Dynamic Meteorology

Lecture 15

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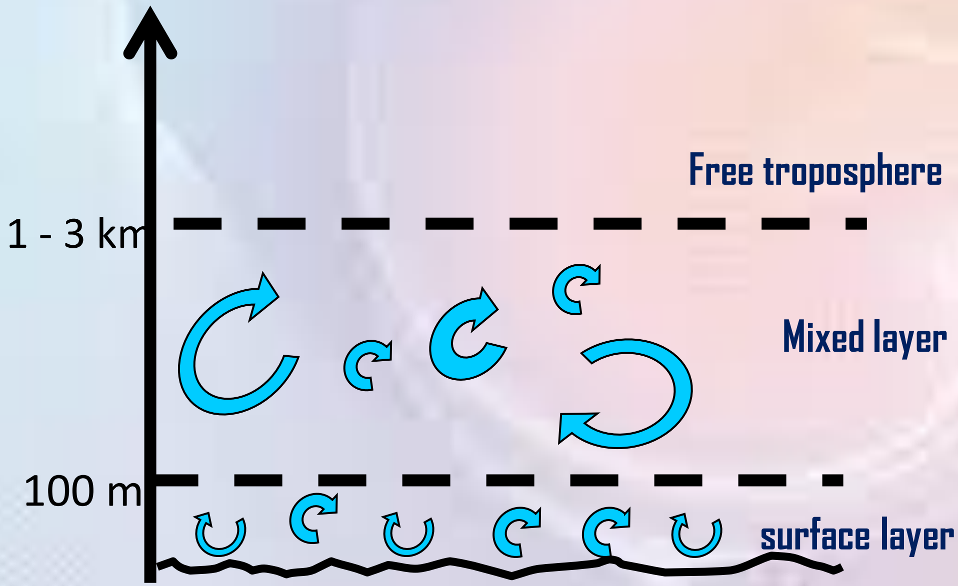
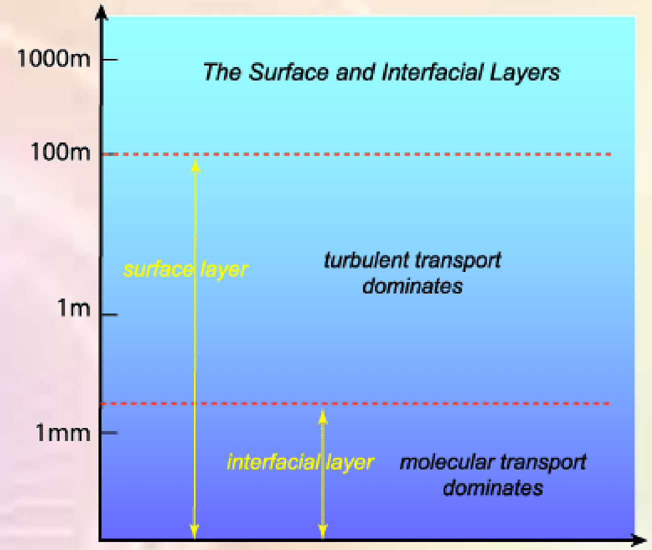
Planetary bondary Layer

The PBL is the layer close to the surface within which vertical transports by turbulence play dominant roles in the momentum, heat and moisture budgets.





The atmospheric boundary layer is formed as a consequence of strong interactions between the atmosphere and the underlying surface (land or water).

Micrometeorology is the study of the meteorology in PBL.

From bottom up:

1. The viscous sub-layer (0-1 cm):

Characterized by: molecular diffusion, extreme shear, wind at surface (molecular scale) = 0, no turbulence

2. The surface layer

The layer extending from the top of the interfacial layer to about 10% of the depth of the PBL.

Characterized by: strong gradient, vertical momentum transfer by turbulent eddies, not directly dependent on Coriolis and PG forces

3. The Ekman layer

The layer extending from the top of the surface layer to the top of the PBL

Characterized by: well-mixed, vigorous turbulence, turning wind with height as the effect of friction diminishes and the wind approaches its geostrophic value

Turbulence

Turbulence: refers to the apparently chaotic nature of many flows, which is manifested in the form of irregular, almost random fluctuations in velocity and temperature around their mean values in time and space.

The motions in the PBL are almost always turbulent

Sources of turbulence

Much of the boundary layer turbulence is generated by forcings from the ground.

1. Solar heating of the ground during sunny days causes thermals of warmer air to rise.

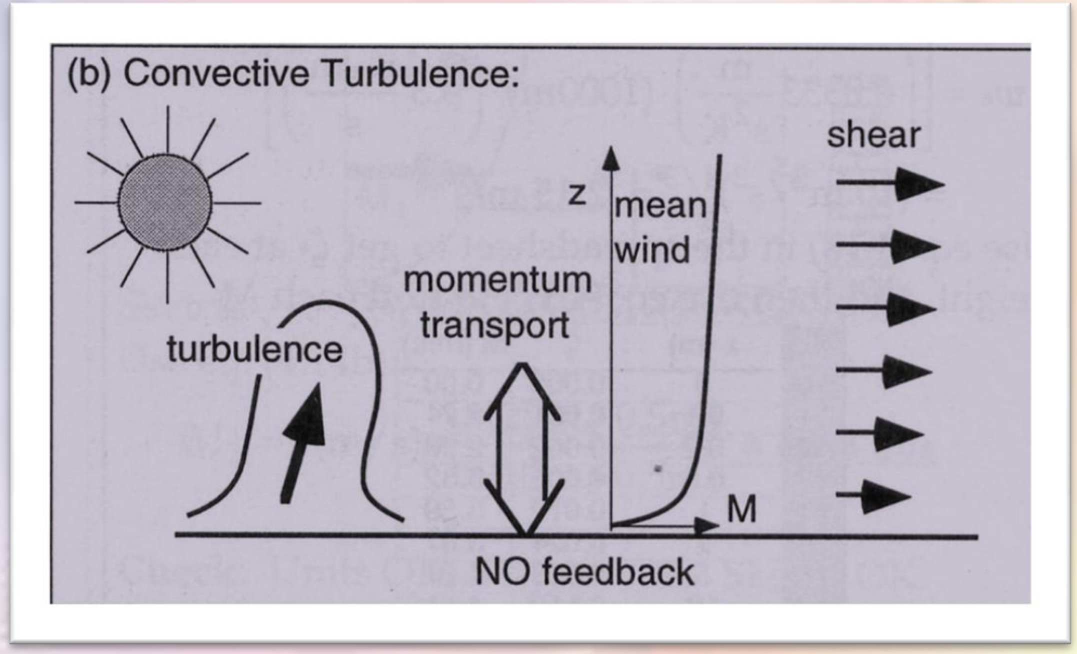
(thermals: buoyant eddies forced by solar heating of the surface)

Thermal Turbulence

Caused by heating/cooling of the earth's surface

Flows are typically vertical

Convection cells of upwards of 1000 - 1500 meters



Turbulence

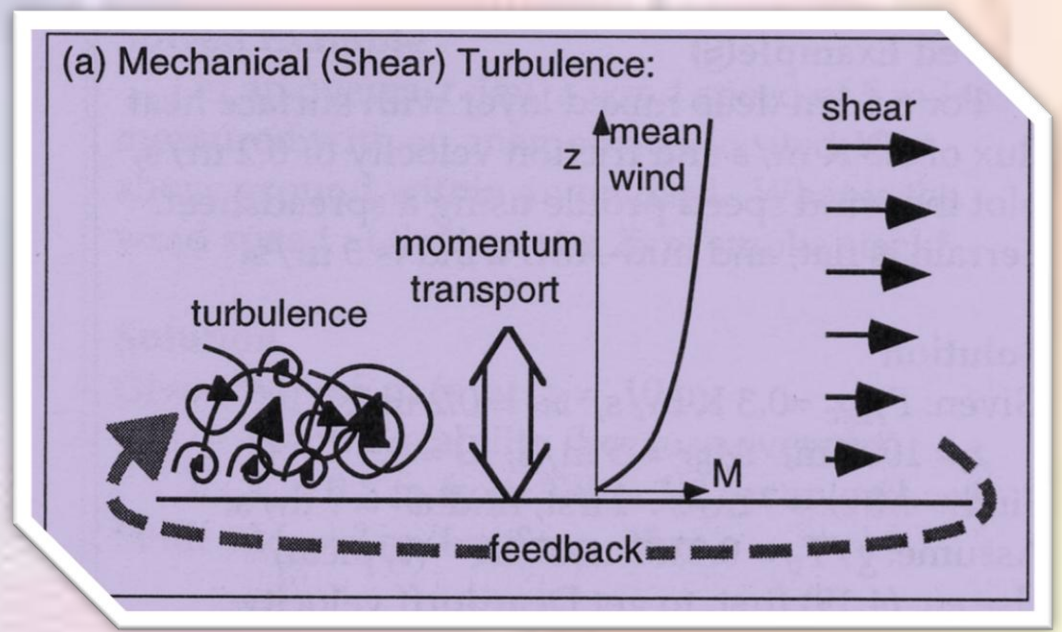
Circular eddies of air movements over short timescales than those that determine wind speed (unstable)

