Dynamic Meteorology 2

Lecture 11

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A Wave Solution Strategy

Approximations to the full equations governing atmospheric dynamics will be solved for wave motions many times.

Even though aspects of each individual case are different, a guide to the general approach to solving these problems isas follows:

1. Choose a basic state

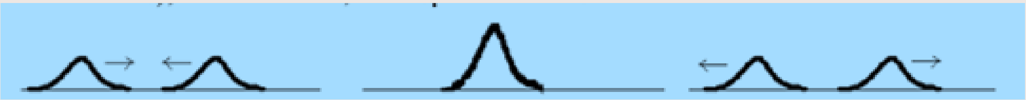
2. Linearize the governing equations

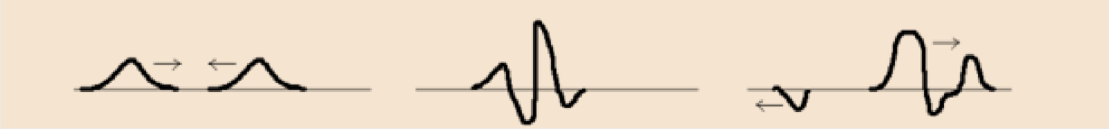
3. Assume wave solutions of the form in equation

$$f(x, y, t) = \mathrm{Re}(Ae^{i(kx+ly-\omega t)}) = \mathrm{Re}(Ae^{i\phi})$$

$$F_y e^{i(kx-\omega t)}$$

4. Solve for the dispersion and polarization relationships





SIMPLE WAVE TYPES

Waves in fluids result from the action of restoring forces on fluid parcels that have been displaced from their equilibrium positions.

The restoring forces may be due to compressibility, gravity, rotation, or electromagnetic effects.

This section considers the two simplest examples of linear waves in fluids:

1) acoustic waves

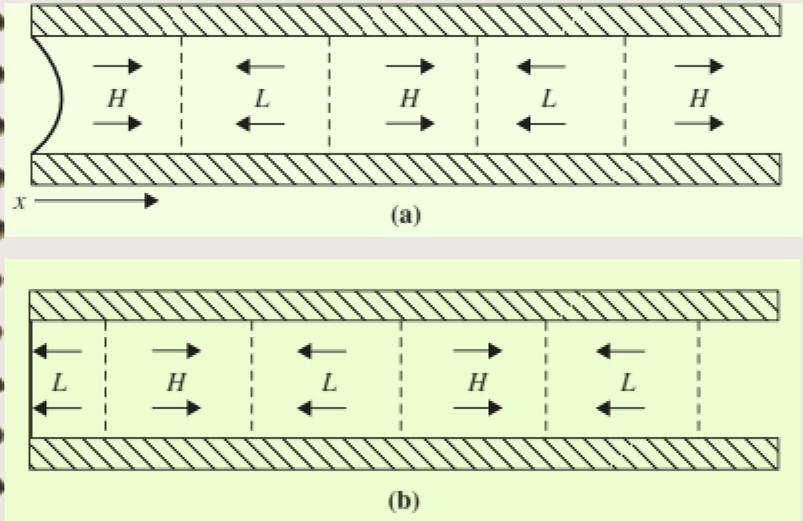
2) shallow water gravity waves

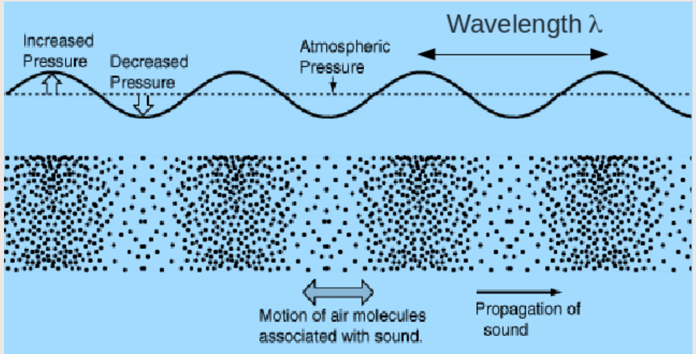
Acoustic (Sound) Waves

1) Sound waves, or acoustic waves, are *longitudinal waves*

2) Sound is propagated by the alternating adiabatic compression and expansion of the medium.

As an example,





Describe mathematiclly

1) the perturbation method

2) adiabatic

3) waves propagating in a straight pipe parallel to the x axis (one-dimensional sound)

4) To exclude the possibility of transverse oscillations (for simplicity) we assume:

$$\vec{V} = (u,0,0) \quad and \quad u = u(x,t)$$

With these restrictions, $\left\{ \begin{array}{l} \text{the momentum equation,} \\ \text{thermodynamic energy equation} \\ \text{continuity equation} \end{array} \right.$

for adiabatic motion are

$$\frac{du}{dt}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0$$

momentum equation

$$\frac{d\rho}{dt}+\rho\frac{\partial u}{\partial x}=0$$

continuity equation

$$\frac{d\ln\theta}{dt}=0$$

thermodynamic energy equation

For adiabatic process

$$c_p\frac{dT}{dt}-\alpha\frac{dp}{dt}=\dot{q}$$

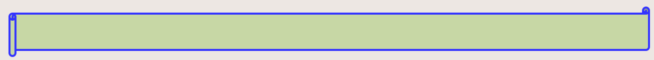
Dividing through by *T and again using the equation of state, we obtain* the entropy form of the first law of thermodynamics:

$$c_p\frac{d\ln T}{dt}-R\frac{d\ln p}{dt}=\frac{\dot{q}}{T}\equiv\frac{ds}{dt} \quad *$$

the rate of change of entropy per unit mass following the thermodynamically *reversible process.*

$$\theta = T\left(\frac{p_0}{p}\right)^{R/c_p} \qquad c_p \frac{d\ln\theta}{dt} = c_p \frac{d\ln T}{dt} - R\frac{d\ln p}{dt}$$

