دانشگاه رازی

Dynamic Meteorology 2

Lecture 14

Sahraei

Physics Department

Razi University

http://www.razi.ac.ir/sahraei

Gravity waves

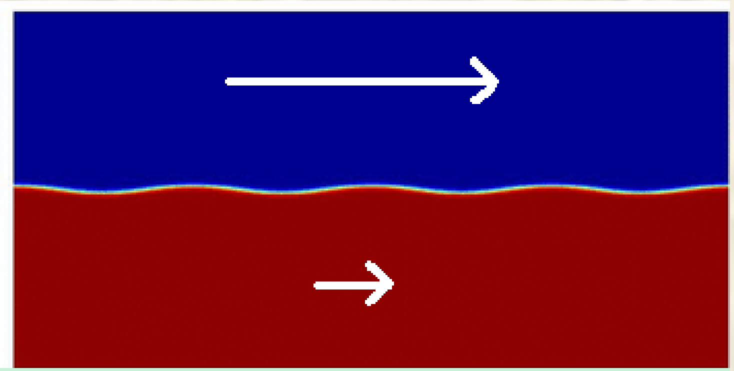
Gravity waves are not something outside your daily experience. Have you ever watched the wake that forms behind a boat? The waves you see are gravity waves.

Every noticed the clouds which form in regular bands of cloud and clear sky? These clouds are the result of gravity waves.

A gravity wave is an oscillation caused by the displacement of an air parcel which is restored to its initial position by gravity.

The lifting force is buoyancy, while the restoring force is gravity, so a few scientists feel they should be called buoyancy waves!

The buoyancy force is proportional to the difference in air temperature inside and outside an air parcel.

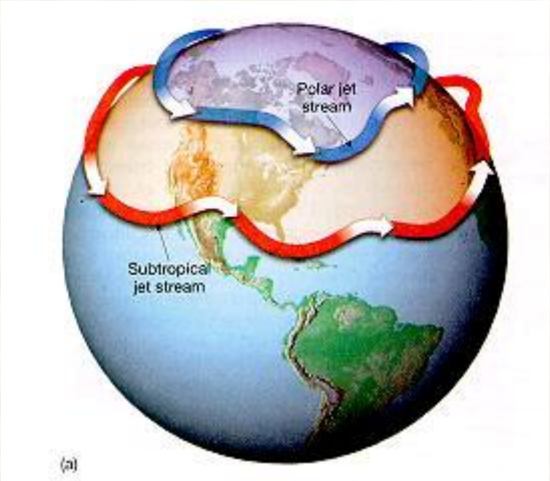


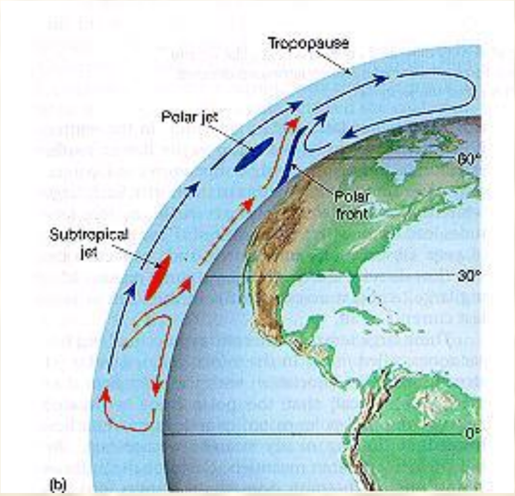
The air in the upper (blue) layer is moving faster than the air in the lower (red) layer, and with enough instability (temperature decreasing with height), these waves form.

Rossby waves

In meteorology, large horizontal atmospheric undulation that is associated with the polar-front jet stream and separates cold polar air from warm tropical air.

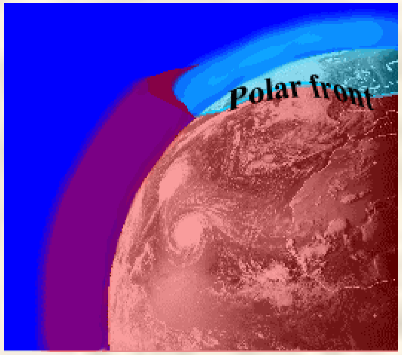
Rossby waves are formed when polar air moves toward the Equator while tropical air is moving poleward.



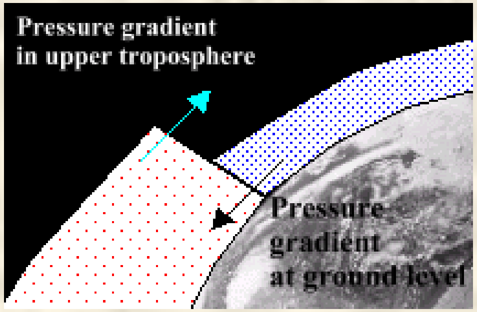


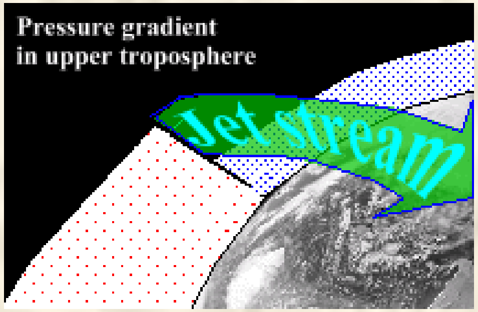
This diagram shows the thicker atmosphere of the tropics converging against the thinner polar atmosphere.

The band on the Earth's surface where this takes place is known as the polar front.



From the diagram, it is clear that this wind would start as a south to north flow but the spin of the earth would deflect it to a westerly wind flow which meanders around the earth at the temperate latitudes. The net effect is a high speed ribbon of high altitude wind. This flows entirely around the globe and - like all fluid motions - has a tendency to meander.



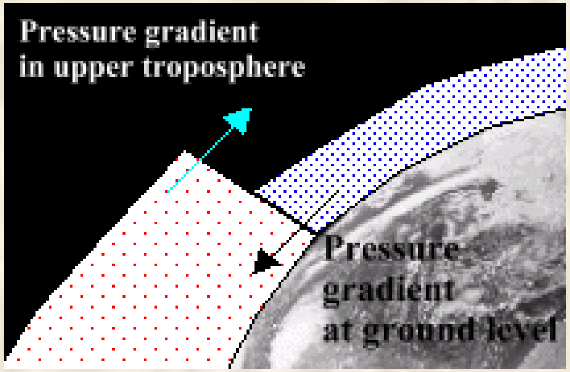


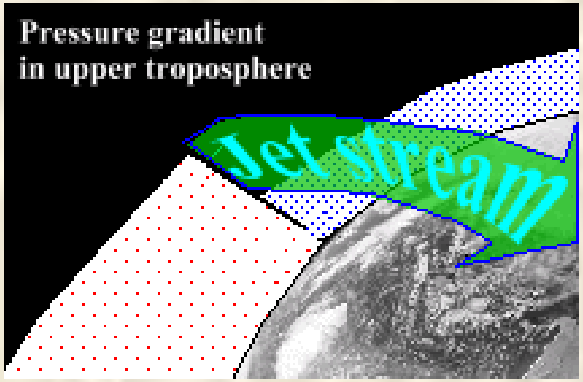
The nature, size and spacing of these meanders has a profound influence on the weather experienced in temperate latitudes.