2. Mechanical Turbulence:

Caused by air moving over and around structures/vegetation

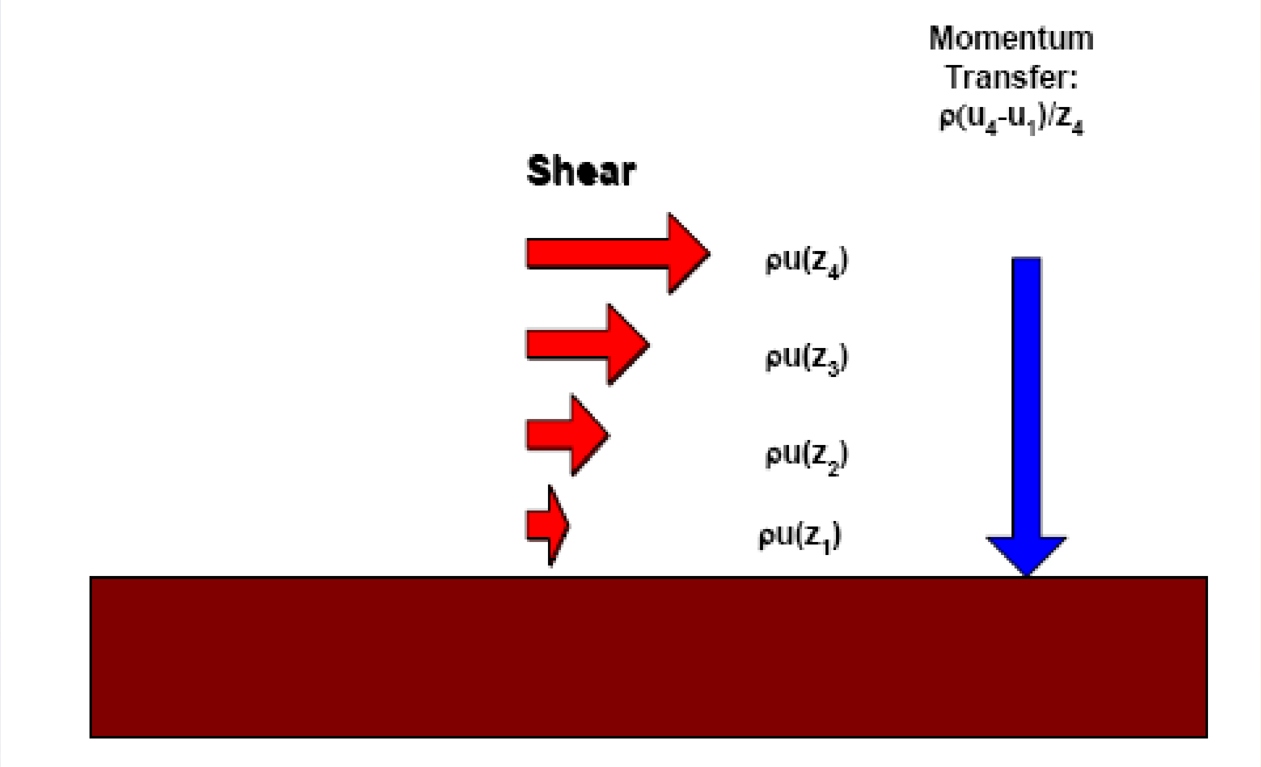
Increases with wind speed

Affected by surface roughness



Vertical wind shear

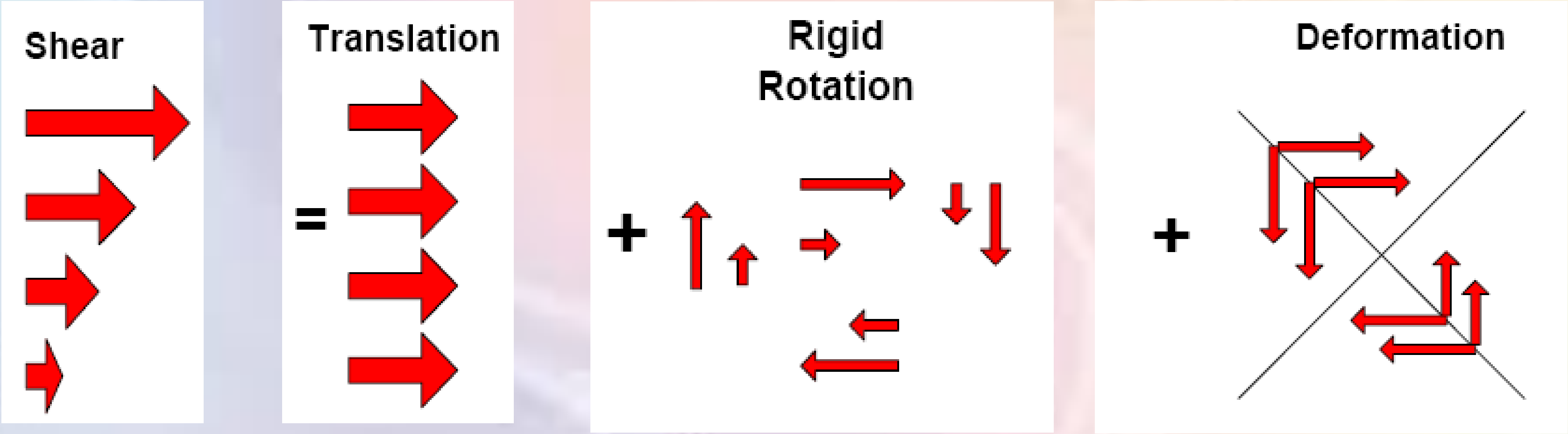
Frictional drag on the air flowing over the ground causes wind shears to develop, which frequently become turbulent.



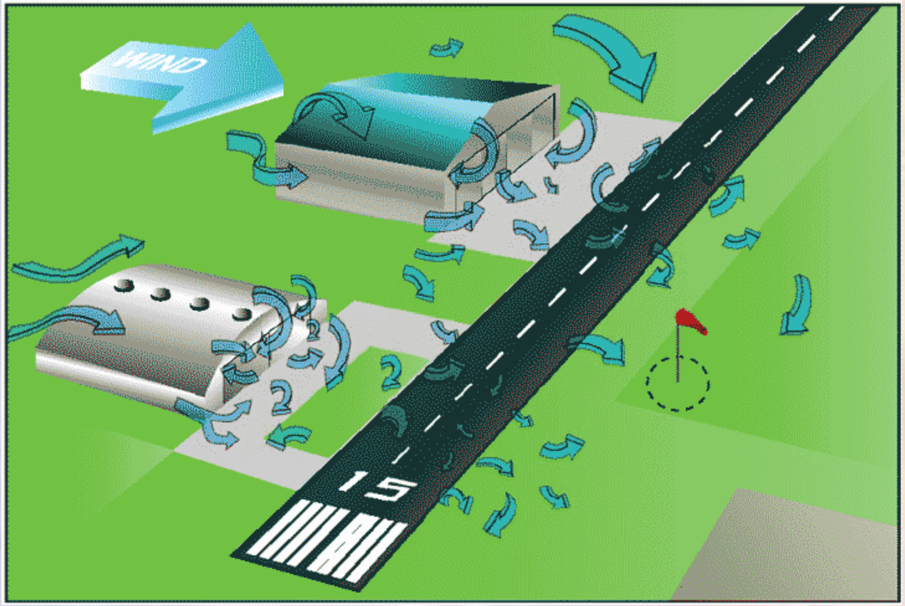
vertical shear $\frac{\partial u(z)}{\partial z}$ is inversely related to height, it is greatest near the ground and decreases with height.

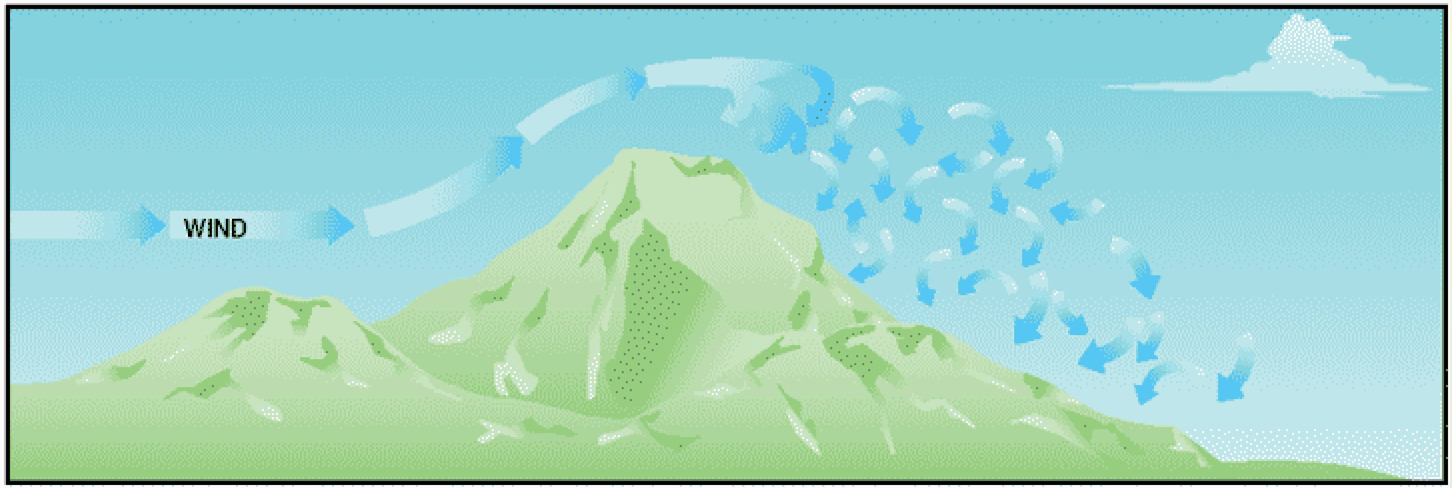
Wind profiles above vegetation

Conceptually, shear is defined as the sum of translation, rigid rotation, and pure deformation (Blackadar, 1997).



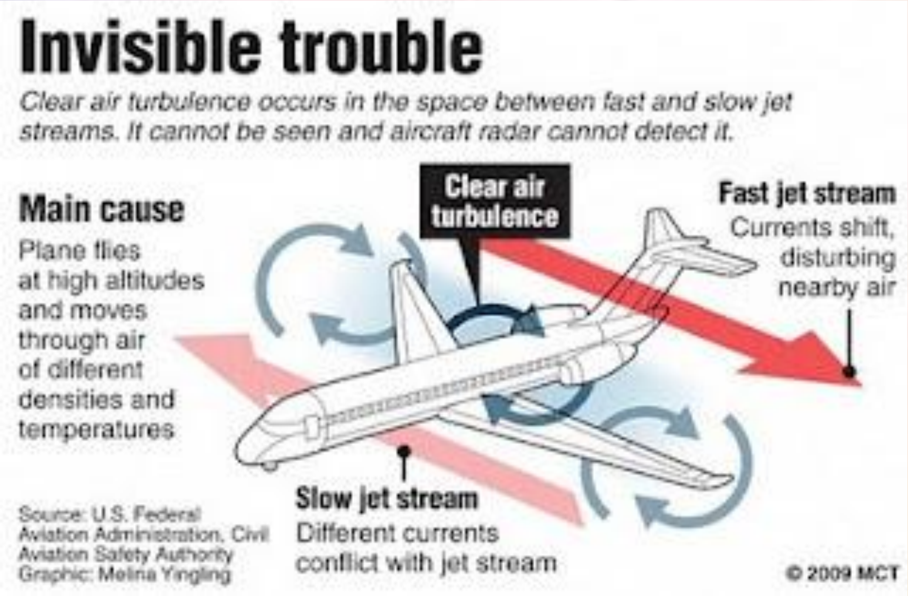
3. Obstacles like trees and buildings deflect the flow, causing turbulent wakes adjacent to, and downwind of the obstacle.





