Dynamical Meteorology 1

Lecture 5

Sahraei

Physics Department,
Razi University

http://www.razi.ac.ir/sahraei

Structure of the Static Atmosphere

جو ایستا: در صورتی که در جو هیچگونه ناپایداری وجود نداشته باشد یعنی همه جو در حال سکون باشد جو را ایستا گویند.

حالت ترمودینامیکی جو در هر نقطه با مقادیر فشار- دما و چگالی (حجم ویژه) در آن نقطه تعیین می شود رابطه ای که ارتباط این کمیتها را با هم نشان می دهد معادله حالت نامیده می شود.

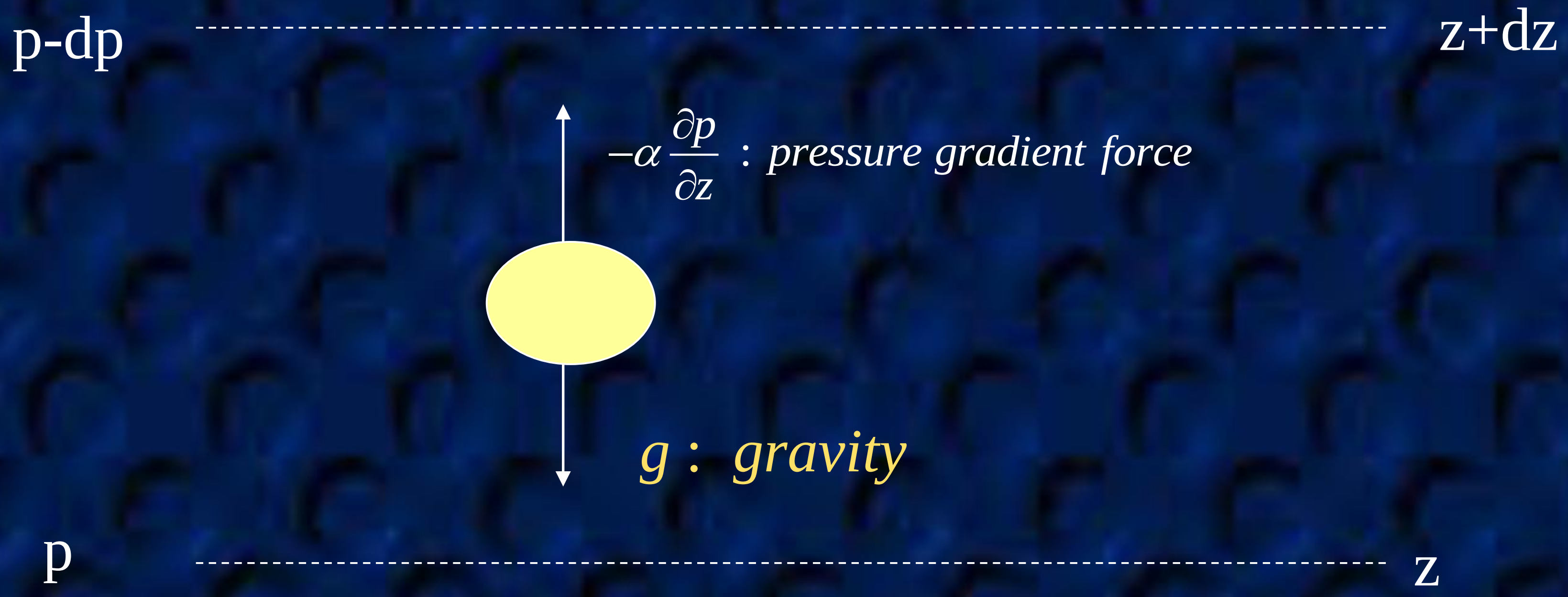
We can express the equation of state for dry air as:

$$P\alpha = RT \qquad P = \rho RT$$

Where R is the gas constant for dry air ($R=287\ Jkg^{-1}K^{-1}$)

The Hydrostatic Equilibrium

In the absence of atmospheric motions the gravity force must be exactly balanced by the vertical component of the pressure gradient force.



$$-\alpha \frac{dp}{dz} = g \quad \text{or} \quad \frac{dp}{dz} = -\rho g$$

(one of the best approximations in meteorology)

$$\frac{dp}{dz} = -\rho g \qquad dp = -\rho g \, dz$$

$$p(z) = \int_z^\infty \rho g \, dz$$

Pressure at any point is the weight per square meter of the atmospheric column overlying that point.

For average conditions,

$$p(0) = \int_0^\infty \rho g \, dz = 101.325 \, kPa$$

This is the mean sea-level pressure.