The meanders in the upper westerly circulation are known as ***Rossby waves***.

Within these upper wind belts particularly strong ribbons of wind form.

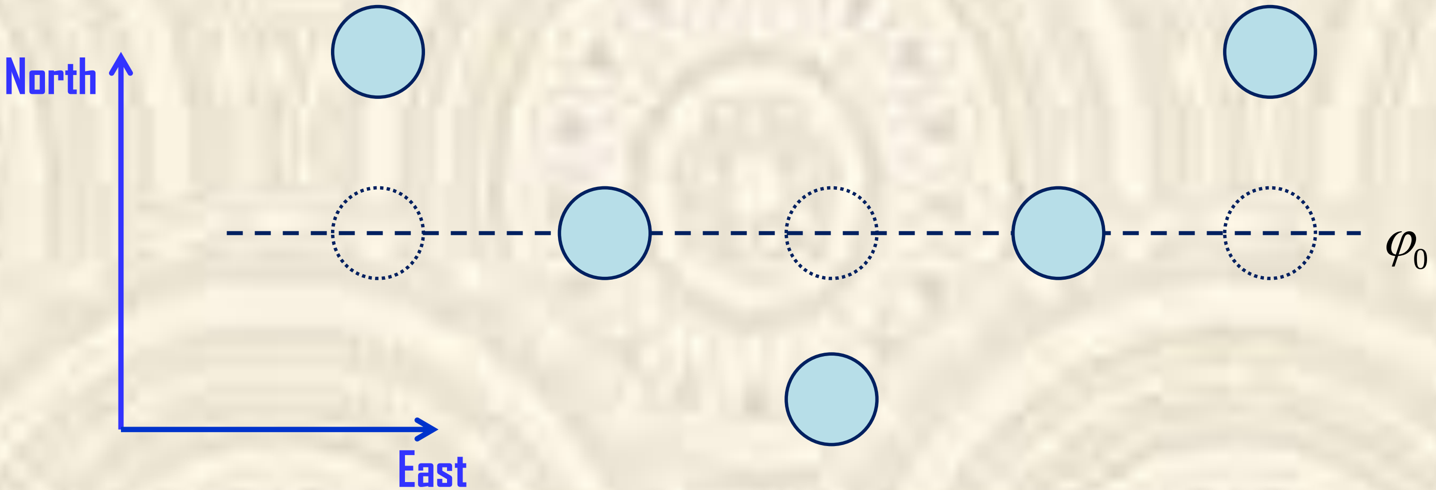
These are known as ***jet streams***. They are strong, narrow currents of wind thousands of kilometres in length, hundreds of kilometres wide and 2 - 4 kilometres deep.



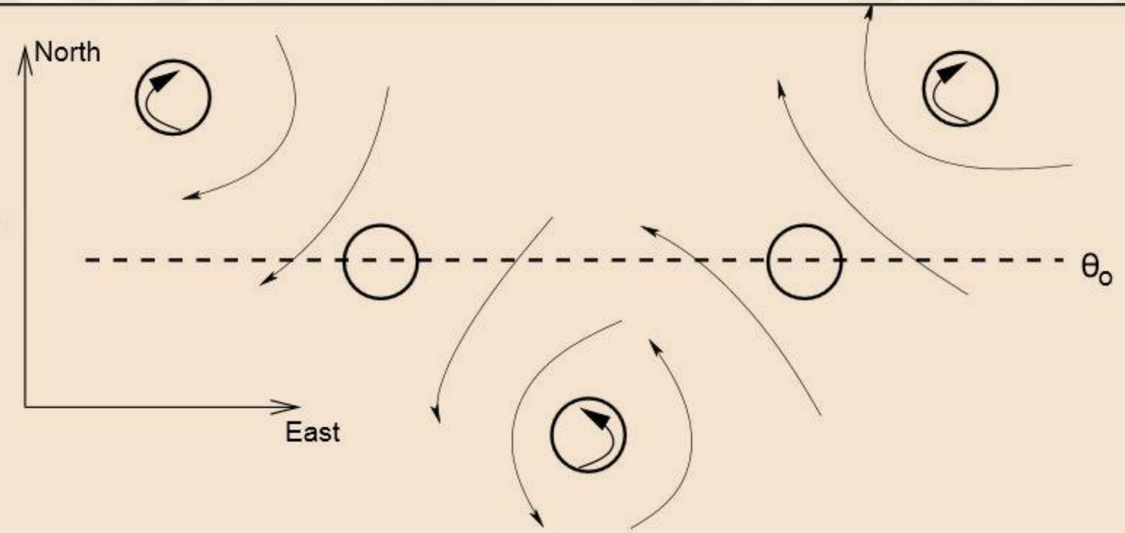


Mechanism of Rossby Wave Propagation

Consider five columns of water all initially at the same latitude, on a β-plane. Move two the columns north and one to south, conserving potential vorticity. As we are assuming the water depth is constant, and thus the fluid column lengths remain constant, conservation of potential vorticity is just conservation of total vorticity $\zeta + f$.

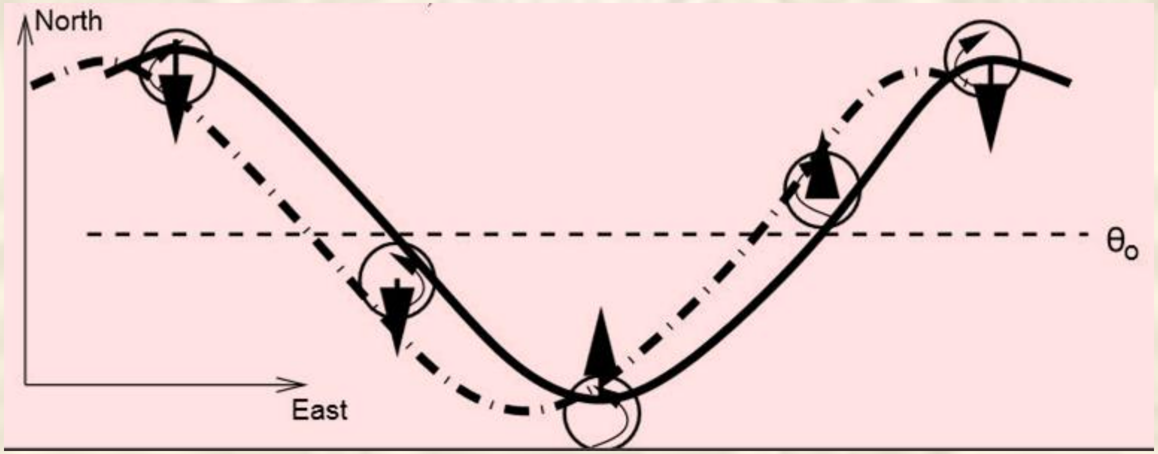


The two columns moved north have increased planetary vorticity and so get anti-cyclonic relative vorticity. The column moved south has decreased planetary vorticity and thus gets cyclonic relative vorticity. This vorticity field generates a flow field.



