Dynamic Meteorology 2

Lecture 5

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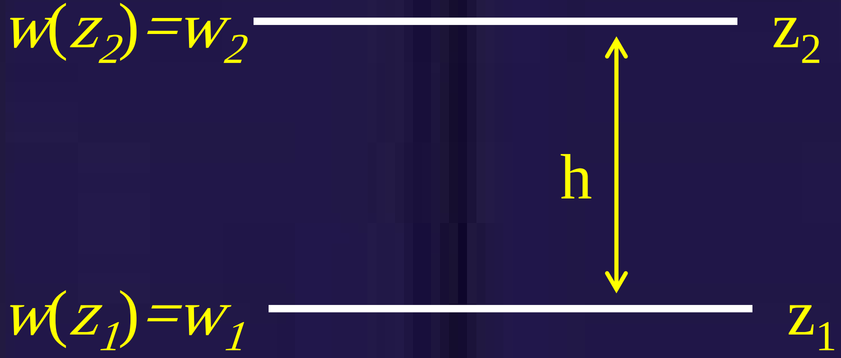
Potential Vorticity

$$\frac{1}{\rho}\frac{d\rho}{dt} + \nabla.\vec{V} = 0 \rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z}$$



$$\frac{d}{dt}(z + f) = -(z + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$$\frac{d}{dt}(z + f) = (z + f)\frac{\partial w}{\partial z} \quad *$$

$$\frac{\partial w}{\partial z} \approx \frac{\delta w}{\delta z} = \frac{w_2 - w_1}{z_2 - z_1} = \frac{1}{h}(w_2 - w_1) \qquad \frac{dz}{dt} = w$$

$$\frac{\partial w}{\partial z} \approx \frac{1}{h}\left(\frac{dz_2}{dt} - \frac{dz_1}{dt}\right) = \frac{1}{h}\frac{d}{dt}(z_2 - z_1) = \frac{1}{h}\frac{dh}{dt}$$

$$\frac{d(\zeta + f)}{dt} = (\zeta + f)\frac{1}{h}\frac{dh}{dt}$$

Absolute Vorticity

$$\frac{1}{(\zeta + f)}\frac{d(\zeta + f)}{dt} - \frac{1}{h}\frac{dh}{dt} = 0$$

$$\frac{d}{dt}\left[\ln(\zeta + f) - \ln h\right] = 0$$

$$\frac{d}{dt}\left[\ln\left(\frac{\zeta + f}{h}\right)\right] = 0$$

$$\frac{1}{\left(\frac{\zeta + f}{h}\right)}\frac{d}{dt}\left(\frac{\zeta + f}{h}\right) = 0$$

$$\frac{\zeta + f}{h} \neq 0$$

$$\frac{d}{dt}\left(\frac{\zeta + f}{h}\right) = 0$$

Barotropic (Rossby) Potential Vorticity Equation

$$\boxed{\frac{\zeta + f}{h} = const}$$

For a homogeneous, incompressible fluid flow

$$PV_R = \frac{z + f}{h} \qquad \frac{d}{dt}(z + f) = 0 \qquad z + f = const$$

Which states the absolute vorticity is consèved following the horizontal motion.

Example (1)

$$(z + f)_0 = (z + f)_1$$

$$z_0 + f_0 = z_1 + f_1$$

$$5 ´ 10^{-5} + 2W\sin 30 = z_1 + 2W\sin 90$$

$$z_1 = -2.3 ´ 10^{-5} s^{-1}$$

Example (2)

$$\left(\frac{z + f}{h}\right)_0 = \left(\frac{z + f}{h}\right)_1$$

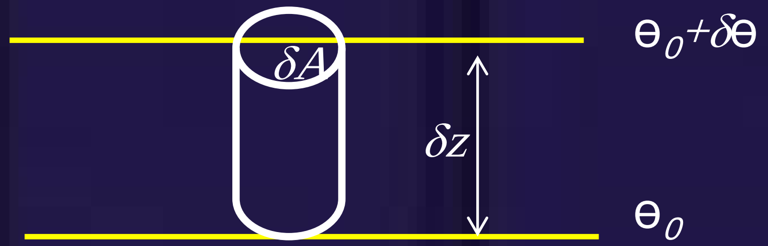
$$\frac{0 + 2W\sin 60}{10\,km} = \frac{z_1 + 2W\sin 45}{(10 - 2.5)\,km}$$