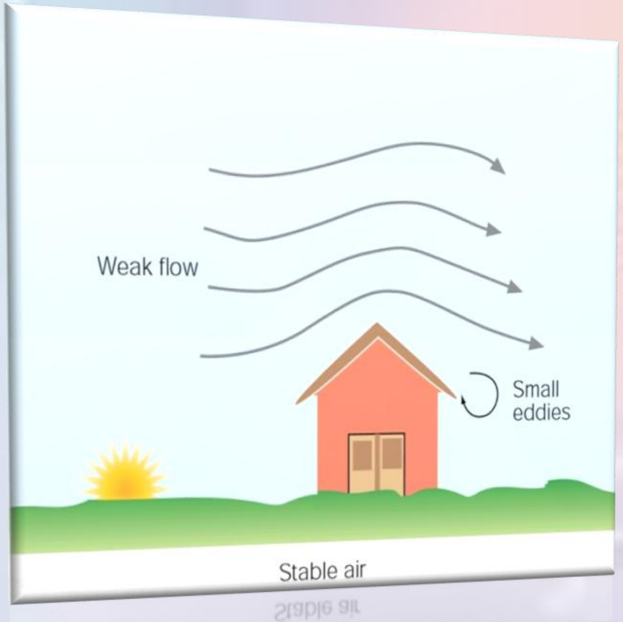
Importance: responsible for the efficient mixing and exchange of mass, heat, and momentum throughout the PBL. Without turbulence, such exchanges would have been at the molecular scale in magnitudes 10^{-3}~10^{-6} times the turbulent transfers that commonly occur.

In the FA, turbulence usually occurs in clouds except CAT (Clear Air Turbulence)



Eddies

When the wind encounters a solid object, a whirl of air or eddy forms on the object's downwind side. The size and shape of the eddy often depends upon the size and shape of the obstacle and the speed of the wind.



Turbulence

Group of eddies of different size.

Eddies range in size from a couple of millimeters to the size of the boundary layer.

Simply defined as perturbation from the mean

$$u = \overline{u} + u'$$

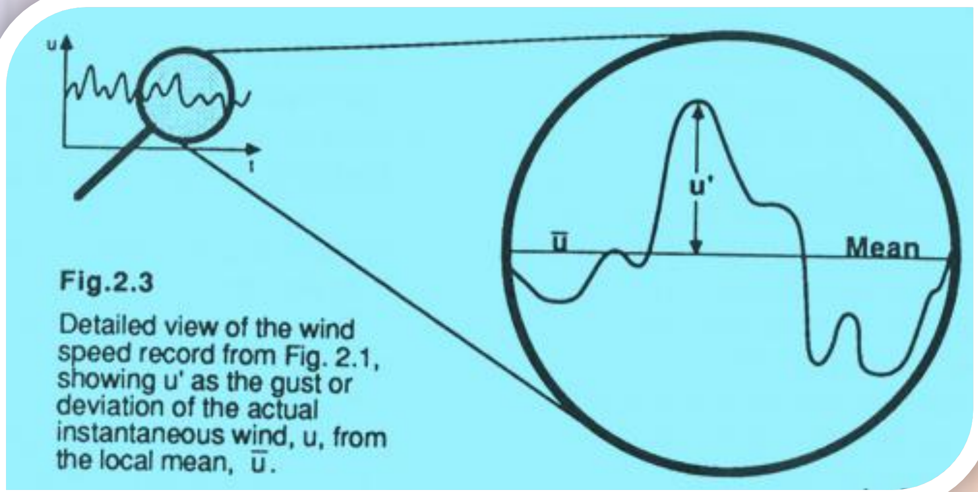


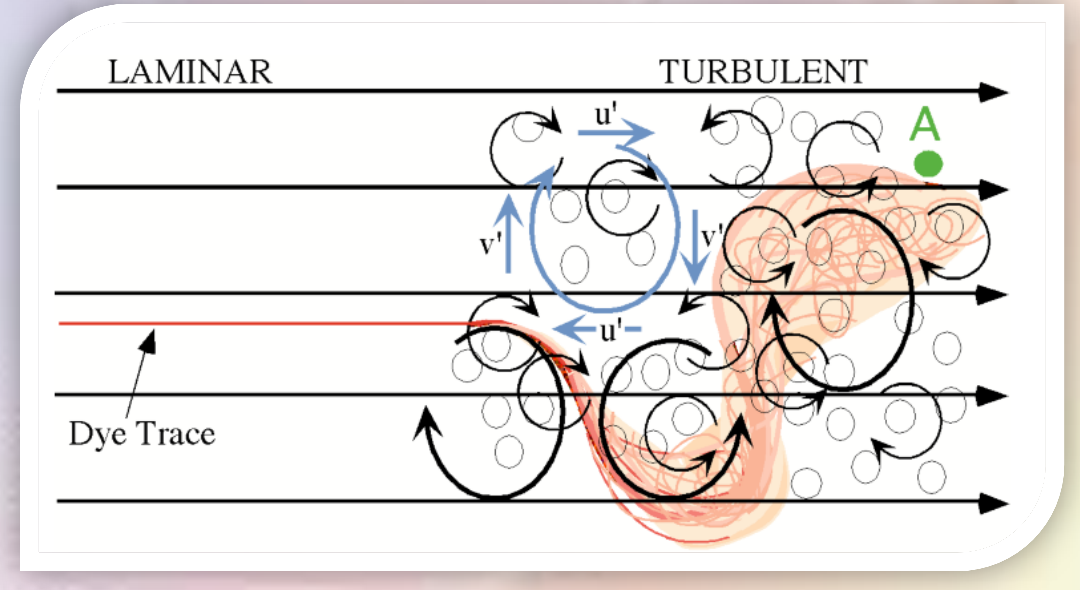
Fig.2.3

Detailed view of the wind speed record from Fig. 2.1, showing u' as the gust or deviation of the actual instantaneous wind, u, from the local mean, ū.

Tracer transport in laminar and turbulent flow. The straight, parallel black lines are streamlines, which are everywhere parallel to the mean flow.

In laminar flow the fluid particles follow the streamlines exactly, as shown by the linear dye trace in the laminar region.

In turbulent flow eddies of many sizes are superimposed onto the mean flow. When dye enters the turbulent region it traces a path dictated by both the mean flow (streamlines) and the eddies. Larger eddies carry the dye laterally across streamlines. Smaller eddies create smaller scale stirring that causes the dye filament to spread (diffuse).



Horizontal Momentum Equations

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + K\frac{\partial^2 u}{\partial z^2} + fv$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + K\frac{\partial^2 v}{\partial z^2} - fu$$

Let us assume that:

(1) the flow is steady state
(2) for simplicity that the flow above the boundary layer is west-east

(3) the flow above the PBL is geostrophic
(4) K is constant (actually varies with z)
(5) f is constant

Making use of the geostrophic relationships:

$$u_g = -\frac{1}{\rho f}\frac{\partial p}{\partial y} \qquad v_g = \frac{1}{\rho f}\frac{\partial p}{\partial x}$$

With these assumptions, our equations become

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + K\frac{\partial^2 u}{\partial z^2} + fv \longrightarrow 0 = K\frac{\partial^2 u}{\partial z^2} + f\left(v - v_g\right) \quad (1)$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + K\frac{\partial^2 v}{\partial z^2} - fu \longrightarrow 0 = K\frac{\partial^2 v}{\partial z^2} - f\left(u - u_g\right) \quad (2)$$

We will now do a mathematical trick to solve for the Ekman spiral

Lets multiply (2) by the imaginary number $i = \sqrt{-1}$ and add it to (1)

$$K\frac{\partial^2 (u + iv)}{\partial z^2} - if\,(u + iv) = -if\left(u_g + iv_g\right)$$

This is a second order inhomogeneous ordinary differential equation

We will solve the equation with boundary conditions:

$$u = 0\ , \quad v = 0, \quad at \quad z = 0$$

$$u \rightarrow u_g\,,\ v \rightarrow v_g\,, \quad at \quad z = \infty$$

$$K\frac{\partial^2(u+iv)}{\partial z^2} - if(u+iv) = -if\left(u_g + iv_g\right)$$

Let's simplify the look of the equation by using

$$N = u + iv \qquad N_g = u_g + iv_g \qquad A = \frac{if}{K}$$

$$\frac{\partial^2 N}{\partial z^2} - \frac{if}{K}N = -\frac{if}{K}N_g$$

This is a simple second order inhomogeneous ordinary differential equation

To find the general solution, we must find a single particular solution to the equation and a complimentary solution to the corresponding homogeneous equation:

$$\frac{\partial^2 N}{\partial z^2} - \frac{if}{K}N = 0$$

Let's seek a particular solution first:

$$\frac{\partial^2 N}{\partial z^2} - \frac{if}{K} N = -\frac{if}{K} N_g$$

Clearly, one solution of the inhomogeneous equation is obtained by assuming that N is independent of z. This reduces the equation to

$$N = N_g$$

Now let's seek a complementary solution to the homogeneous equation

$$\frac{\partial^2 N}{\partial z^2} - \frac{if}{K} N = 0$$

If we seek a solution of the form $N = A\, \mathrm{e}^{\lambda z}$ we get

$$\lambda^2 = \frac{if}{K}$$

Then λ can have two values:

$$\lambda^2 = \frac{if}{K} \qquad \sqrt{i} = \frac{1+i}{\sqrt{2}}$$

$$\lambda_1 = \frac{1+i}{\sqrt{2}}\sqrt{\frac{f}{K}} \qquad \lambda_2 = \frac{-1-i}{\sqrt{2}}\sqrt{\frac{f}{K}}$$

Let's define $\gamma = \sqrt{\frac{f}{2K}}$

$$\lambda_1 = (1+i)\gamma \qquad \lambda_2 = -(1+i)\gamma$$

The general solution to the homogeneous equation is therefore

$$N = A\,e^{\lambda_1 z} + B\,e^{\lambda_2 z}$$

$$u + iv = A\,e^{\lambda_1 z} + B\,e^{\lambda_2 z}$$

The first term on the right, $Ae^{\lambda_1 z} \to \infty \;\; as \;\; z \to \infty$