This flow field tends to move the other two columns. But as soon as they are displaced, they too have vorticity. This vorticity tends to restore the initial three columns back to their original position.

Also note that the second column from the left moves toward deep water whereas the fourth column moves toward shallow water. Thus the pattern is moving to the left (from the solid curve to the dash-dot curve).



Propagate zonally from east to west

Results from conservation of potential vorticity, PV = $(\xi + f)/h$

Long-waves in upper-level flow

Rossby waves

4~6 waves

Wavelength
4000~8000 km

ROSSBY WAVES

The variation of the Coriolis parameter with latitude can be approximated by expanding the latitudinal dependence of f in a Taylor series about a reference latitude φ_0 and retaining only the first two terms to yield:

The Coriolis paramter $f = 2\Omega \sin \varphi$ can be expanded using a taylor series around $\varphi = \varphi_o$

$$f = f_0 + \frac{\partial f}{\partial \varphi}\Big|_{\varphi_0} \delta\varphi + \frac{\partial^2 f}{\partial \varphi^2}\Big|_{\varphi_0} \frac{\delta\varphi^2}{2} + \ldots$$

$$f = 2\Omega \sin\varphi_0 + 2\Omega \cos\varphi_0 \delta\varphi + \frac{\partial^2 f}{\partial \varphi^2}\Big|_{\varphi_0} \frac{\delta\varphi^2}{2} + \ldots$$

$$f = f_0 + \beta y$$

$$\beta = \left(\frac{\partial f}{\partial y}\right)_{\varphi_0} = \frac{\partial f}{\partial \varphi}\frac{\partial \varphi}{\partial y} = 2\Omega \cos\varphi_0 / a \qquad y = a\delta\varphi \qquad y = 0 \; at \; \varphi_0$$

(a is the radius of earth)

The approximation is usually referred to as the midlatitude β-plane approximation

The Coriolis parameter changes linearly with latitude

The f-plane approximates *f* as the first term of the Taylor Series, *f* is taken as a constant.

Rossby wave propagation can be understood in a qualitative fashion by considering a closed chain of fluid parcels initially aligned along a circle of latitude.

At the equator, $f_0 = 0$ where upon $$f = \beta y$$

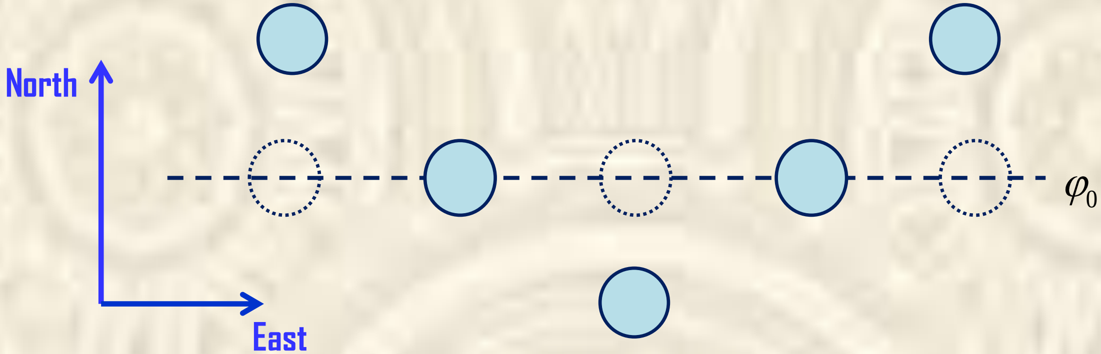
We call this an equatorial beta-plane approximation.

Planetary Waves are Inertial Waves

A planetary wave in its pure form is a type of inertial wave which owes its existence to the variation of the Coriolis parameter with latitude.

An inertial wave is one in which energy transfer is between the kinetic energy of relative motion and kinetic energy of absolute motion.

Such waves may be studied within the framework of the Cartesian equations described above by making the so-called "beta-plane" or "beta-plane" approximation.



Recall that the absolute vorticity $\eta = \zeta + f$

Assume that $\zeta = 0 \; at \; time \; t_0$

Now suppose that at t_1 , δy is the meridional displacement of a fluid parcel from the original latitude.

Then at t_1 we have $(\zeta + f)_{t_1} = f_{t_0}$

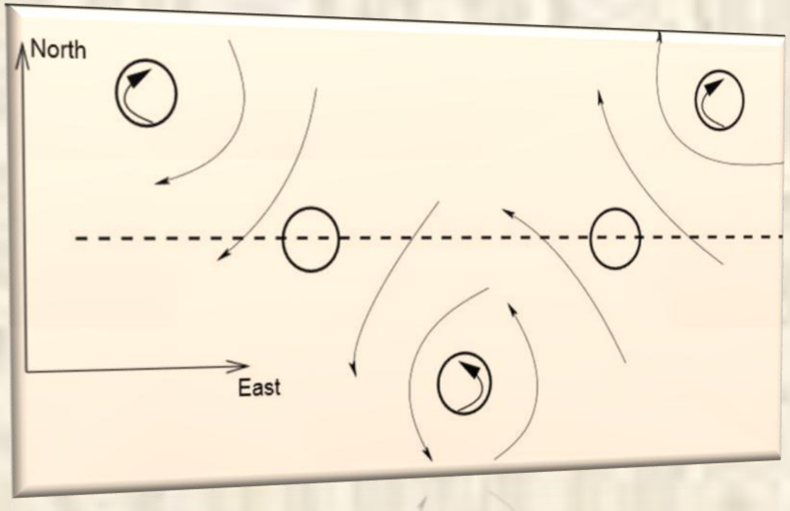
$$\zeta_{t_1} = f_{t_0} - f_{t_1} = -\beta \delta y$$

$\beta = df \, / \, dy$ *is the planetary vorticity gradient at the original latitude*

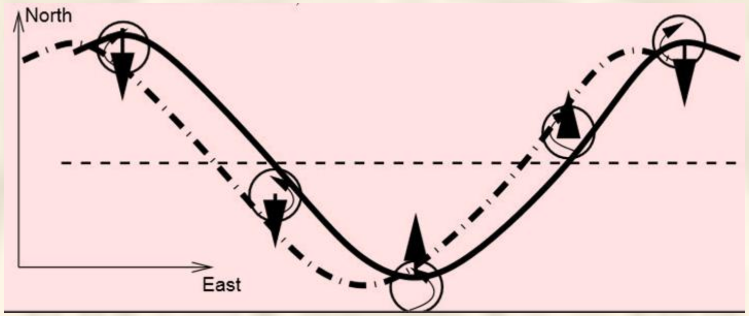
Variation of the Coriolis parameter with latitude, the so called β effect