$$z_1 = -8.4 \times 10^{-6} s^{-1}$$

$$z_{a1} = z_1 + f_1 = 9.5 \times 10^{-5} s^{-1}$$

Ertel potential Vorticity

$$\delta m = \rho \delta V = \rho \delta A \delta z$$



$$\frac{\delta p}{\delta z} = -\rho g$$

$$\delta m = \rho \delta A(-\frac{\delta p}{\rho g})$$

$$\frac{\delta m}{\delta \theta} = const$$

$$\delta A = -g \frac{\delta m}{\delta \theta} \frac{\delta \theta}{\delta p}$$

$$\delta A = (-g \frac{\delta \theta}{\delta p}) \times const$$

Kelvin's Circulation Theorem

Adiabatic flow can be described by Kelvin's circulation theorem:

$$\frac{d}{dt}\left(\delta C + 2\Omega\,\delta A \sin\varphi\right) = 0$$

where δC is evaluated for a closed loop encompassing the area δA on an isentropic surface.

The vertical component of vorticity is given by $\zeta_\theta = \dfrac{\delta C}{\delta A}$,

thus if the isentropic surface is approximately horizontal, for an infinitesimal parcel of air:

$$\frac{d}{dt}\left(\delta A\left(\zeta_\theta + f\right)\right) = 0 \rightarrow \delta A\left(\zeta_\theta + f\right) = const$$

relative vorticity on an isentropic surface (ζ_θ)

Coriolis parameter (f)

$$\begin{cases} \delta A = (-g\dfrac{\delta\theta}{\delta p}) \times const \\ (\zeta_\theta + f)\,\delta A = const \end{cases}$$

$$(\zeta_\theta + f)(-g\frac{\delta\theta}{\delta p}) \times const = const$$

$$(\zeta_\theta + f)(-g\frac{\delta\theta}{\delta p}) = const$$

$$g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p}) = const$$

$$\zeta_\theta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)_\theta$$

$$PV_E = g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p})$$

Ertel potential Vorticity

Isentropic Potential Vorticity

Potential vorticity is conserved following adiabatic, frictionless flow

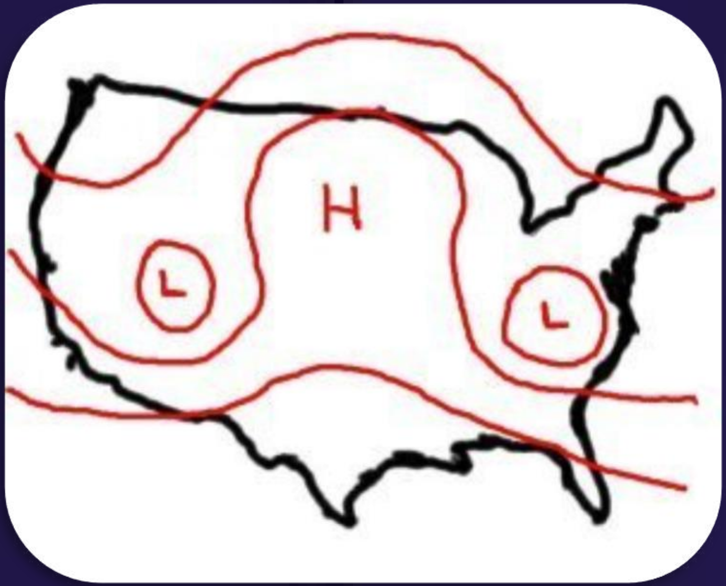
$$\frac{\partial \theta}{\partial p} \rightarrow 0 \quad (adiabatic), \quad \varsigma_\theta \rightarrow very \quad large$$

$$\frac{\partial \theta}{\partial p} \rightarrow \quad (increasing), \quad \varsigma_\theta \rightarrow (decreasing)$$

$$\frac{\partial \theta}{\partial p} \rightarrow 0 \quad (decreasing), \quad \varsigma_\theta \rightarrow (increasing)$$

There are several reasons why many meteorologists think that the consideration of IPV charts are useful.

First of all, PV is a conserved quantity in adiabatic, frictionless flow. The conservation of potential vorticity is a powerful constraint on the large scale motions of the atmosphere. PV centres may be identified on a series of analyses and can be used to describe the evolution of flow patterns during significant synoptic events such as rapid cyclogenesis, blocking and retrogression of longwaves.