Geopotential Height

Definition:

The geopotential height (Z) is the actual height normalized by the globally averaged acceleration due to gravity at the Earth's surface (g_0 = 9.81 m s^{-2}), and is defined by:

$$Z = \frac{\Phi}{g_0}$$

Used as the vertical coordinate in most atmospheric applications in which energy plays an important role (i.e. just about everything)

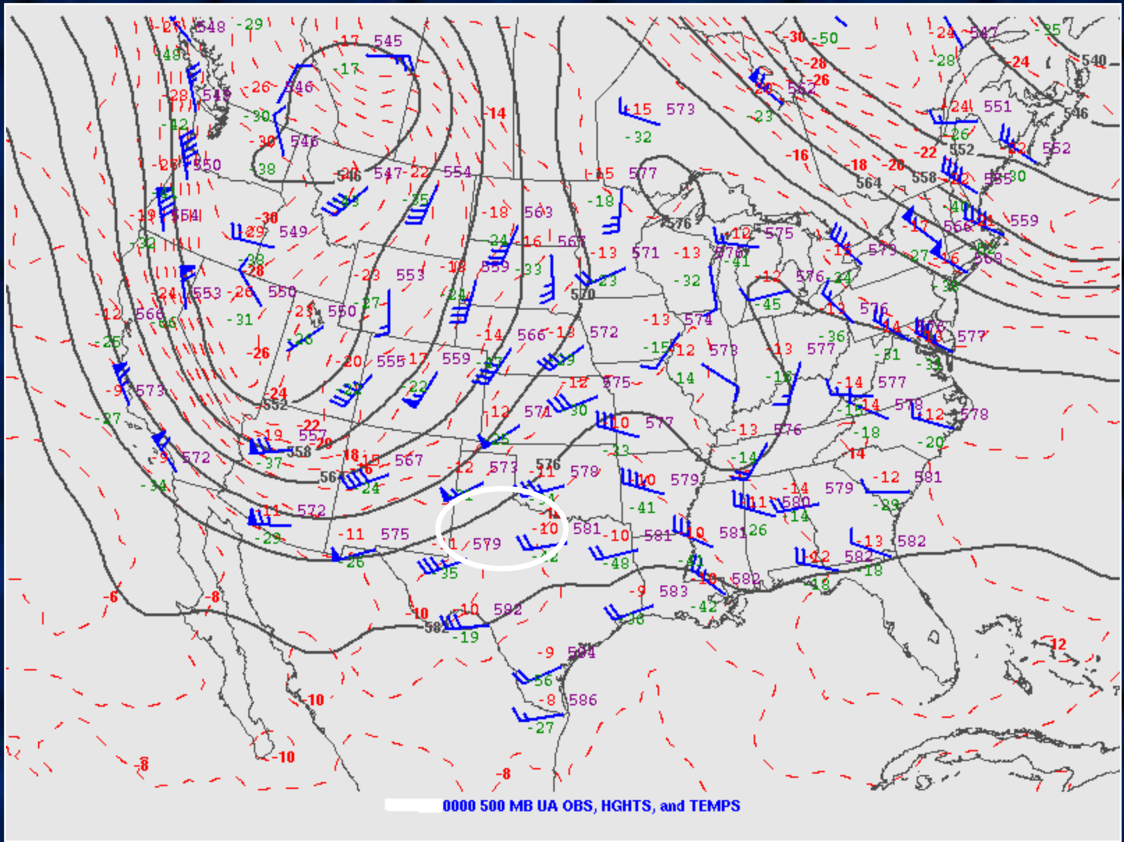
Lucky for us → $g \approx g_0$ in the troposphere

Height z (km)	Geopotential Height Z (km)	Gravity g (m s^{-2})
0	0.00	9.81
1	1.00	9.80
10	9.99	9.77
100	98.47	9.50

Geopotential Height

Application:

The geopotential height (Z) is the standard "height" parameter plotted on isobaric charts constructed from daily soundings:



500 mb

Geopotential heights (Z) are solid black contours (Ex: Z = 5790 meters)

Air temperatures (T) are red dashed contours (Ex: T = -11°C)

Winds are shown as barbs

Hypsometric Equation

Derivation:

If we combine the Hydrostatic Equation with the Ideal Gas Law for moist air and the Geopotential Height, we can derive an equation that defines the thickness of a layer between two pressure levels in the atmosphere

1. Substitute the ideal gas law into the Hydrostatic Equation

$$\frac{dp}{dz} = -\rho g$$

$$p = \rho R_d T_v$$

$$\frac{dp}{dz} = \frac{-p\, g}{R_d T_v}$$

Virtual Temperature: The temperature that a parcel of dry air would have if it were at the same pressure and had the same density as moist air.