The only way the solution can be finite is for $A = 0$

$$u + iv = B\,e^{-(1+i)\gamma z}$$

$$u + iv = B\,e^{-\gamma z}\, e^{-i\gamma z}$$

The complete solution is the sum of the particular solution to the inhomogeneous equation and the general solution to the homogeneous equation

$$u + iv = B\,e^{-\gamma z}\, e^{-i\gamma z} + (u_g + iv_g)$$

We will solve the equation with boundary conditions:

$$u = 0, \quad v = 0, \quad at \;\; z = 0$$

$$u \to u_g, \quad v \to v_g, \quad at \;\; z = \infty$$

Setting z = 0

$$0 = B + (u_g + iv_g)$$

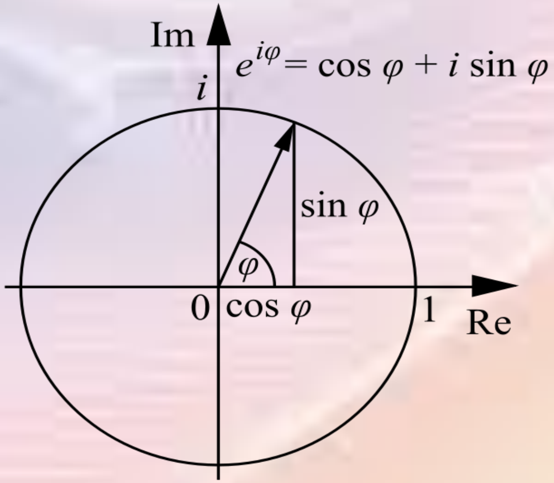
$$B = -(u_g + iv_g)$$

$$u + iv = -(u_g + iv_g)\mathrm{e}^{-\gamma z}\,\mathrm{e}^{-i\gamma z} + (u_g + iv_g)$$

$$u + iv = (u_g + iv_g)(1 - \mathrm{e}^{-\gamma z}\,\mathrm{e}^{-i\gamma z})$$

Dividing into the real and imaginary parts

Euler's formula



$$u + iv = (u_g + iv_g)\left[1 - e^{-\gamma z}\left(\cos\gamma z - i\sin\gamma z\right)\right]$$

$$u + iv = (u_g + iv_g)\left[1 - e^{-\gamma z}\cos\gamma z + i\,e^{-\gamma z}\sin\gamma z\right]$$

For simplicity, we will now assume that the geostrophic wind is zonal (west-east) so the $v_g = 0$

$$u + iv = u_g\left[1 - e^{-\gamma z}\cos\gamma z + i\,e^{-\gamma z}\sin\gamma z\right]$$

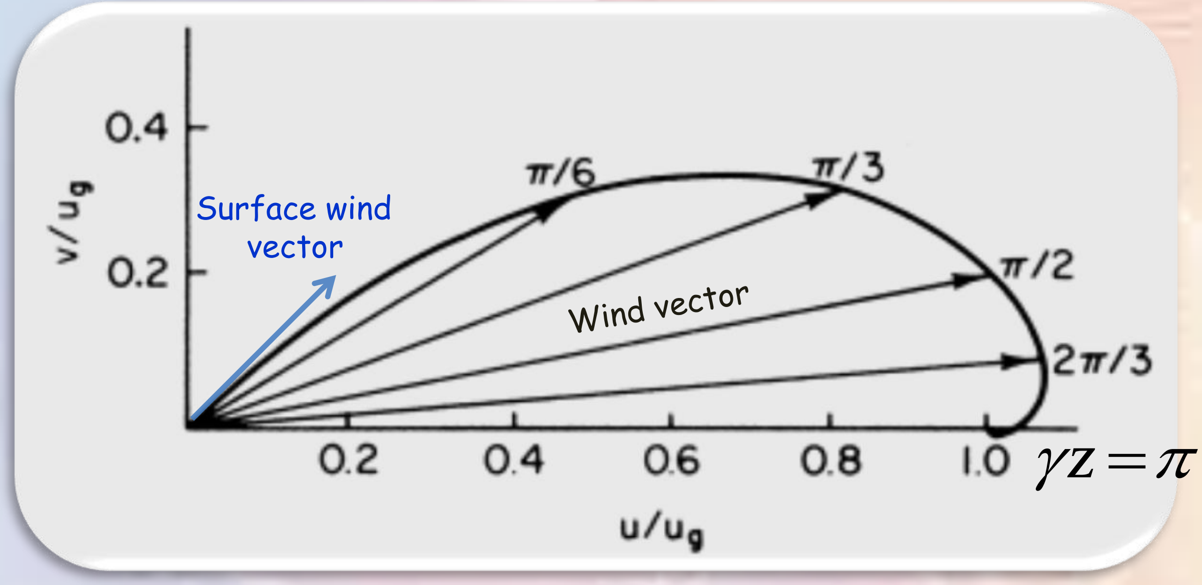
$$u = u_g(1 - e^{-\gamma z}\cos\gamma z)$$

$$\gamma^2 = \frac{f}{2K}$$

$$v = u_g(e^{-\gamma z}\sin\gamma z)$$

These equations describe the Ekman Spiral

$$u = u_g(1 - e^{-\gamma z}\cos\gamma z) \qquad v = u_g(e^{-\gamma z}\sin\gamma z)$$



When $z = \pi/\gamma$, the wind is parallel to and nearly equal to the geostrophic value.

Hodograph of wind components in the Ekman spiral solution. Arrows show velocity vectors for several levels in the Ekman layer, whereas the spiral curve traces out the velocity variation as a function of height. Points labeled on the spiral show the values of γz, which is a nondimensional measure of height.

Effective depth of the boundary layer

We assume the values $f = 10^{-4} s^{-1}$ and $K \cong 10\ m^2 s^{-1}$

The effective height $z_0 = \pi/\gamma$

$$z_0 = \frac{\pi}{\gamma} = \pi\sqrt{\frac{2K}{f}} = \pi\sqrt{\frac{2\times 10}{10^{-4}}} \approx 1400m$$

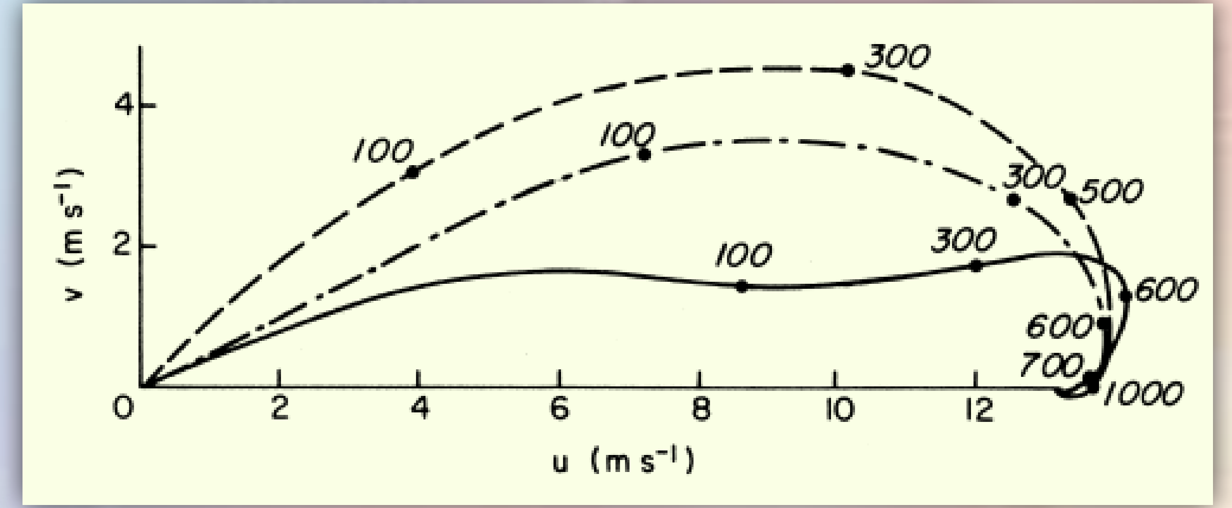
Thus, the effective depth of the Ekman boundary layer is about 1.4 km.

The Ekman theory predicts a cross-isobar flow of 45° at the lower boundary. This is not in agreement with observations.

Better agreement can be obtained by coupling the Ekman layer to a surface layer where the wind direction is unchanging and the speed varies logarithmicaly.

This can be done by taking a boundary condition.

The solution is then called a modified Ekman spiral.



Mean wind hodograph for Jacksonville, Florida (~=30° N), April 4, 1968 (solid line) compared with the Ekman spiral (dashed line) and the modified Ekman spiral (dash-dot line) computed with De~=1200 m.

Heights are shown in meters. (Adapted from Brown, 1970).

Description of the solution in qualitative terms

There is cross-isobar flow towards low pressure.

The velocity vanishes at the lower boundary.

The velocity tends to the geostrophic flow at high levels.

For small z, we have $u \cong u_g(z)$ and $v \cong u_g(z)$.

Thus,the flow near the surface is 45 to the left of the limiting geostrophic flow (purely zonal).

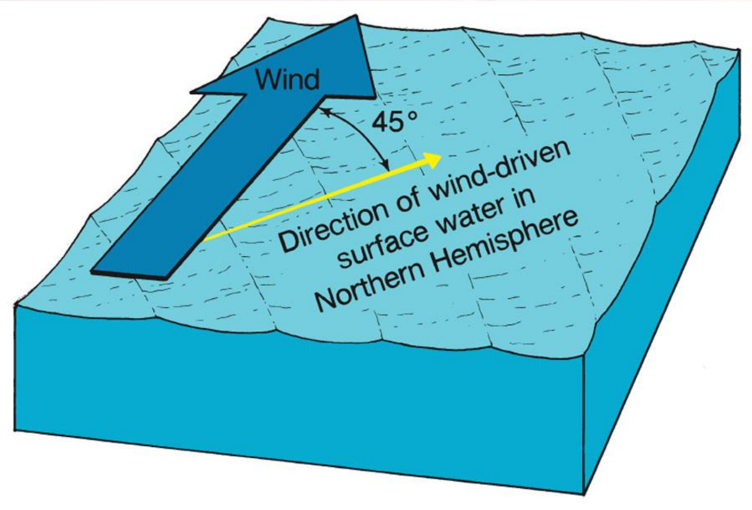
The hodograph of the velocity against height is a clockwise spiral converging to $(u_g, 0)$.

The velocity reaches a maximum at the first zero of v, which is at $\gamma z = \pi$.

The flow is super-geostrophic at this point.

The height where this occurs may be taken as the effective height of the Ekman layer.

The wind is close to geostrophic above this height.