Secondly, it is possible to deduce the T, p and wind fields from the PV distribution if a number of assumptions are made.

For example, one assumption involves the specification of a balance condition which relates the mass field to the motion field.

The simplest balance condition is the quasi-geostrophic approximation.
One must also specify an initial reference state and appropriate boundary conditions.
Once this is done, however, the spatial distribution of PV then becomes a source term in the equations, the flow field being derived entirely from this term.
Later, an analogy will be made with static electric charge distributions and their associated electric fields.

Finally, certain atmospheric processes may be described in terms of the interaction of PV anomalies with the background structure of the atmosphere.

For example, when a strong upper-level PV anomaly moves over a low-level baroclinic zone, cyclogenesis usually results.

There is no need to invoke secondary circulations (vertical motions) as drivers of the development. In addition, a superposition principle may be used to describe the interaction of PV anomalies at different levels in the atmosphere, interactions which lead to changes in the circulations at these levels.

Conservation of Potential Vorticity

The conservation of potential vorticity couples changes in depth, relative vorticity, and changes in latitude. All three interact.

Changes in the depth *h* of the flow causes changes in the relative vorticity. The concept is analogous with the way figure skaters decreases their spin by extending their arms and legs.

$$\frac{z + f}{h} = const \qquad PV_E = g(\zeta_\theta + f)(-\frac{\partial \theta}{\partial p})$$

The conservation of potential vorticity is the air's equivalent of the conservation of angular momentum.

$$PV_E = g(\zeta_\theta + f)(-\frac{\partial \theta}{\partial p})$$

Ertel potential Vorticity

Isentropic Potential Vorticity

Relative vorticity is zero for stationay atmosphere

$$\zeta_\theta + f = 0 + f = f$$

$$PV_s = -gf\frac{\partial \theta}{\partial p}$$

$$g = 10m/s^2 \qquad f = 10^{-4}s^{-1} \qquad \frac{\partial \theta}{\partial p} = \frac{10K}{100hpa}$$

$$PV_s \cong 10m/s^2 \times 10^{-4}s^{-1} \times \frac{10K}{1000Pa} = 10^{-6}m^2s^{-1}Kkg^{-1}$$

Isentropic potential vorticity is of the order of:

$$P_s = 10^{-6} m^2 s^{-1} K kg^{-1} = 1 PVU$$

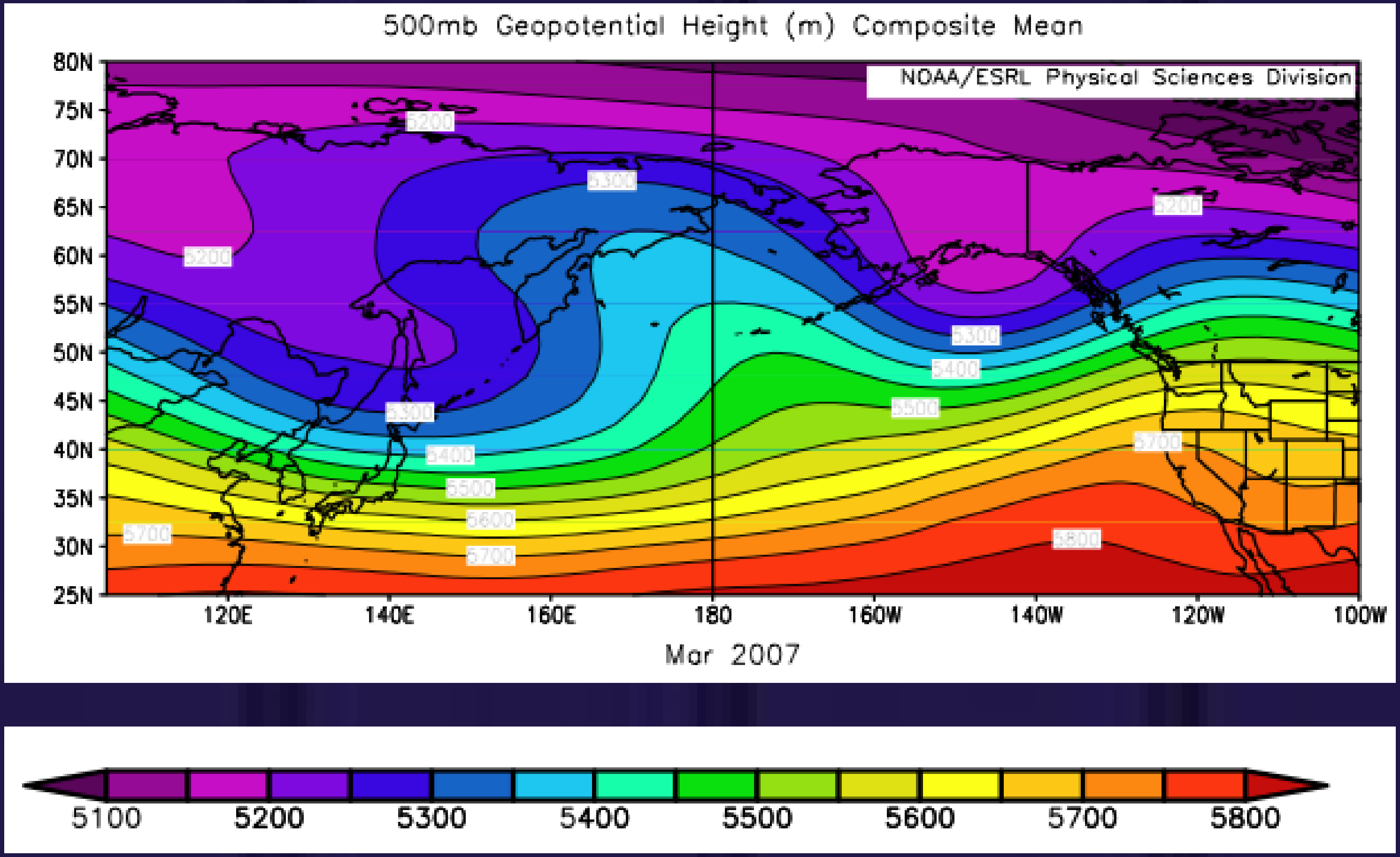
Potential Vorticity Unit

Values of IPV < 1.5 PVU are generally associated with tropospheric air

Values of IPV > 1.5 PVU are generally associated with stratospheric air

Comparison to Isobaric Analyses:

Regions of low geopotential heights correspond to regions with large PV values



Consevation of absolute vorticity

Conservation of absolute vorticity following the motion provides a strong constraint on the flow, as can be shown by a simple example that again illustrates an asymmetry between westerly and easterly flow.

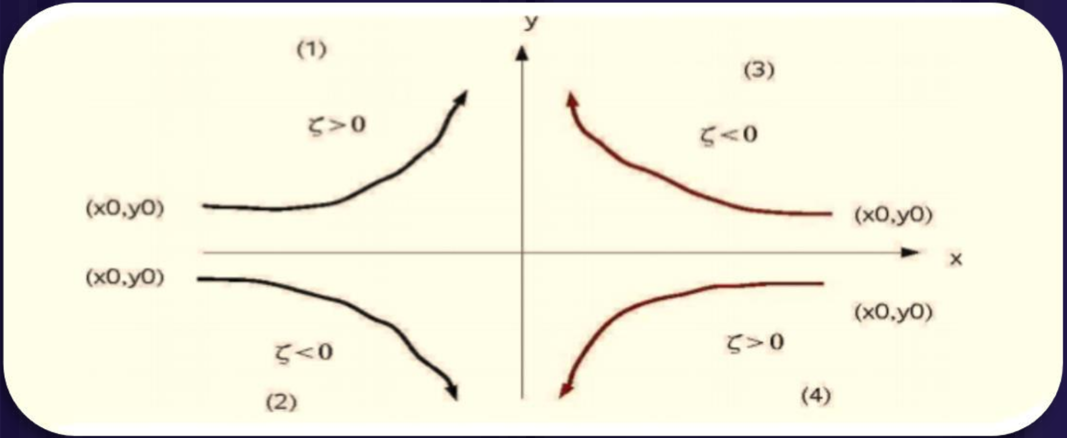
Suppose that at a certain point (x_0, y_0) the flow is in the zonal direction and the relative vorticity vanishes $(\zeta = 0)$ so that

$$\text{absolute vorticity} \quad \eta = \zeta + f \quad \rightarrow \quad \eta(x_0, y_0) = f_0$$

Then, if absolute vorticity is conserved, the motion at any point along a parcel trajectory that passes through (x_0, y_0) must satisfy

$$\zeta + f = f_0$$

$$\eta(x_0, y_0) = f_0$$



(1) $\zeta > 0$; f increases; $\zeta + f > f_0$; η not conserved

(2) $\zeta < 0$; f decreases; $\zeta + f < f_0$; η not conserved

(3) $\zeta < 0$; f increases; $\zeta + f = f_0$; η conserved

(4) $\zeta > 0$; f decreases; $\zeta + f = f_0$; η conserved

- for westerly flows (wind blows from west to east) the motion has to remain purely zonal if the absolute vorticity is conserved.