Comparing *, we ontain $$c_p \frac{d\ln\theta}{dt} = \frac{\dot{q}}{T} = \frac{ds}{dt}$$



where for this case $$\frac{d}{dt} = \frac{\partial}{\partial t} - u\frac{\partial}{\partial x}$$

$$\theta = \left(\frac{p}{\rho R}\right)\left(\frac{p_0}{p}\right)^{R/c_p} \qquad \rightarrow \ln\theta = \ln\frac{p_0^{R/c_p}}{R} + \ln p^{1-R/c_p} - \ln\rho$$

$$\frac{d\ln\theta}{dt} = -\frac{d\ln\rho}{dt} + \frac{1}{\gamma}\frac{d\ln p}{dt} = 0 \qquad \text{where} \qquad 1 - R/c_p = \frac{c_v}{c_p} = \frac{1}{\gamma}$$

where p_0 =1000 hPa, we may eliminate θ in $\frac{d\ln\theta}{dt}=0$

$$\frac{1}{\gamma}\frac{d\ln p}{dt}-\frac{d\ln\rho}{dt}=0 \qquad \frac{d\rho}{dt}+\rho\frac{\partial u}{\partial x}=0$$

Eliminating ρ $\quad \frac{1}{\gamma}\frac{d\ln p}{dt}+\frac{\partial u}{\partial x}=0$

The perturbation method

$$u(x,t)=\bar{u}+u'(x,t)$$

$$p(x,t)=\bar{p}+p'(x,t)$$

$$\rho(x,t)=\bar{\rho}+\rho'(x,t)$$

Substituting into $\frac{du}{dt}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0$ and $\frac{1}{\gamma}\frac{d\ln p}{dt}+\frac{\partial u}{\partial x}=0$

$$\frac{\partial}{\partial t}(\bar{u}+u')+(\bar{u}+u')\frac{\partial}{\partial x}(\bar{u}+u')+\frac{1}{(\bar{\rho}+\rho')}\frac{\partial}{\partial x}(\bar{p}+p')=0$$

$$\frac{\partial}{\partial t}(\bar{p}+p')+(\bar{u}+u')\frac{\partial}{\partial x}(\bar{p}+p')+\gamma(\bar{p}+p')\frac{\partial}{\partial x}(\bar{u}+u')=0$$

$|\rho'/\bar{\rho}|\ll 1$, we can use the binomial expansion to approximate the density term as

$$\frac{1}{(\bar{\rho}+\rho')}=\frac{1}{\bar{\rho}}\left(1+\frac{\rho'}{\bar{\rho}}\right)^{-1}\approx\frac{1}{\bar{\rho}}\left(1-\frac{\rho'}{\bar{\rho}}\right)$$

we obtain the linear perturbation equations

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)u'+\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x}=0 \qquad \left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)p'+\gamma\bar{p}\frac{\partial u'}{\partial x}=0$$

Eliminate u' by applying $\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)$ on *

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)u' = -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x}$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)p' + \gamma\bar{p}\frac{\partial u'}{\partial x} = 0 \quad *$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)^2 p' + \gamma\bar{p}\frac{\partial\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)u'}{\partial x} = \left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)^2 p' - \frac{\gamma\bar{p}}{\bar{\rho}}\frac{\partial^2 p'}{\partial x^2} = 0$$

which is a form of the standard *wave equation familiar from electromagnetic* theory. *A* simple solution representing a plane sinusoidal wave propagating in *x is*

$$p' = Ae^{ik(x-ct)}$$

the assumed solution

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)^2 p' - \frac{\gamma\bar{p}}{\bar{\rho}}\frac{\partial^2 p'}{\partial x^2} = 0$$

$$p' = A e^{ik(x-ct)}$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)^2 p' = \left(\frac{\partial^2}{\partial t^2}+2\bar{u}\frac{\partial^2}{\partial x\partial t}+\bar{u}^2\frac{\partial^2}{\partial x^2}\right)p' = \left(-ikc+ik\bar{u}\right)^2 p'$$

$$\frac{\gamma\bar{p}}{\bar{\rho}}\frac{\partial^2 p'}{\partial x^2} = \frac{\gamma\bar{p}}{\bar{\rho}}\left(ik\right)^2 p'$$

$$\left(-ikc+ik\bar{u}\right)^2 - \frac{\gamma\bar{p}}{\bar{\rho}}\left(ik\right)^2 = 0$$

Dispersion relation

Solving for c gives

$$c = \bar{u} + \sqrt{\frac{\gamma\bar{p}}{\bar{\rho}}} = \bar{u} + \sqrt{\gamma R\bar{T}}$$

the phase speed

the speed of wave propagation relative to the zonal current is

$$c - \bar{u} = \pm c_s$$

where $c_s = \sqrt{\gamma R \bar{T}}$ is called the *adiabatic speed of sound*

The mean zonal velocity here plays only a role of *Doppler shifting the sound* wave so that the frequency relative to the ground corresponding to a given wave number *k is*

$$\omega = kc = k(\bar{u} \pm c_s)$$

Thus, in the presence of a wind, the frequency as heard by a fixed observer depends on the location of the observer relative to the source.

$$if \quad \bar{u} > 0$$

The frequency of a stationary source will appear to be higher for an observer to the east (downstream) of the source

$$c = \bar{u} + c_s$$

than for an observer to the west (upstream) of the source

$$c = \bar{u} - c_s$$