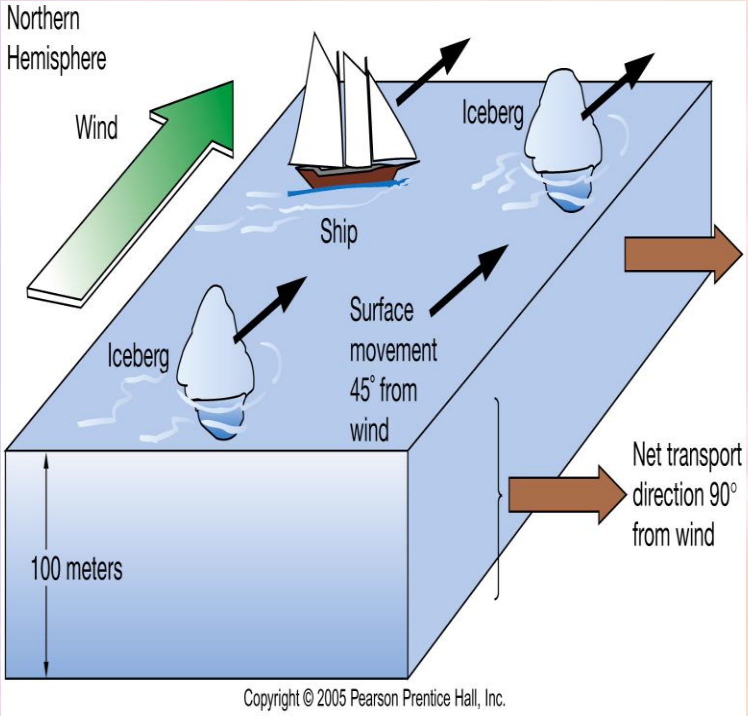


Ekman Motion

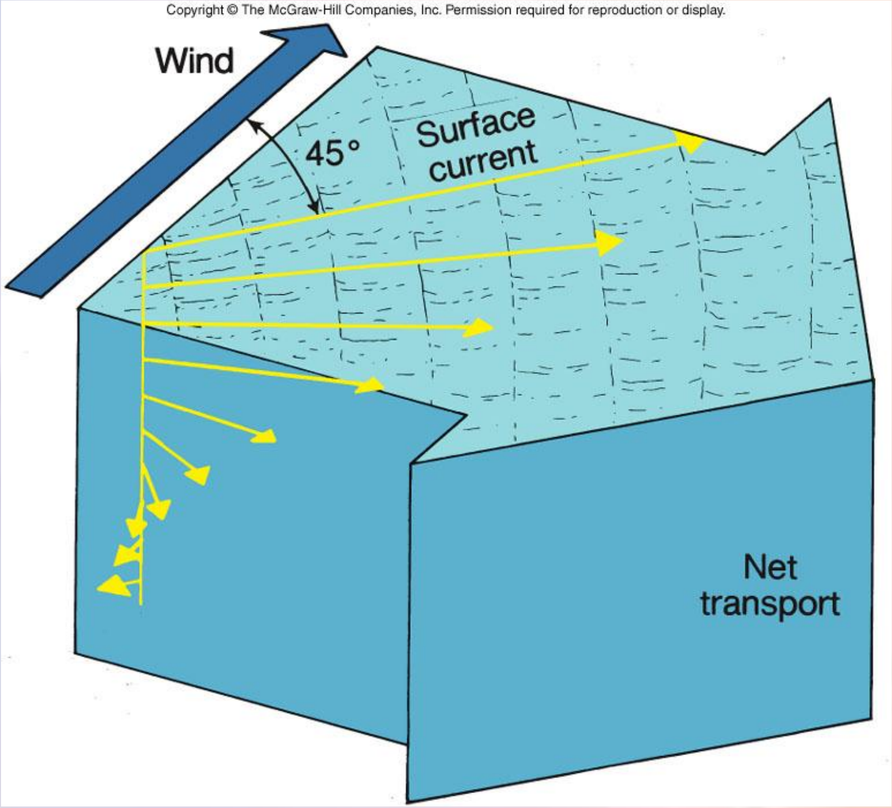
Ekman Motion

Under ideal conditions, the surface layer is deflected 45° to the right of the wind.

In practice, the deflection is usually less.

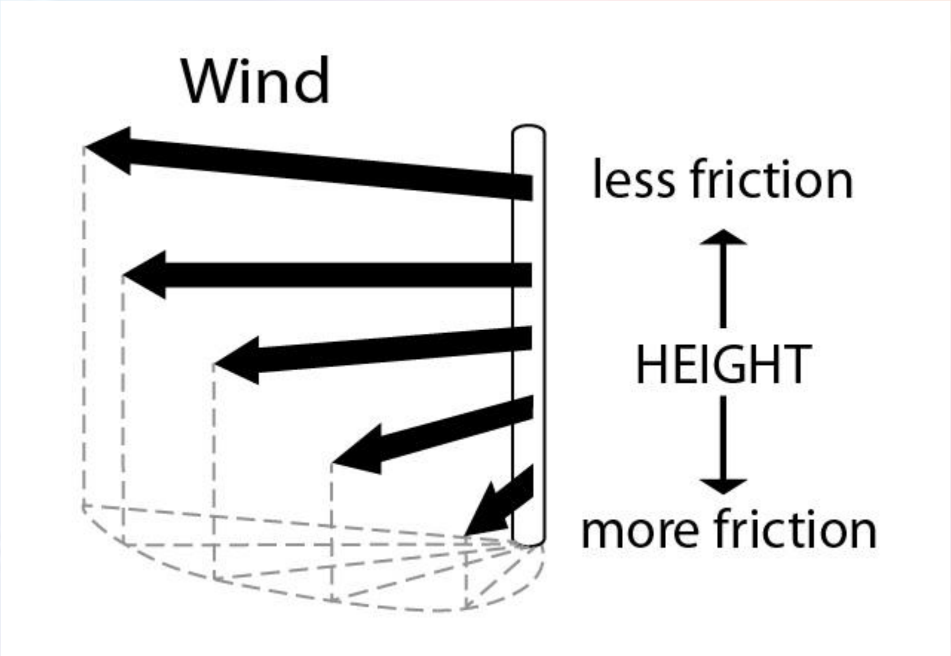


Ekman Motion



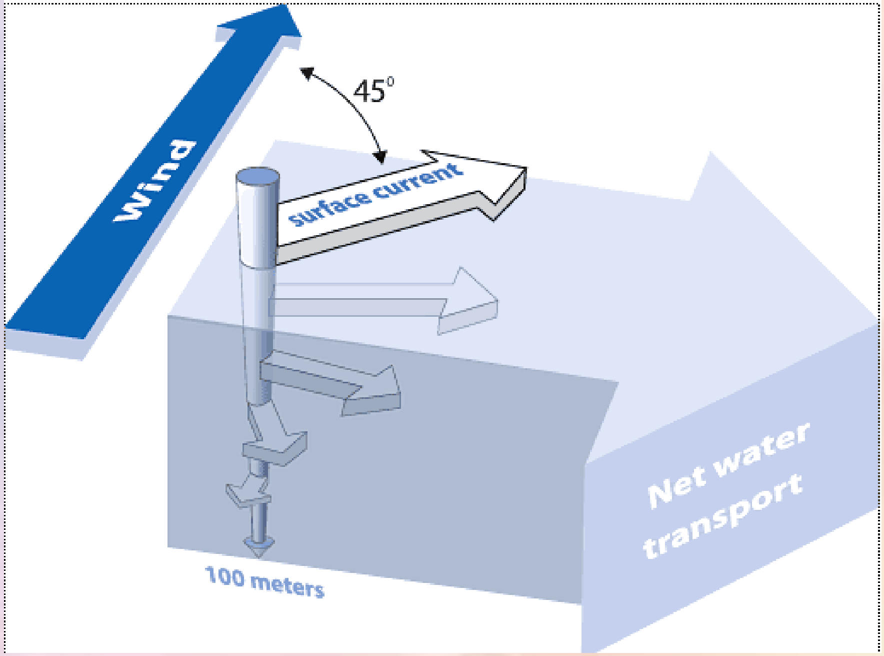
How does the atmosphere move?

Ekman spiral

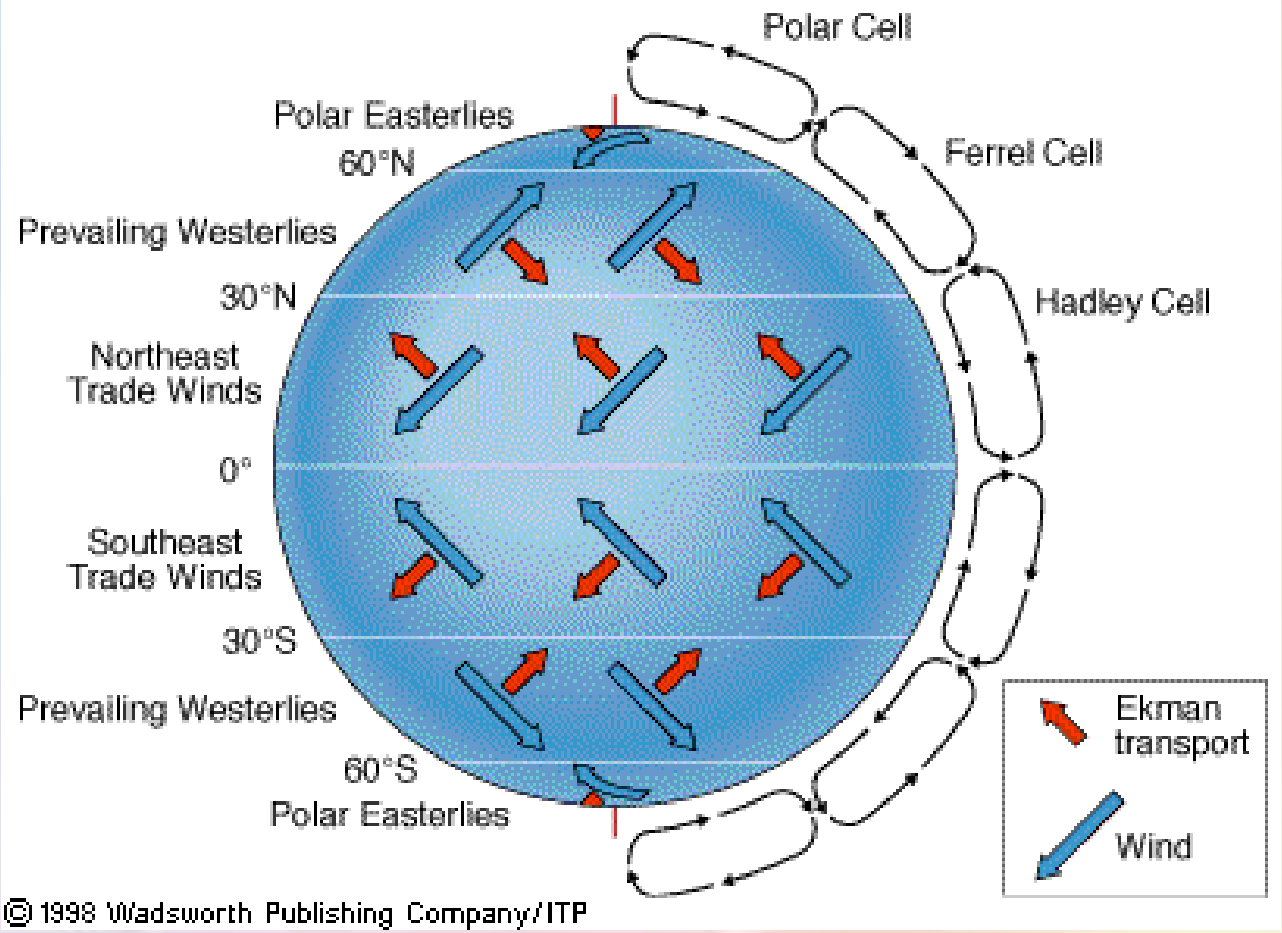


The Ekman spiral occurs as a consequence of the Coriolis effect. When surface water molecules are moved by the wind, they drag deeper layers of water molecules below them.