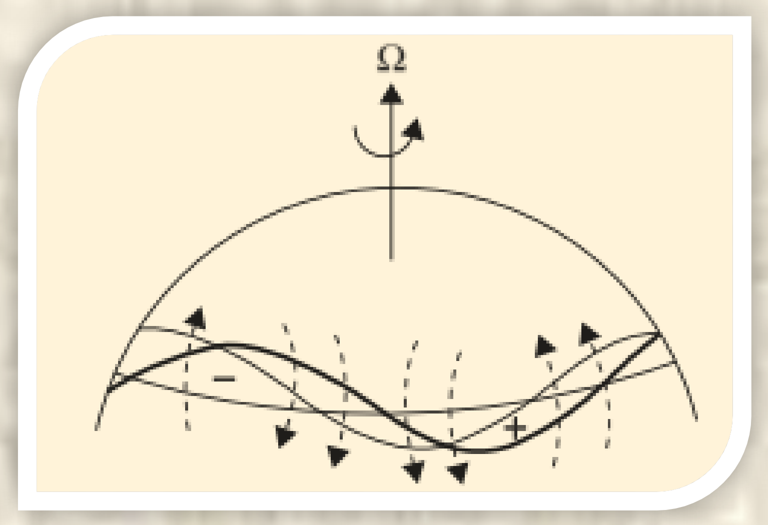
$$\zeta_{t_1} = f_{t_0} - f_{t_1} = -\beta\delta y$$

It is evident that if the chain of parcels is subject to a sinusoidal meridional displacement under absolute vorticity conservation, the resulting perturbation vorticity will be positive for a southward displacement and negative for a northward displacement.



This perturbation vorticity field will induce a meridional velocity field, which advects the chain of fluid parcels southward west of the vorticity maximum and northward west of the vorticity minimum, as indicated in Fig.





$\zeta_{t_1} = -\beta \delta y < 0$ $\qquad$ $\zeta_{t_1} = -\beta \delta y > 0$

The speed of westward propagation, c, can be computed for this simple example by letting

$$\delta y = a\sin\left[k(x-ct)\right]$$

where a is the maximum northward displacement

$$v = \frac{d(\delta y)}{dt} = -kca\cos\left[k(x-ct)\right]$$

$$\zeta = \frac{\partial v}{\partial x} = -k^2 ca\sin\left[k(x-ct)\right]$$

Substitution for δy *and ξ in*

$$\zeta_{t_1} = f_{t_0} - f_{t_1} = -\beta\delta y$$

$$k^2 ca\sin\left[k(x-ct)\right] = -\beta a\sin\left[k(x-ct)\right] \qquad c = -\frac{\beta}{k^2}$$

Thus, the phase speed is westward relative to the mean flow and is inversely proportional to the square of the zonal wave number.

Free Barotropic Rossby Waves

The dispersion relationship for barotropic Rossby waves may be derived formally by finding wave-type solutions of the linearized barotropic vorticity equation.

The barotropic vorticity equation $$\frac{d_h(\zeta + f)}{dt} = 0 \rightarrow (\zeta + f) = \cos\tan t$$

states that the vertical component of absolute vorticity is conserved following the horizontal motion.

For a midlatitude -plane this equation has the form:

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\zeta + \beta v = 0$$

since $$\frac{d_h f}{dt} = \frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} = v\frac{\partial f}{\partial y} \equiv \beta v$$

We now assume that the motion consists of a constant basic state zonal velocity plus a small horizontal perturbation:

$$u = \bar{u} + u' \qquad v = \bar{v} + v' \qquad \zeta = \bar{\zeta} + \zeta'$$

The perturbation form: $$\frac{\partial \zeta'}{\partial t} + \bar{u}\frac{\partial \zeta'}{\partial x} + \bar{v}\frac{\partial \zeta'}{\partial y} + \beta v' = 0$$

If we assume $\bar{v} = 0$: $$\frac{\partial \zeta'}{\partial t} + \bar{u}\frac{\partial \zeta'}{\partial x} + \beta v' = 0$$

To solve this PDE, we recall: $$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \rightarrow \zeta' = \frac{\partial v'}{\partial x} - \cancel{\frac{\partial u'}{\partial y}} = \frac{\partial v'}{\partial x}$$

We define a perturbation streamfunction $\psi'(x, y, t)$ according to:

$$\frac{\partial \psi'}{\partial x} = v' \qquad \frac{\partial \psi'}{\partial y} = -u'$$

$$\zeta' = \nabla^2 \psi'$$

$$\frac{\partial \zeta'}{\partial t} + \bar{u}\frac{\partial \zeta'}{\partial x} + \beta v' = 0$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\nabla^2 \psi' + \beta \frac{\partial \psi'}{\partial x} = 0 \quad *$$

where as usual we have neglected terms involving the products of perturbation quantities. We seek a solution of the form

$$\psi' = \Psi\, \mathrm{e}^{i(kx+ly-\omega t)} = \Psi\, \mathrm{e}^{i\phi} \qquad \phi = (kx + ly - \omega t)$$

Here k and l are wave numbers in the zonal and meridional directions, respectively

Substituting for ψ' in * $\qquad \nabla^2 \psi' = -\left(k^2 + l^2\right)\psi'$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\nabla^2 \psi' = -\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\left(k^2 + l^2\right)\psi' = -\left(k^2 + l^2\right)\left(-i\omega + \bar{u}ik\right)\psi'$$

$$\beta \frac{\partial \psi'}{\partial x} = \beta i k \psi'$$

This gives the dispersion relation

$$-\left(k^2 + l^2\right)\left(-\omega + \bar{u}k\right) + \beta k = 0$$

$$\boxed{\omega = \bar{u}k - \frac{\beta k}{k^2 + l^2}}$$

$$\boxed{\omega = \bar{u}k - \frac{\beta k}{K^2}}$$

$K^2 \equiv k^2 + l^2$ is the total horizontal wave number squared.

Phase speed in the x direction is $c_x = \dfrac{\omega}{k} = \bar{u} - \dfrac{\beta}{K^2}$

$c_R = -\dfrac{\beta}{K^2}$ Depend on the variation of the Coriolis parameter with latitude and wavelength

The zonal phase speed relative to the mean wind is

$$c_x - \bar{u} = -\frac{\beta}{K^2} < 0$$

when the mean wind vanishes and $l \rightarrow 0$

Thus, the Rossby wave zonal phase propagation is always westward relative to the mean zonal flow.

Furthermore, the Rossby wave phase speed depends inversely on the square of the horizontal wave number.