- for easterly flows (wind blows from east to west) conservation of absolute vorticity is possible both for northward and southward curvature.

Whereas trajectories that curve southward must have

$$\zeta = f_0 - f > 0$$

However, as indicated in the figure if the flow is westerly, northward curvature downstream implies $\zeta > 0$

whereas southward curvature implies $\zeta < 0$

Thus, westerly zonal flow must remain purely zonal if absolute vorticity is to be conserved following the motion.

The easterly flow case, also shown in Fig. Is just the opposite.

Northward and southward curvatures are associated with negative and positive relative vorticities, respectively.

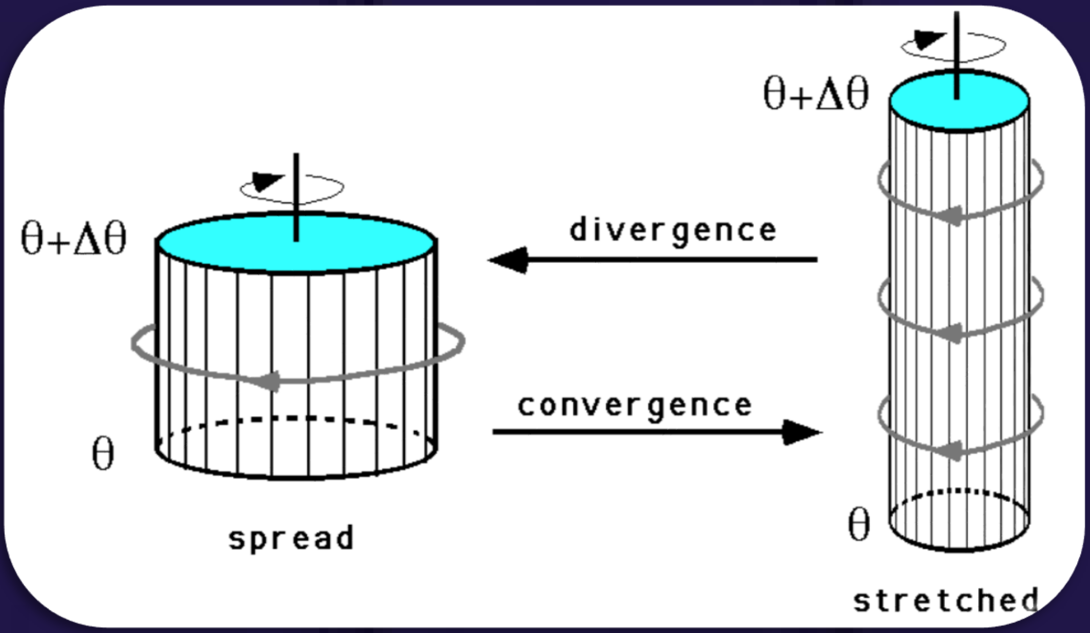
Hence, an easterly current can curve either to the north or to the south and still conserve absolute vorticity.

Because f increases toward the north, trajectories that curve northward in the downstream direction must have

$$\zeta = f_0 - f < 0$$

The potential vorticity can only be changed by diabatic heating or friction

$$\frac{z + f}{h} = const$$



The vortex of air on the left in is broad and slow. When the air converges, the column stretches, i.e. h increases. To maintain potential vorticity, the air spins faster (ξ increases), resulting in the streched vortex on the right.

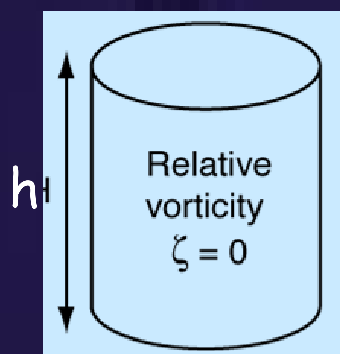
Divergence, on the other hand, causes vortex spreading and slows down the rate of spin.