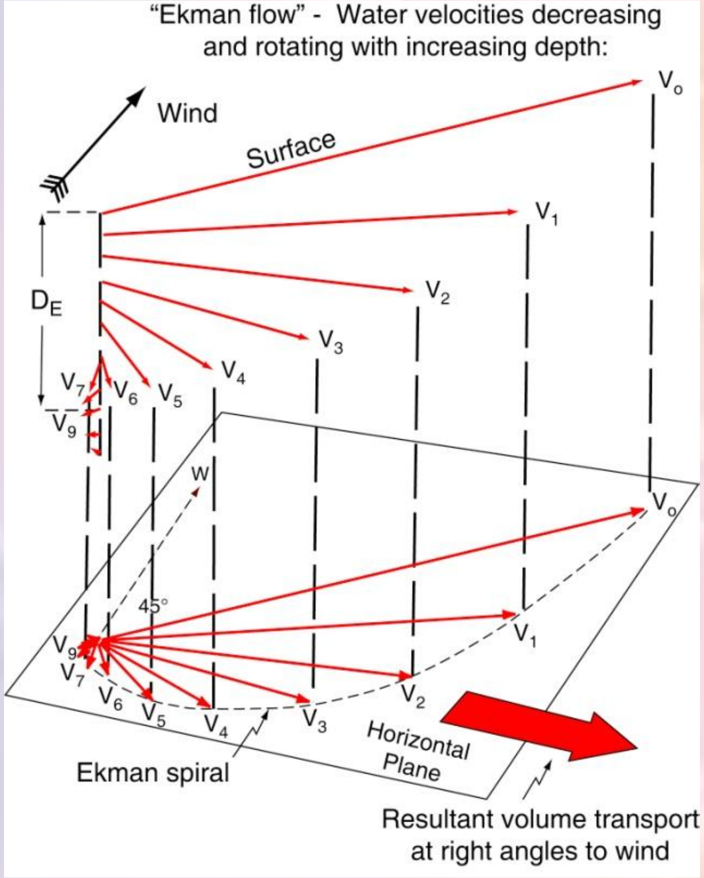
Like surface water, the deeper water is deflected by the Coriolis effect—to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. As a result, each successively deeper layer of water moves more slowly to the right or left, creating a spiral effect.



Open Ocean Surface Currents
Ekman surface transport

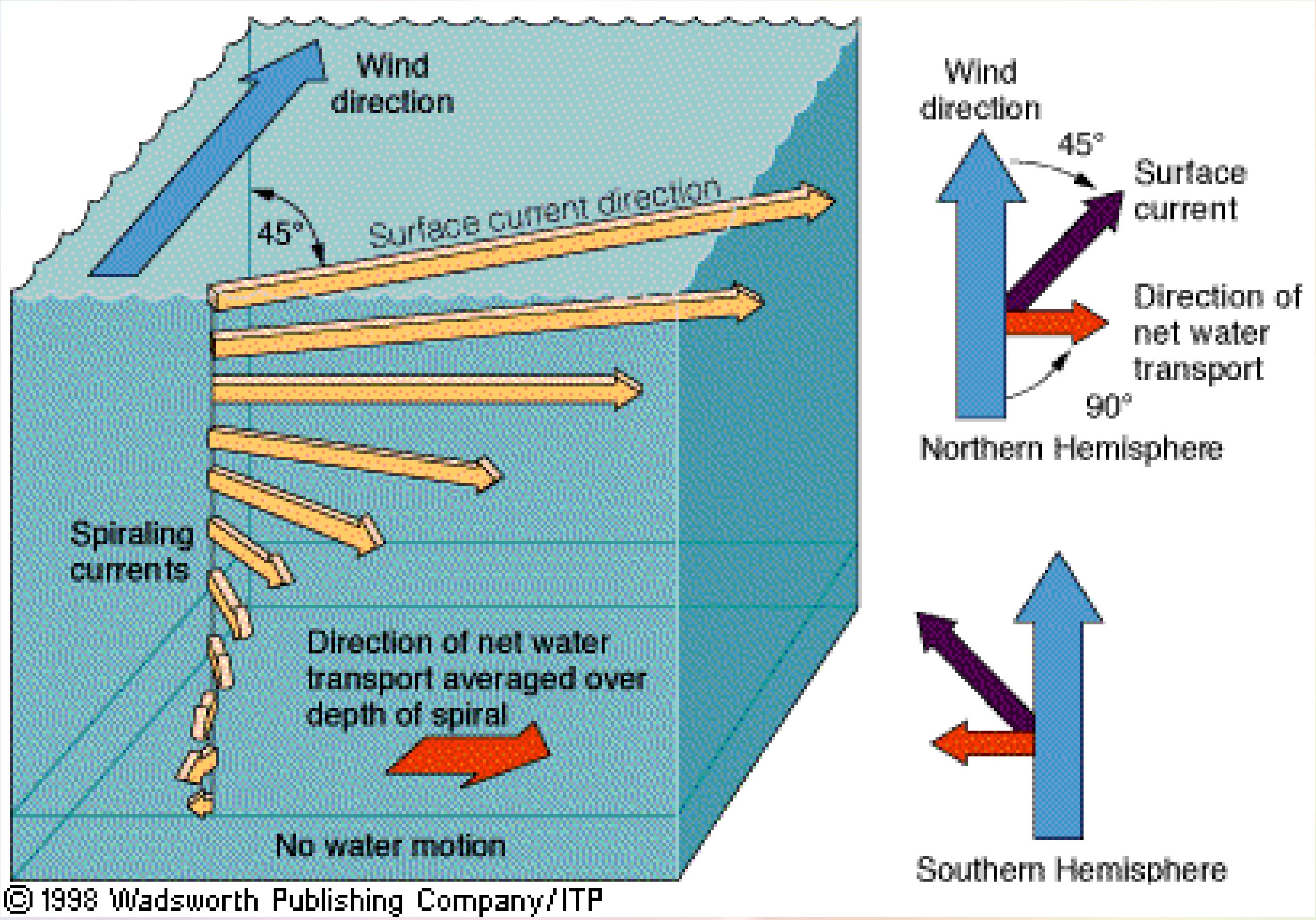


Ekman layer velocities (Northern Hemisphere). Water velocity as a function of depth (upper projection) and Ekman spiral (lower projection).



The large open arrow shows the direction of the total Ekman transport, which is perpendicular to the wind.

Ekman Spiral



Wind Hodograph

A wind hodograph displays the change of wind speed and direction with height (vertical wind shear) in a simple polar diagram.

Wind speed and direction are plotted as arrows (vectors) with their tails at the origin and the point in the direction toward which the wind is blowing. This is backward from our station model.

The length of the arrows is proportional to the wind speed. The larger the wind speed, the longer the arrow.

Normally only a dot is placed at the head of the arrow and the arrow itself is not drawn.

The hodograph is completed by connecting the dots!

Hodograph - Example

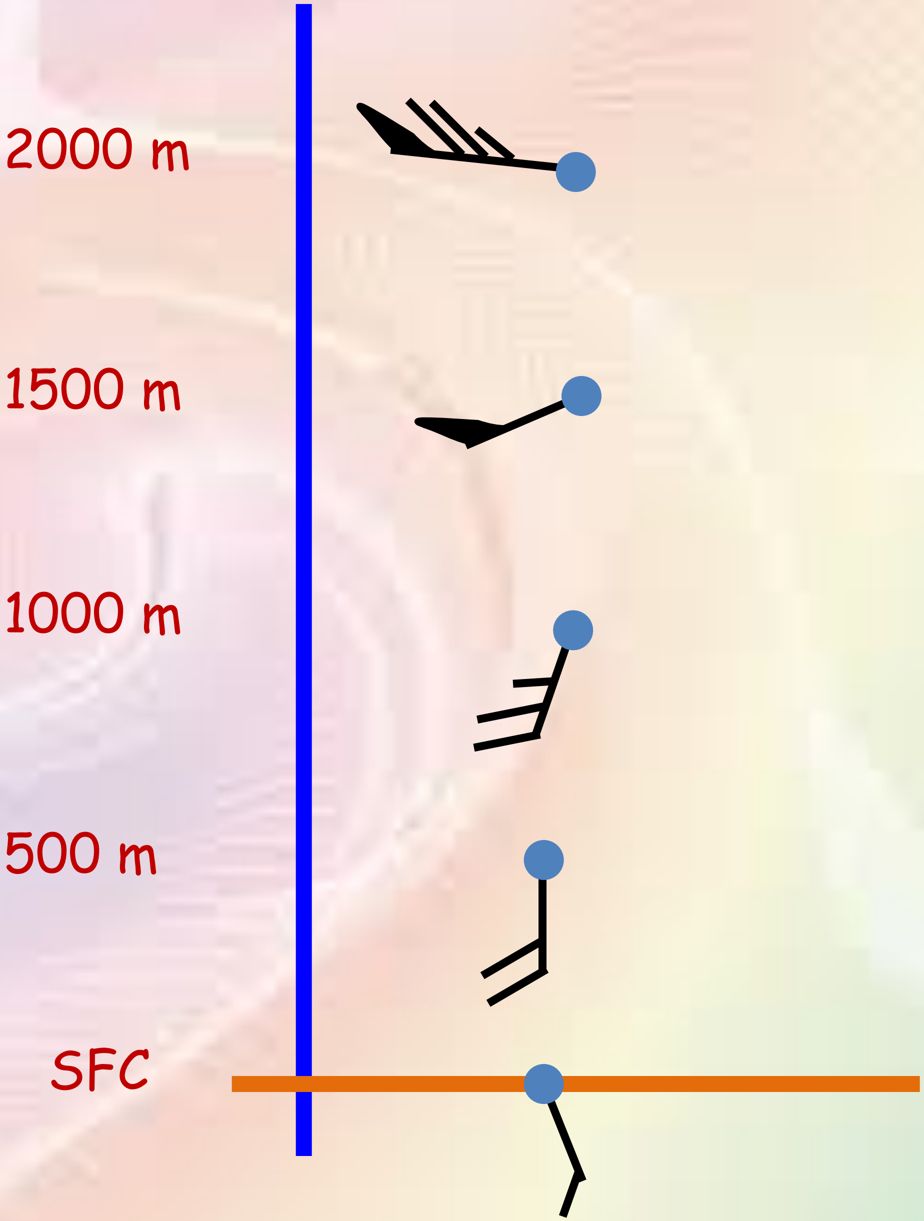
Height (MSL)	Direction	Speed (kt)
250 m (SFC)	160	10
500 m	180	20
1000 m	200	35
1500 m	260	50
2000 m	280	75

Just by looking at this table, it is hard (without much experience) to see what the winds are doing and what the wind shear is.