Therefore, Rossby waves are dispersive waves whose phase speeds increase rapidly with increasing wavelength $(c_x \propto 1/K^2)$

$$c_x = \bar{u} - \frac{\beta}{K^2} \qquad c_x = \bar{u} - \frac{\beta L^2}{4\pi^2}$$

1) $c_x > 0 \quad i.e \quad \bar{u} - \frac{\beta}{K^2} > 0 \quad i.e \quad \bar{u} > \frac{\beta}{K^2}$

$$\longrightarrow \bar{u}$$
$$-c_R \longleftarrow$$
$$\longrightarrow c_x$$

2) $c_x < 0 \quad i.e \quad \bar{u} - \frac{\beta}{K^2} < 0 \quad i.e \quad \bar{u} < \frac{\beta}{K^2}$

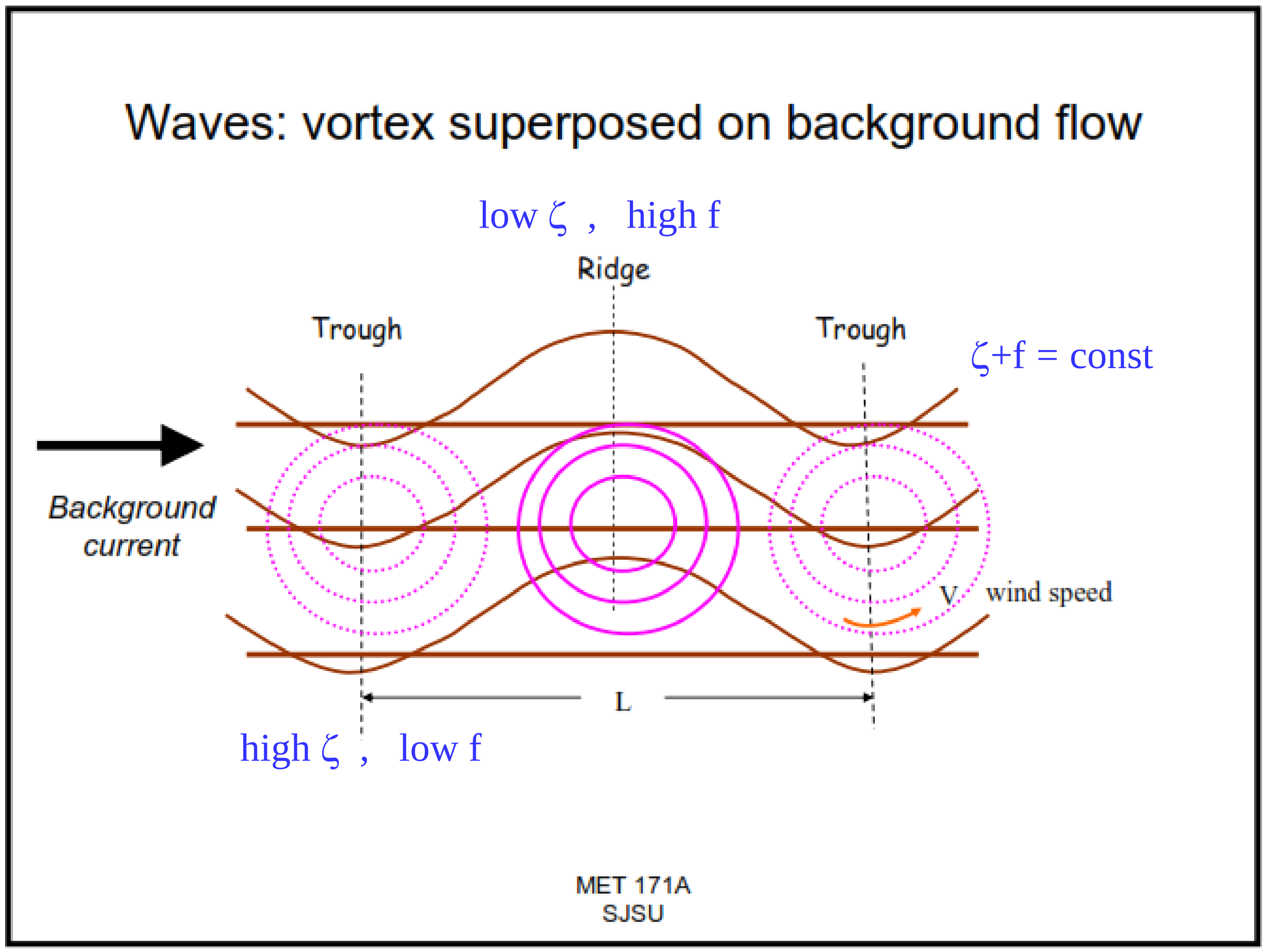
$$\longrightarrow \bar{u}$$
$$-c_R \longleftarrow$$
$$c_x \longleftarrow$$

c_x >0 for small L , c_x < 0 for large L ,

It is noted that the zonal phase speed of Rossby waves is always westward (traveling east to west) relative to mean flow $\bar{u}$, but the zonal group speed of Rossby waves can be eastward or westward depending on wavenumber.

