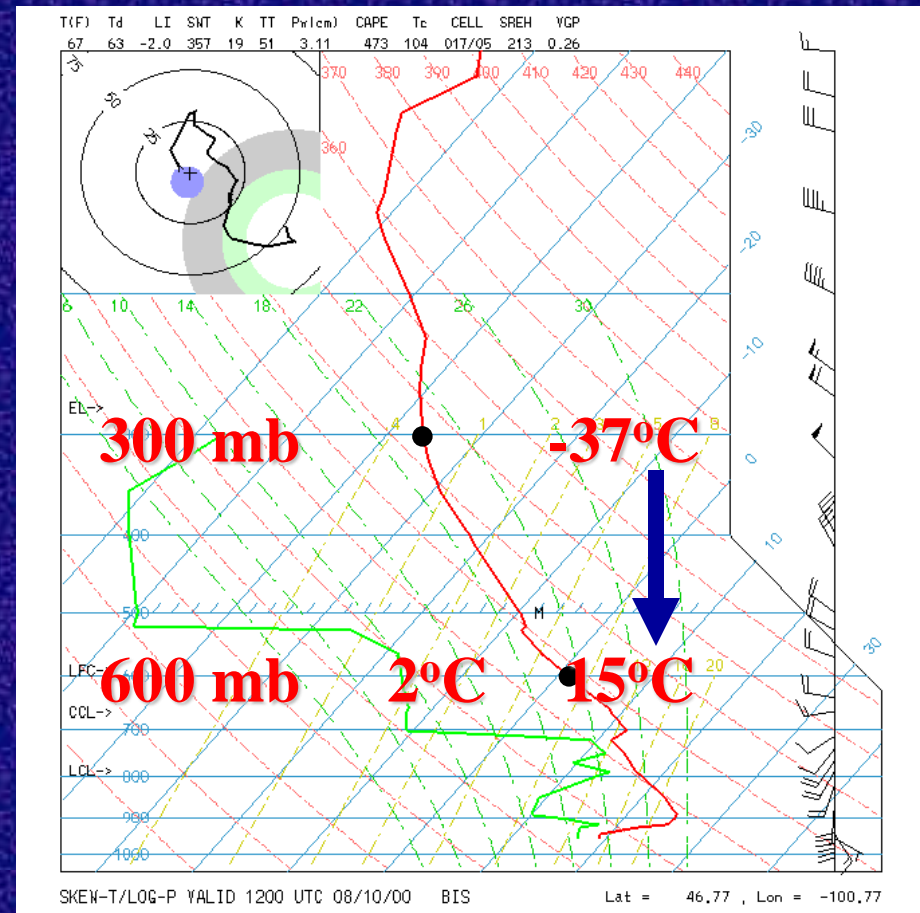
$p_{\text{initial}} = 300 \text{ mb}$

$p_{\text{final}} = 600 \text{ mb}$

$T_{\text{initial}} = -37^\circ\text{C} = 236 \text{ K}$

$T_{\text{final}} = 288 \text{ K} = 15^\circ\text{C}$

Note: We could we have chosen to expand the 600 mb parcel to 300 mb for the comparison



Potential Temperature

Special form of Poisson's Relation:

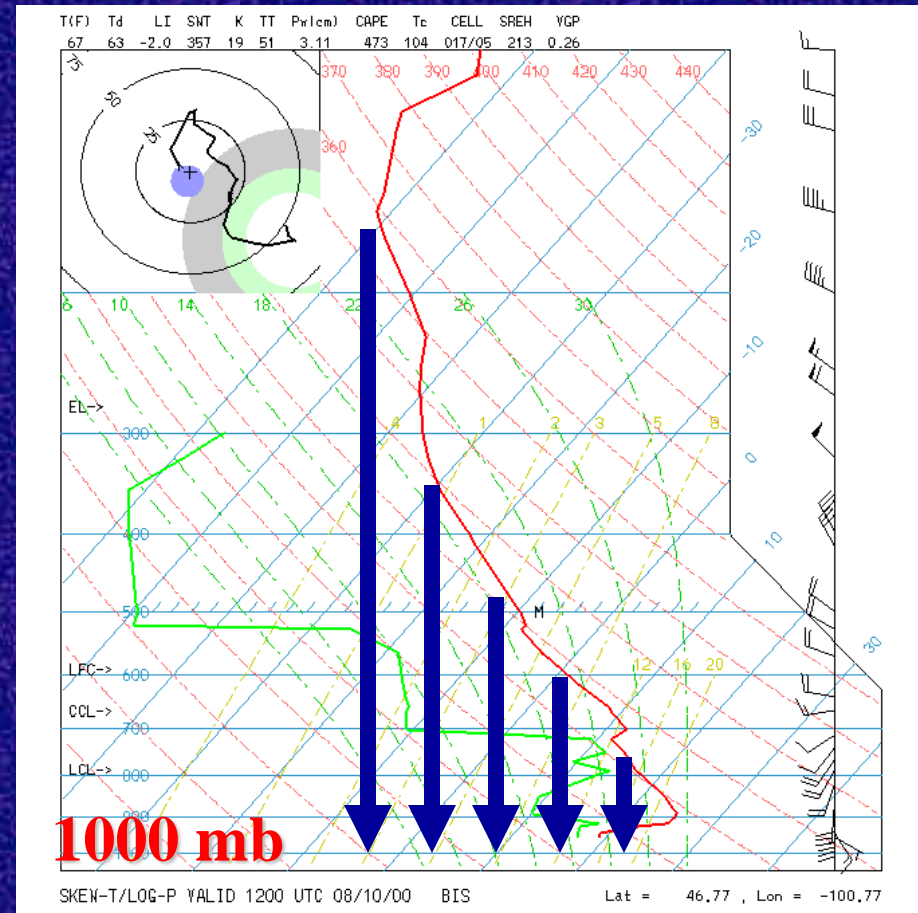
Compress all air parcels to 1000 mb
Provides a "standard"
Avoids using an arbitrary pressure level

Define $T_{\text{final}} = \theta$

θ is the potential temperature

$$\theta = T_{\text{initial}} \left(\frac{1000\text{mb}}{p_{\text{initial}}} \right)^{R_d / c_p}$$

$$\theta = T \left(\frac{p_0}{p} \right)^{R_d / c_p}$$



Applications of Potential Temperature

Air parcel 1: **p = 900 mb**
T = 21°C

Air Parcel 2: **p = 700 mb**
T = 0.6°C

$$\theta = T \left(\frac{p_0}{p} \right)^{\frac{R_d}{c_p}}$$

Which parcel is relatively colder? warmer?

$$\theta = 294\text{K} \left(\frac{1000\text{mb}}{900\text{mb}} \right)^{0.286}$$

$$\theta = 303 \text{ K}$$

$$\theta = 273.6\text{K} \left(\frac{1000\text{mb}}{700\text{mb}} \right)^{0.286}$$

$$\theta = 303 \text{ K}$$

Potential Temperature Conservation:

Air parcels undergoing adiabatic transformations maintain a constant potential temperature (θ)

During adiabatic ascent (expansion) the parcel's temperature must decrease in order to preserve the parcel's potential temperature

Potential Temperature as an Air Parcel Tracer:

Constant θ

During adiabatic descent (compression) the parcel's temperature must increase in order to preserve the parcel's potential temperature