Hodograph - Example

Let us plot the winds using a station model diagram.

This is better but it is time consuming to draw and still is not that helpful.

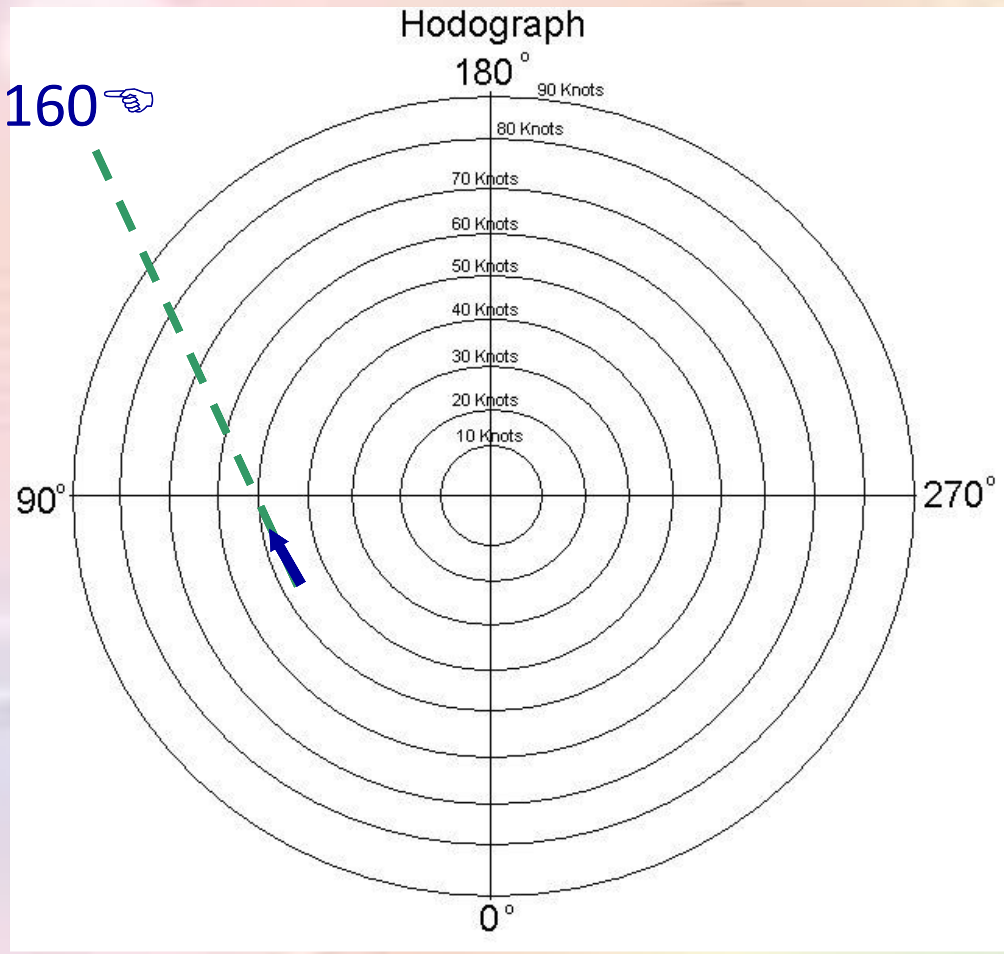


Let us now draw the hodograph!

Let us draw the surface observation. 160° at 10 kts

Since the wind speed is 10 kt, the length of the arrow is only to the 10 knot ring.

The direction points to 160°.

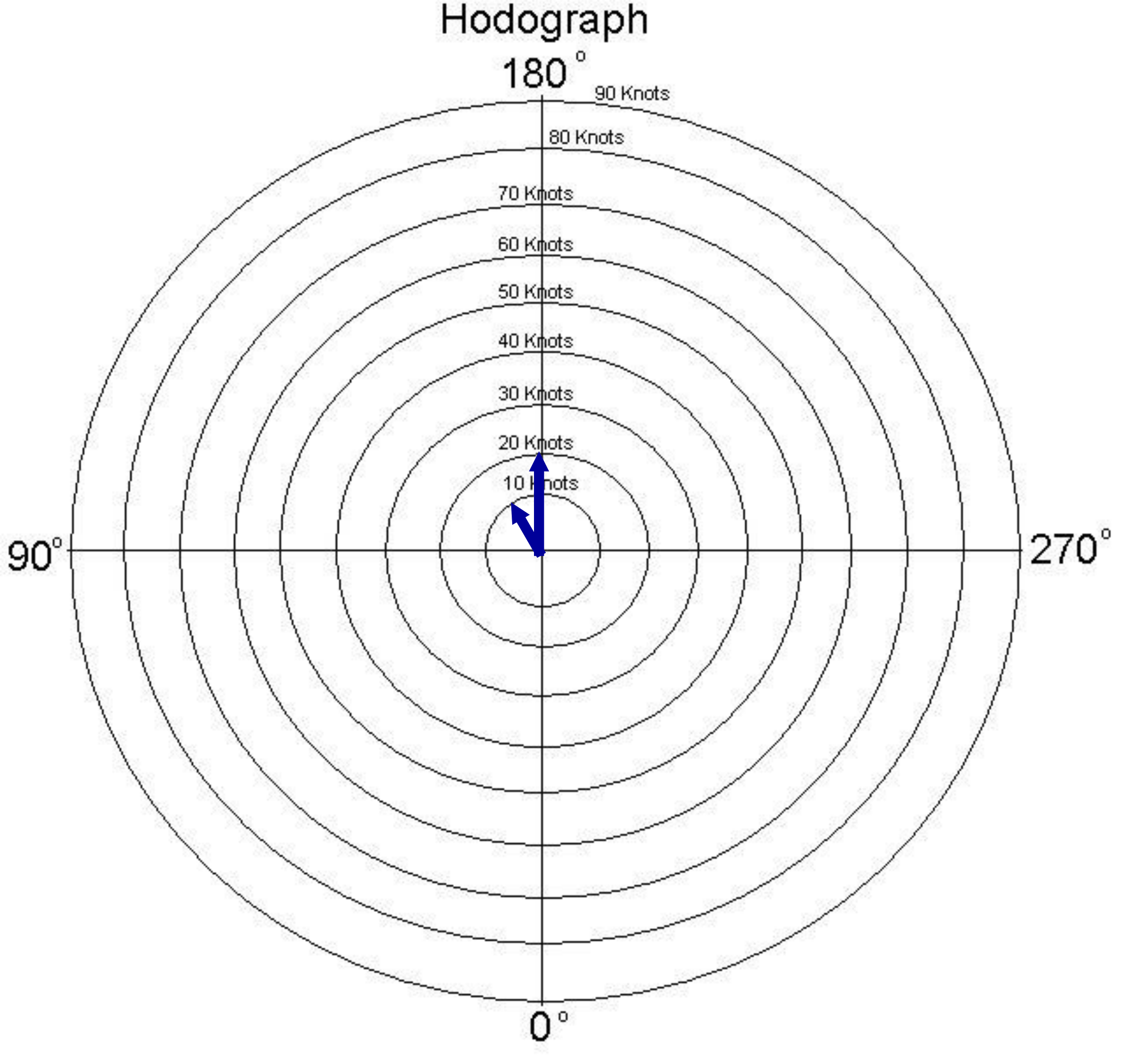


Let us now draw the 500 m observation.