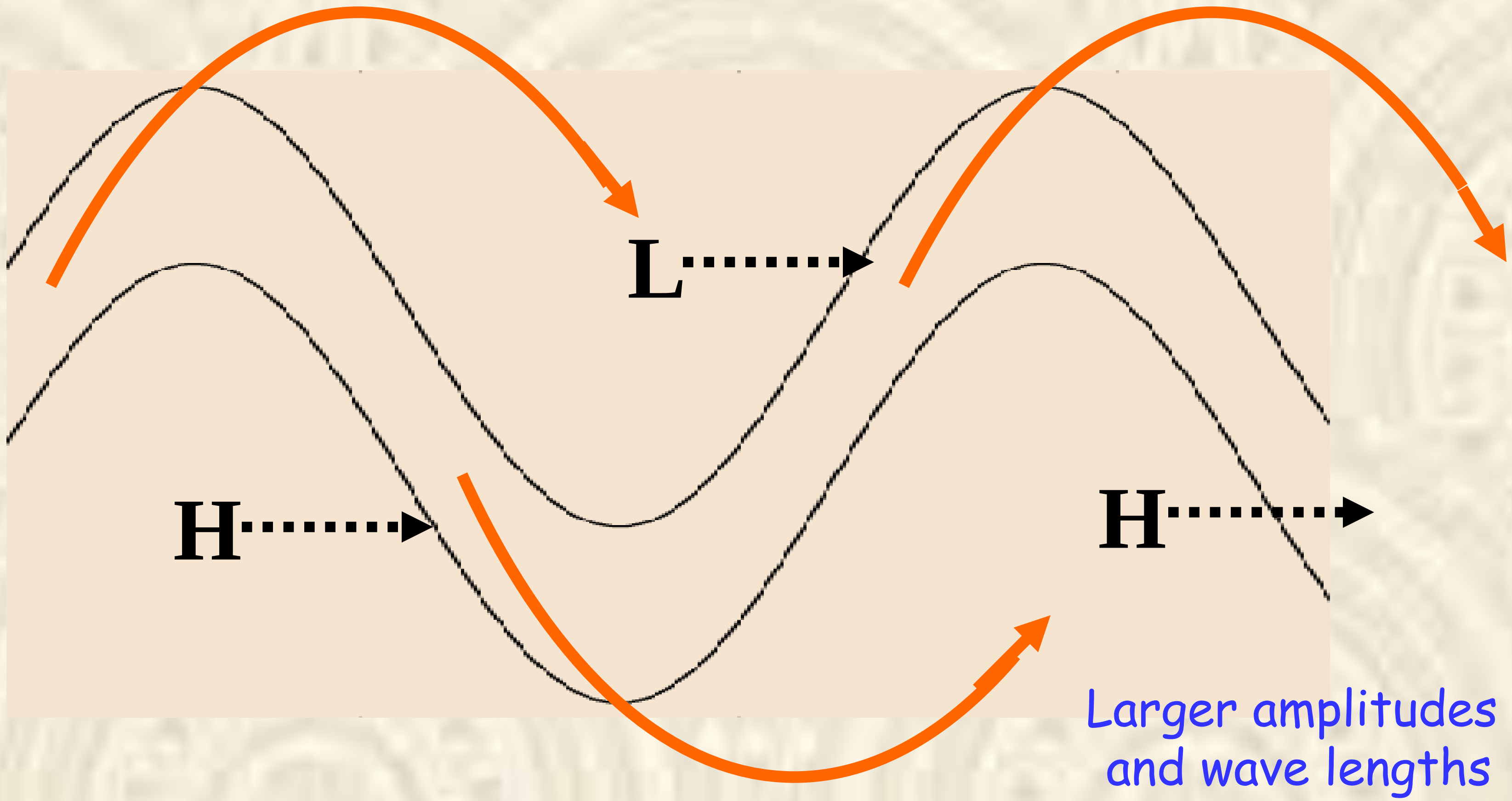
Note that $\beta > 0$ implies that $c_x < 0$ and hence the waves travel towards the west, consistent with physical arguments.

A very common misunderstanding:
This is NOT a Rossby wave!



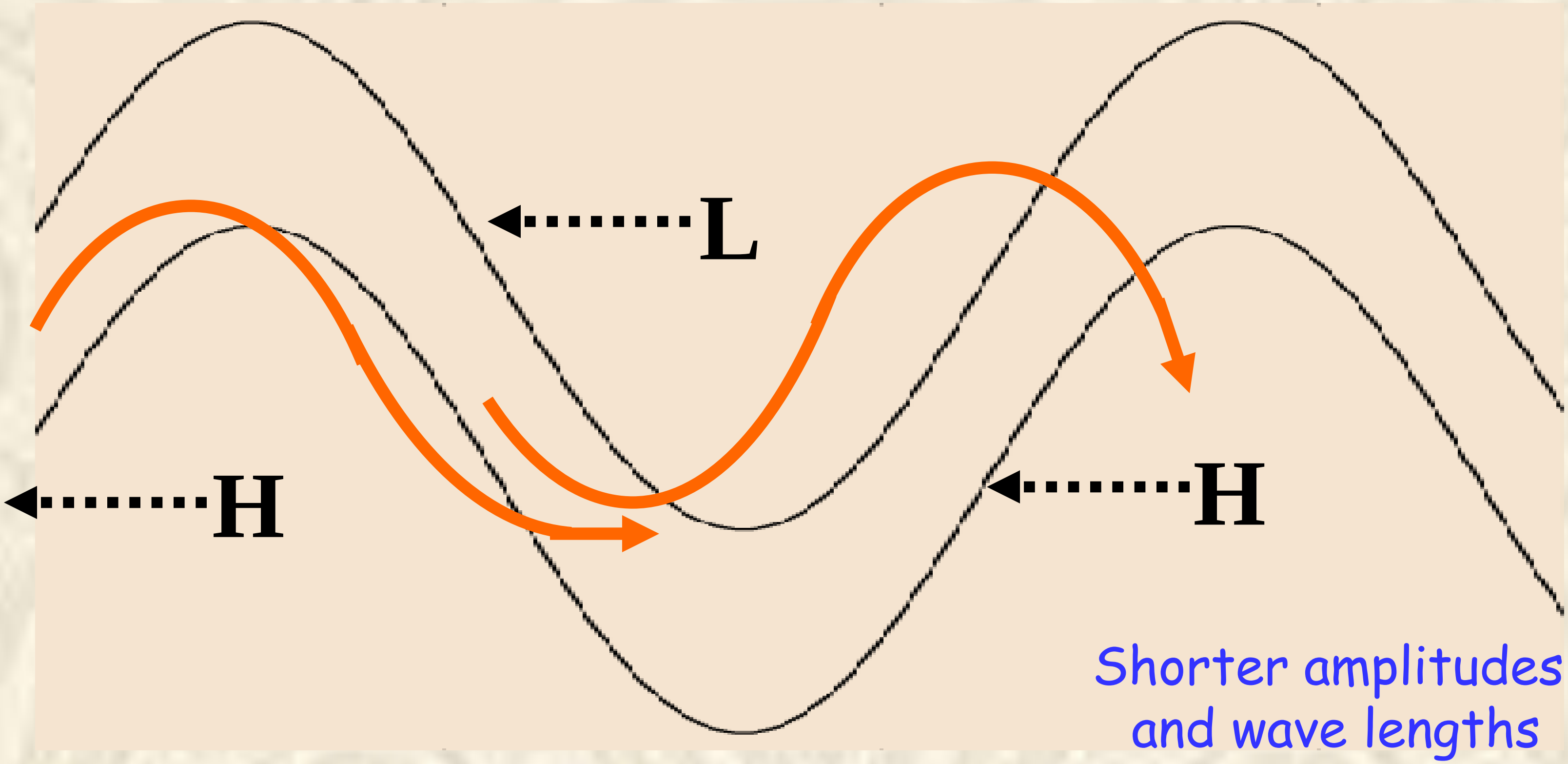
...but a Constant Absolute Vorticity Trajectory!