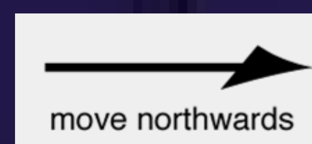
Conservation of potential vorticity (relative plus planetary)

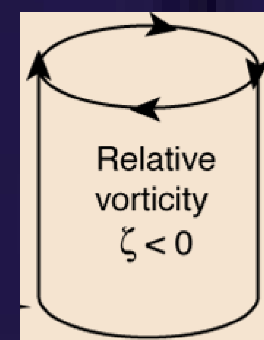
Conservation of potential vorticity in the absence of stretching (N.H.)

(Balance of planetary vorticity and relative vorticity)

$$\frac{z + f}{h} = const$$





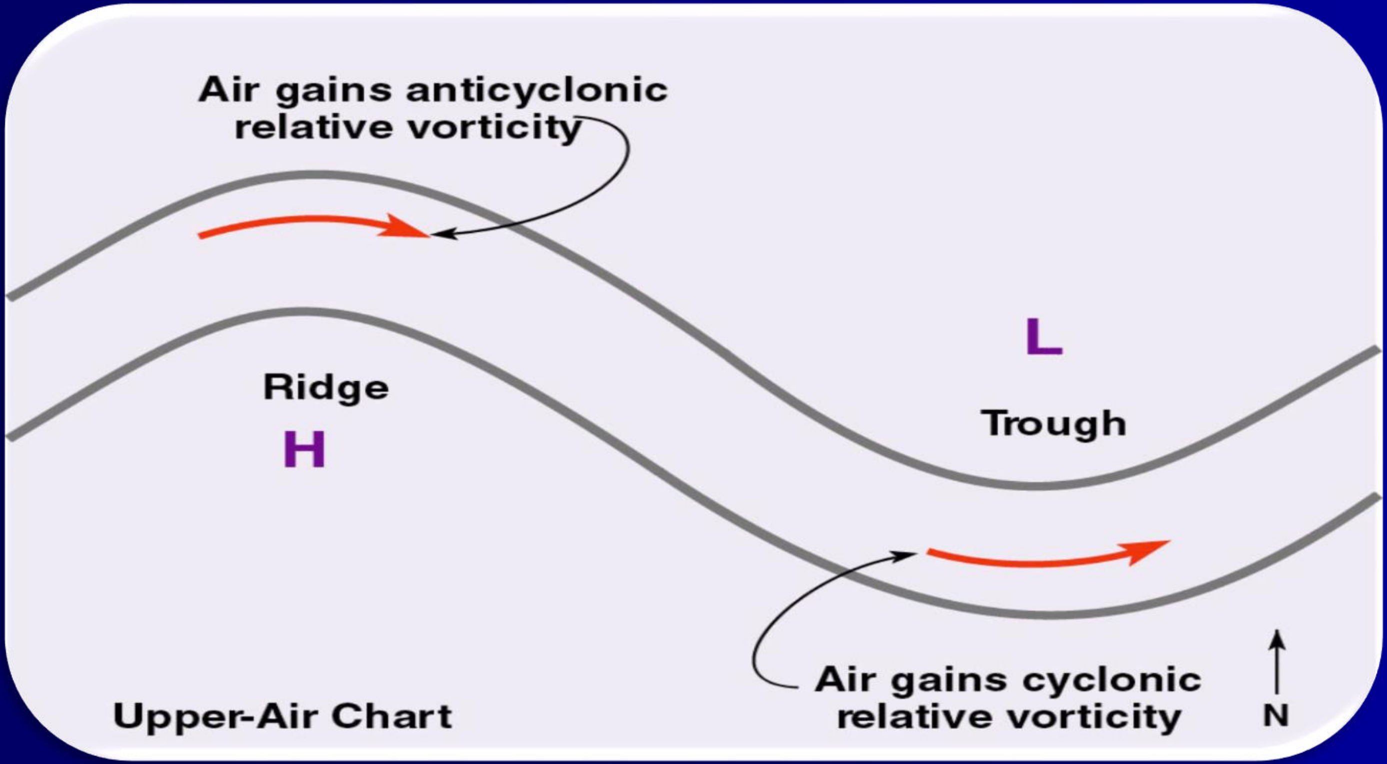


$$PV = \frac{f(j_1)}{h}$$

$$PV = \frac{f(j_2) + z}{h}$$

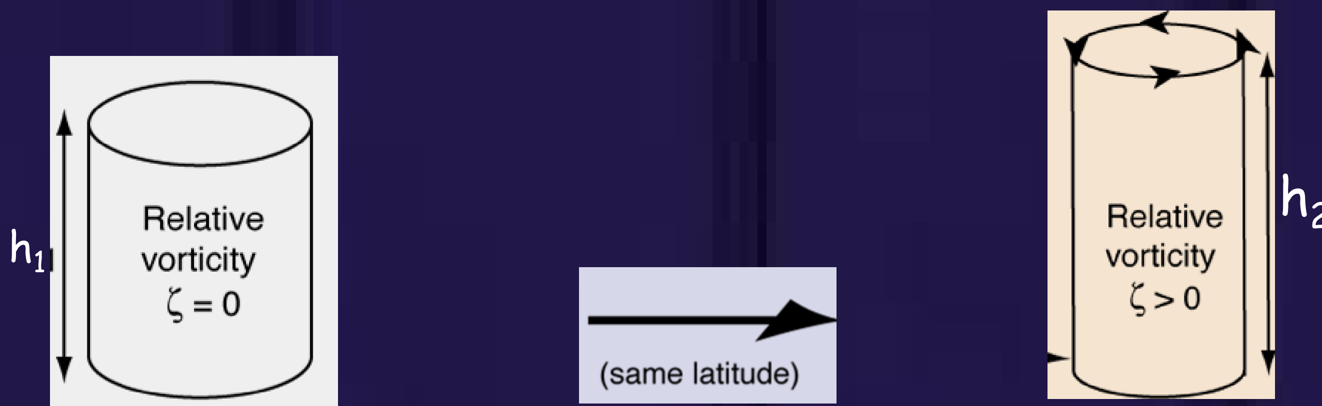
$$\frac{f(j_1)}{h} = \frac{f(j_2) + z}{h}$$

Vorticity


Air gains anticyclonic
relative vorticity
L
Ridge
Trough
H
Air gains cyclonic
relative vorticity
Upper-Air Chart
N

Conservation of potential vorticity (relative and stretching)

Conservation of potential vorticity in the absence of planetary vorticity change (N.H.) (Balance of planetary vorticity and stretching)



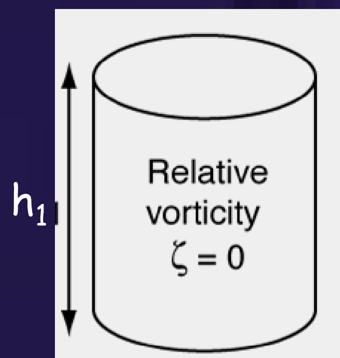
$$PV = \frac{f(\varphi_1)}{h_1}$$

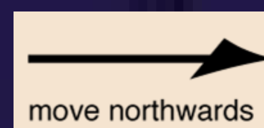