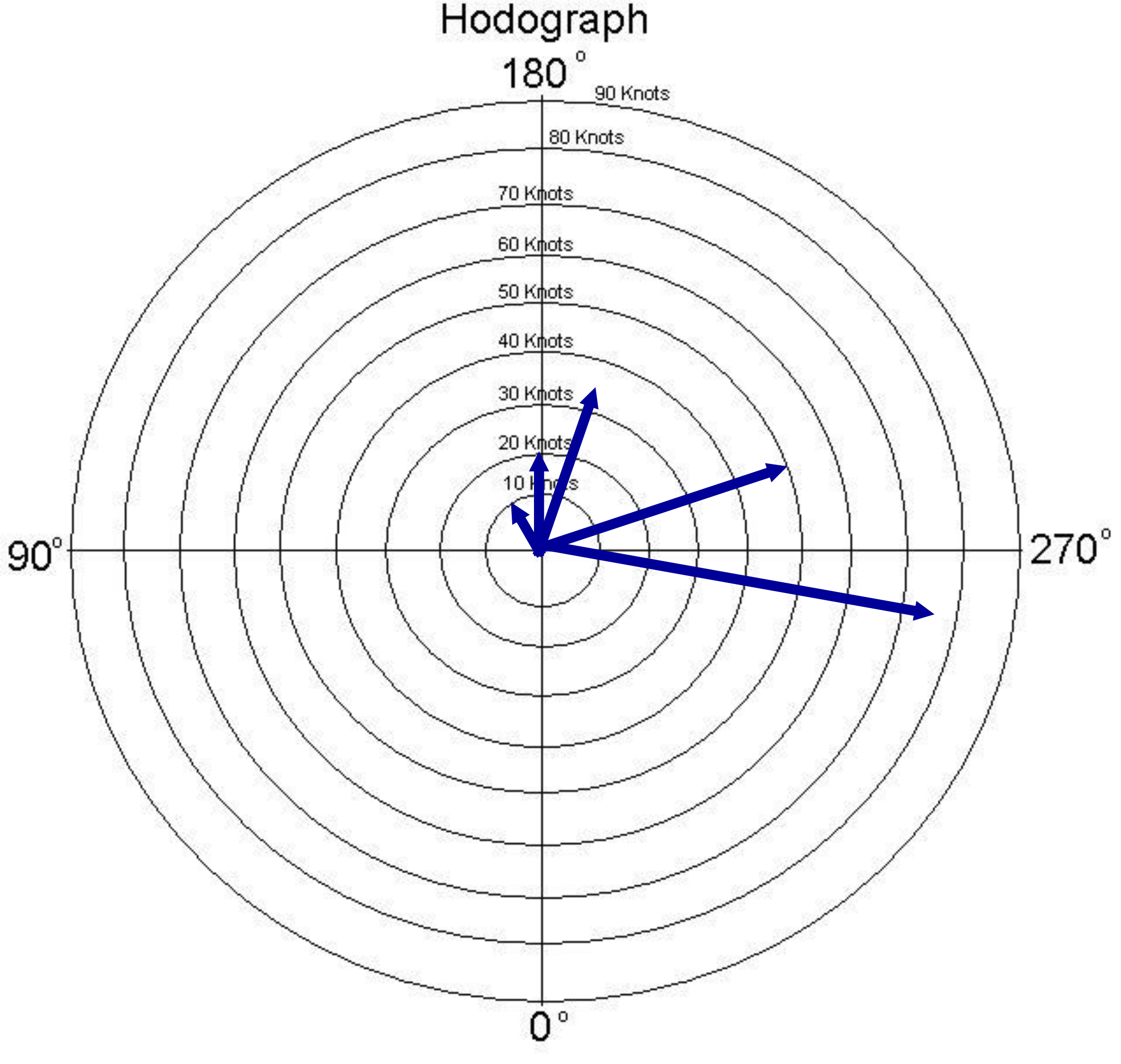
Let us draw the 500 m observation: **180º at 20 kts**

Since the wind speed is 20 kt, the length of the arrow is only to the 20 knot ring.

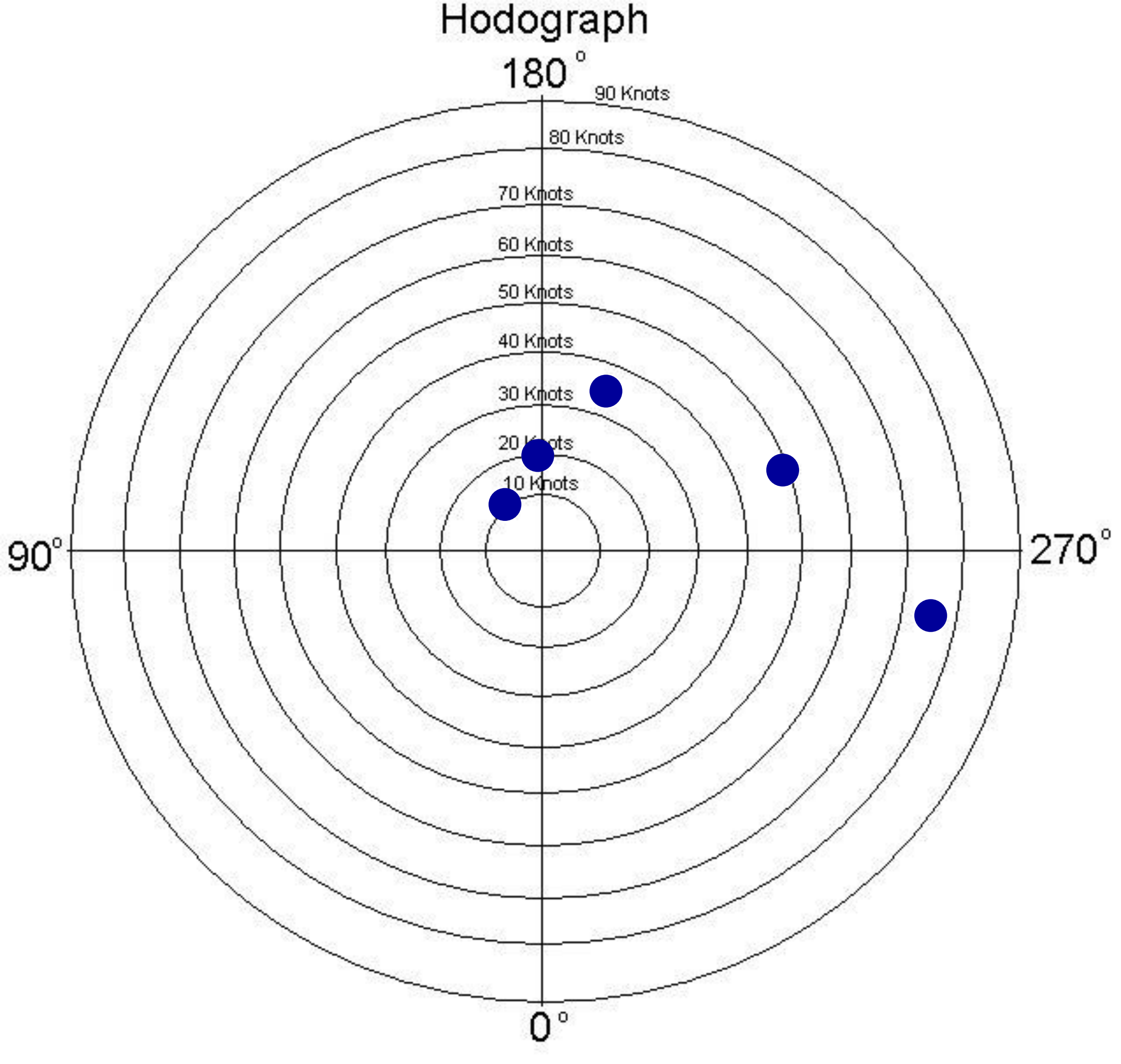
The direction points to 180º.



Let us now draw the remaining observations.

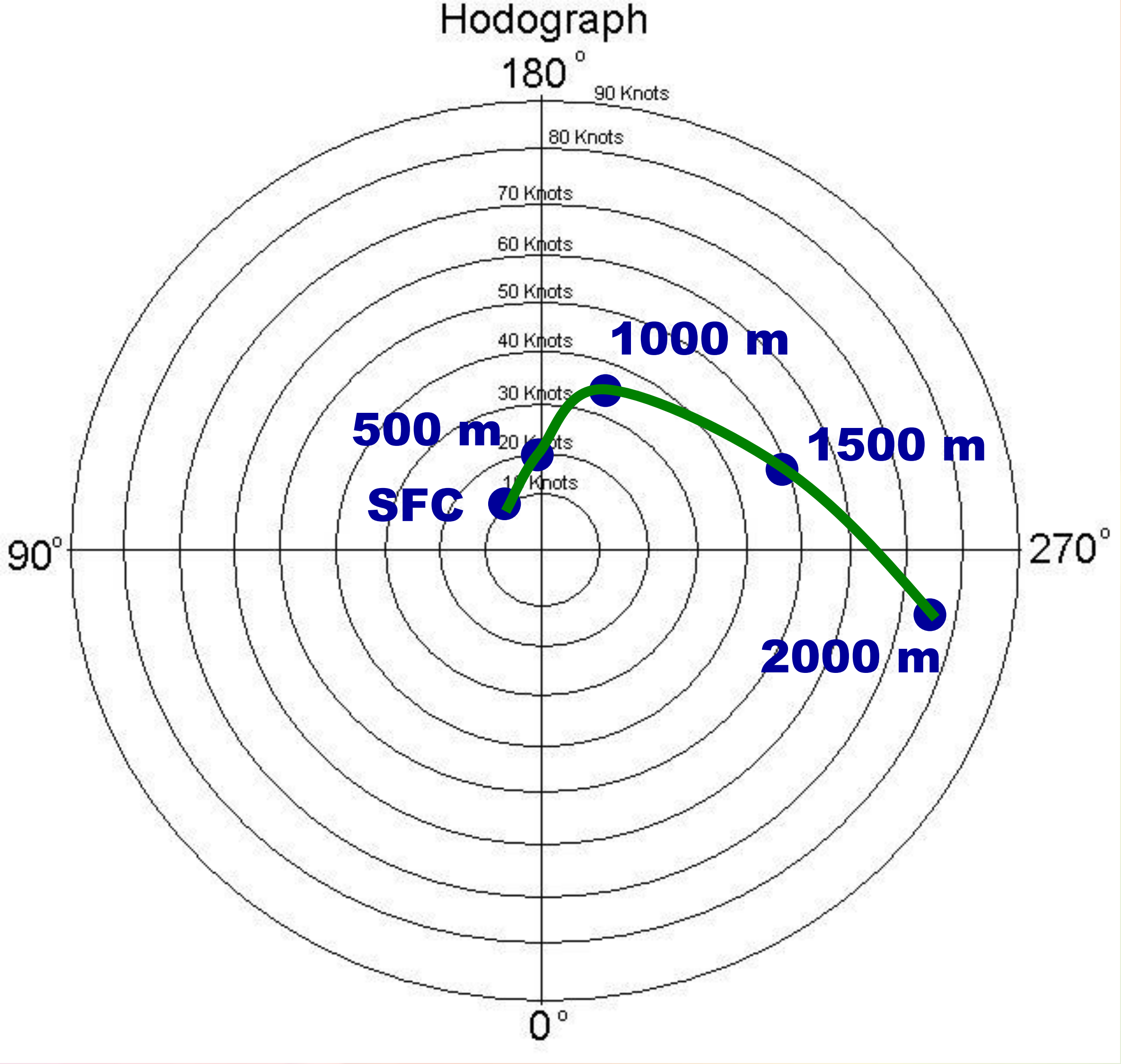


We now place dots at the end of the arrows then erase the arrows



We then connect the dots with a smooth curve and label the points

This is the final hodograph



What can we learn from this diagram?

We see that the wind speeds increase with height.

We know this since the plotted points get farther from the origin as we go up.

We see that the winds change direction with height.

In this example we see that the hodograph is curved and it is curved ***clockwise***.

If we start at the surface (SFC) and follow the hodograph curve, we go in a clockwise direction!