Relation between stream lines and trajectories in a progressive flow



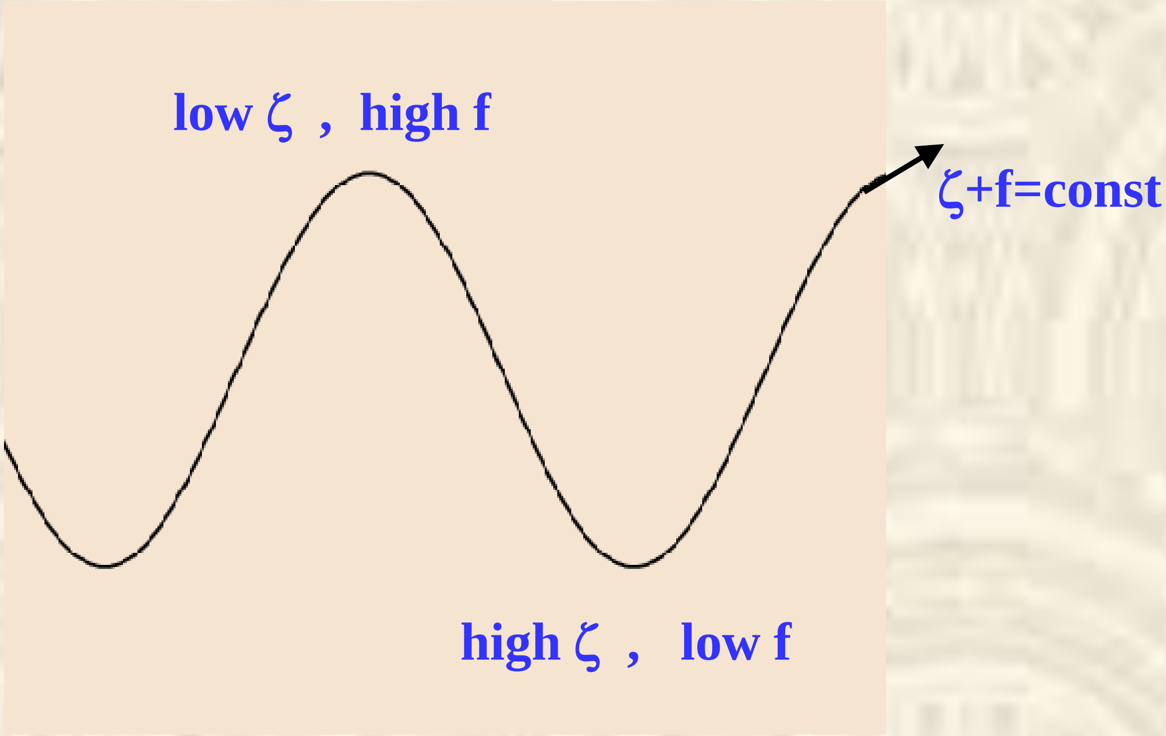
Larger amplitudes and wave lengths

Relation between stream lines and trajectories in a retrogressive flow



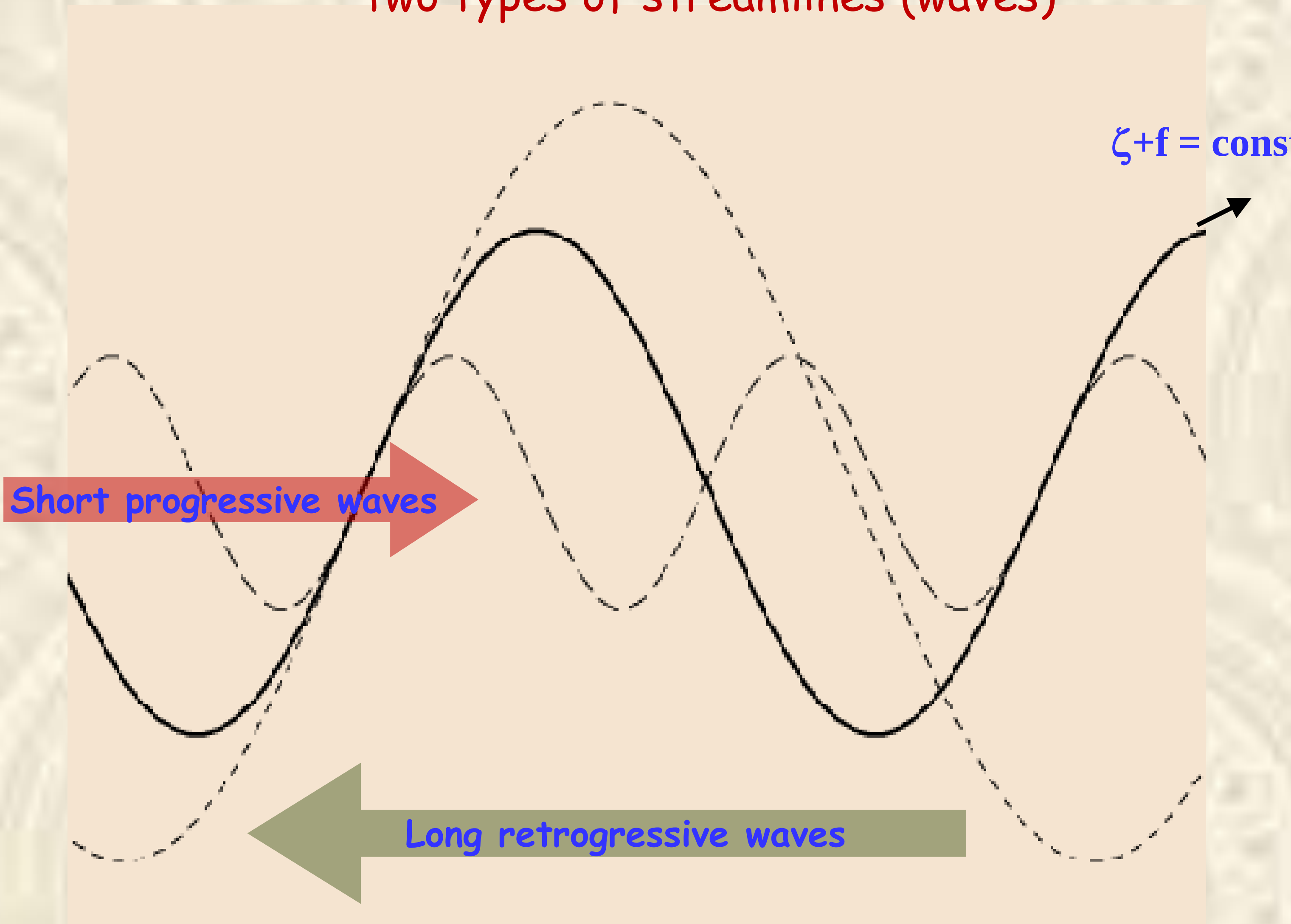
Shorter amplitudes and wave lengths

A very common misunderstanding:
This is NOT a Rossby wave!