$$PV = \frac{f(\varphi_1) + \zeta}{h_2}$$

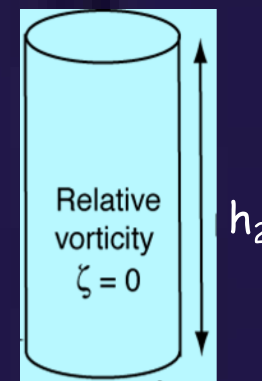
$$\frac{f(\varphi_1)}{h_1} = \frac{f(\varphi_1) + \zeta}{h_2}$$

Conservation of potential vorticity (planetary and stretching)

Conservation of potential vorticity in the absence of relative vorticity change (N.H.) (Balance of planetary vorticity and stretching)







$$PV = \frac{f(\phi_1)}{h_1}$$

$$PV = \frac{f(\phi_2)}{h_2}$$

$$\frac{f(\phi_1)}{h_1} = \frac{f(\phi_2)}{h_2}$$

$$PV_E = g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p})$$

As the air flows over the mountain, the potential temperature is conserved, so the 300K isentrope (ϴ in Fig) bends over the mountain. Air aloft, at the 320K isentrope (ϴ+Δϴ), is lifted much less as it passes the range.

Therefore Δϴ is reduced over the mountain chain, and to keep the potential vorticity constant, the absolute vorticity (f + ξ) must be reduced equally.