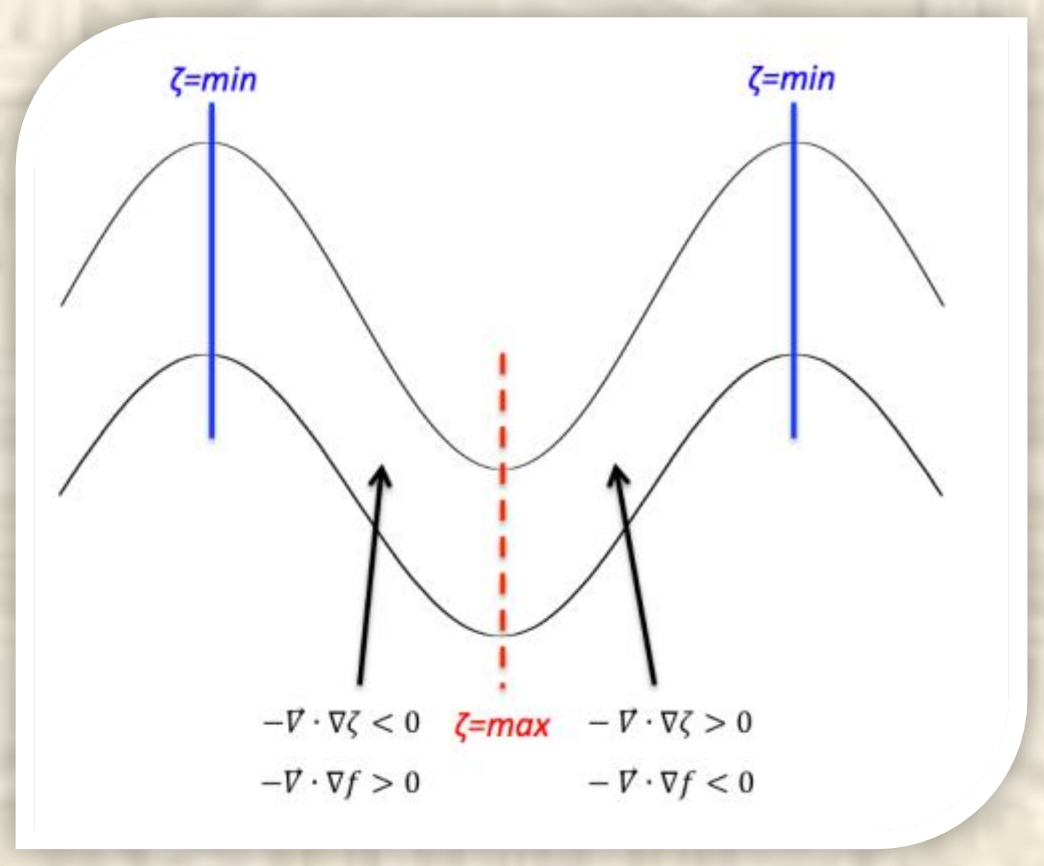


…but a Constant Absolute Vorticity Trajectory!

One and the same CAV trajectory satisfies
two types of streamlines (waves)



File: rossby upper waveeeeeeee

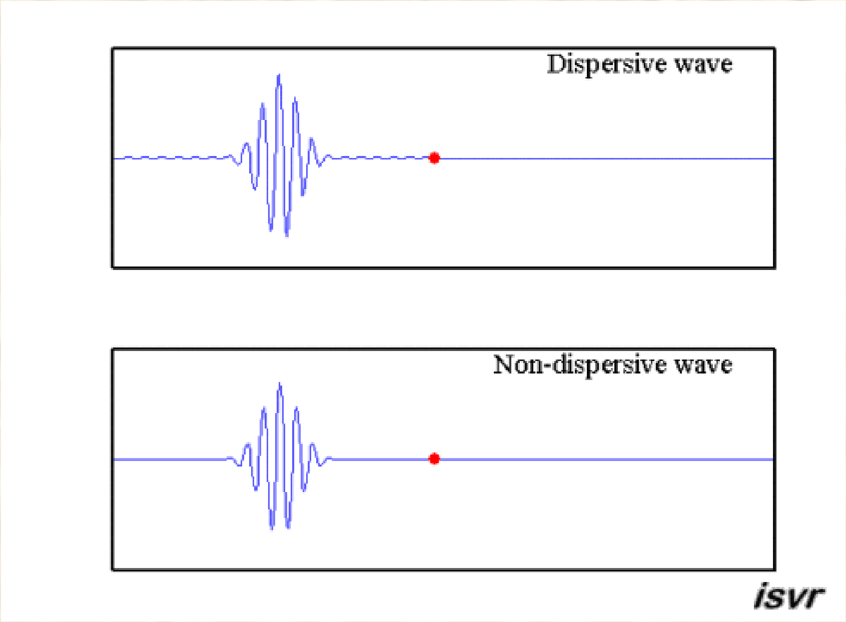


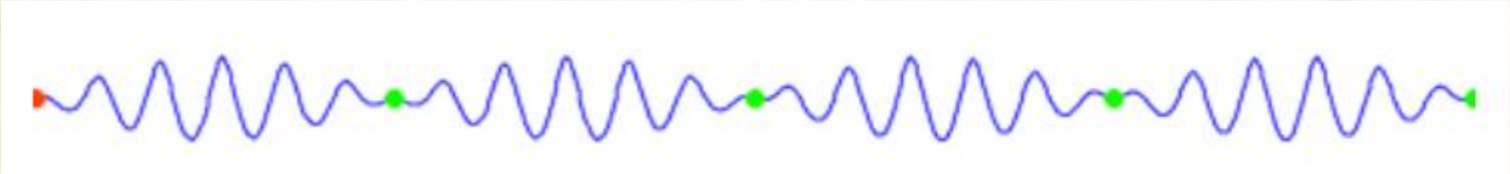
Stationary Rossby Wave

For longer wavelengths the westward Rossby wave phase speed may be large enough to balance the eastward advection by the mean zonal wind so that the resulting disturbance is stationary relative to Earth's surface.

$$3) \quad c_x = 0 \quad i.e \quad \bar{u} = \frac{\beta}{K^2} \quad \rightarrow K^2 = \frac{\beta}{\bar{u}} \equiv K_s^{\,2} \qquad L_s = 2\pi\sqrt{\frac{\bar{u}}{\beta}}$$

Since c_x is a function of k (or λ), the waves are called dispersive.
In this case, the longer waves travel faster than the shorter waves.


Dispersive wave
Non-dispersive wave
isvr



For a typical midlatitude synoptic-scale disturbance, with similar meridional and zonal scales $(k \approx l)$ and zonal wavelength of order 6000 km, the Rossby wave speed relative to the zonal flow calculated from:

$$c_x - \bar{u} = -\frac{\beta}{2k^2} = \frac{L_x^2}{8\pi^2}\frac{d}{dy}2\Omega\sin\varphi = -\frac{L_x^2}{8\pi^2}\frac{d}{Rd\varphi}2\Omega\sin\varphi$$

$$= -\frac{\Omega L_x^2}{4R\pi^2}\frac{d}{d\varphi}\sin\varphi = -\frac{7\times10^{-5}\times36\times10^{12}}{4\times6.4\times10^{6}\times10}\cos 45^\circ \approx -8\, m\, s^{-1}$$

Because the mean zonal wind is generally westerly (to the east) and greater than 8 ms^{-1} .

Synoptic-scale Rossby waves usually move eastward, but at a phase speed relative to the ground at a lower speed.