P = pressure
ρ_d = dry air density
ρ_v = vapor density
ρ= air density
R= gas constant
R_v = vapor gas constant
R_d = dry air gas constant
T = Temperature

Derivation:

Start with ideal gas law for moist air:

$$P = \rho RT$$

$$P = \rho_d R_d T + \rho_v R_v T$$

Now treat moist air as if it were dry by introducing the virtual temperature T_v

$$P = (\rho_d R_d + \rho_v R_v)T = (\rho_d + \rho_v)R_d T_v = \rho R_d T_v$$

What is the relationship between the temperature, T and the virtual temperature T_v?

$$T_v = (1 + 0.61 r_v)T$$

2. Re-arranging the equation and using the definition of geopotenital height:

$$\frac{dp}{dz} = \frac{-p\ g}{R_d T_v}$$

$$d\Phi = g dz \ = -R_d T_v \frac{dp}{p}$$

3. Integrate this equation between two geopotential heights (Φ_1 and Φ_2) and the two corresponding pressures (p_1 and p_2), assuming T_v is constant in the layer

$$\int_{\Phi_1}^{\Phi_2} d\Phi = -R_d T_v \int_{p_1}^{p_2} \frac{dp}{p}$$

4. Performing the integration:

$$\int_{\Phi_1}^{\Phi_2} d\Phi = -R_d T_v \int_{p_1}^{p_2} \frac{dp}{p}$$

$$\Phi_2 - \Phi_1 = -R_d T_v \ln\left(\frac{p_2}{p_1}\right)$$

5. Dividing both sides by the gravitational acceleration at the surface (g_0):

$$\frac{\Phi_2}{g_0} - \frac{\Phi_1}{g_0} = -\frac{R_d T_v}{g_0} \ln\left(\frac{p_2}{p_1}\right)$$

6. Using the definition of geopotential height

$$\frac{\Phi_2}{g_0} - \frac{\Phi_1}{g_0} = -\frac{R_d T_v}{g_0} \ln\left(\frac{p_2}{p_1}\right)$$

$$Z_2 - Z_1 = -\frac{R_d T_v}{g_0} \ln\left(\frac{p_2}{p_1}\right)$$

Hypsometric Equation

Defines the geopotential thickness (Z_2 – Z_1) between any two pressure levels (p_1 and p_2) in the atmosphere.

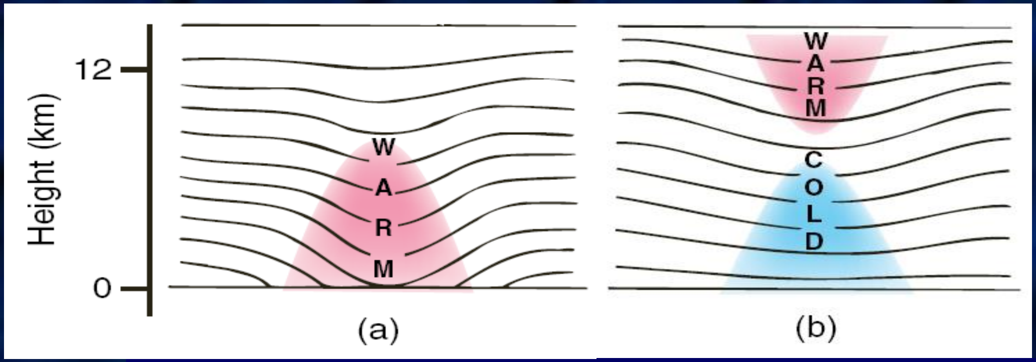
Interpretation:

The thickness of a layer between two pressure levels is proportional to the mean virtual temperature of that layer.

$$Z_2 - Z_1 = -\frac{R_d T_v}{g_0} \ln\left(\frac{p_2}{p_1}\right)$$

If T_v increases, the air between the two pressure levels expands and the layer becomes thicker. Pressure decrease slowly with height

If T_v decreases, the air between the two pressure levels compresses and the layer becomes thinner. Pressure decreases rapidly with height



Hurricane (warm core) Mid-latitude Low (cold core)

Black solid lines are pressure surfaces

We can define a quantity called the geopotential, which is related to gravity.

Gravity can be represented as the gradient of the geopotential.

$$\nabla\Phi = -\vec{g}$$

Because $\vec{g} = -g\hat{k},$ then $\Phi = \Phi(z), \quad \frac{d\Phi}{dz} = g$

If the value of the geopotential is set to zero at mean sea level, the geopotential Φ(z) at height z is the work required to raise a unit mass to height z from mean sea level:

$$\Phi = \int_0^z g\ dz$$

Units of geopotential are J kg^{-1}, which are equivalent to m^2 s^{-2}.

Geopotential is the potential energy acquired by unit mass on being raised through unit distance in a field of gravitational force of unit strength.

The Geopotential meter is related to the Dynamic meter by the expression one geopotential meter=0.98 dynamic meter. It is roughly the height of a pressure surface in the atmosphere above mean sea level.

dynamic meter:

The standard unit of dynamic height, defined as 10 $m^2\ s^{-2}$; it is related to the geopotential φ, the geometric height z in meters, and the geopotential height Z in geopotential meters by where g is the acceleration of gravity in meters per second squared.

(Some sources prefer to give the constants 10 and 9. 8 the units of meters per second squared so that the units of φ and Z would be the same as those of the geometric height.)

The dynamic meter is about 2% longer than the geometric meter and the geopotential meter.

The equation which defines the relationship between geopotential height (Z) and geometric height (z) is

$$dp = -\rho g\, dz \qquad dp = -\rho g_0\, dZ$$

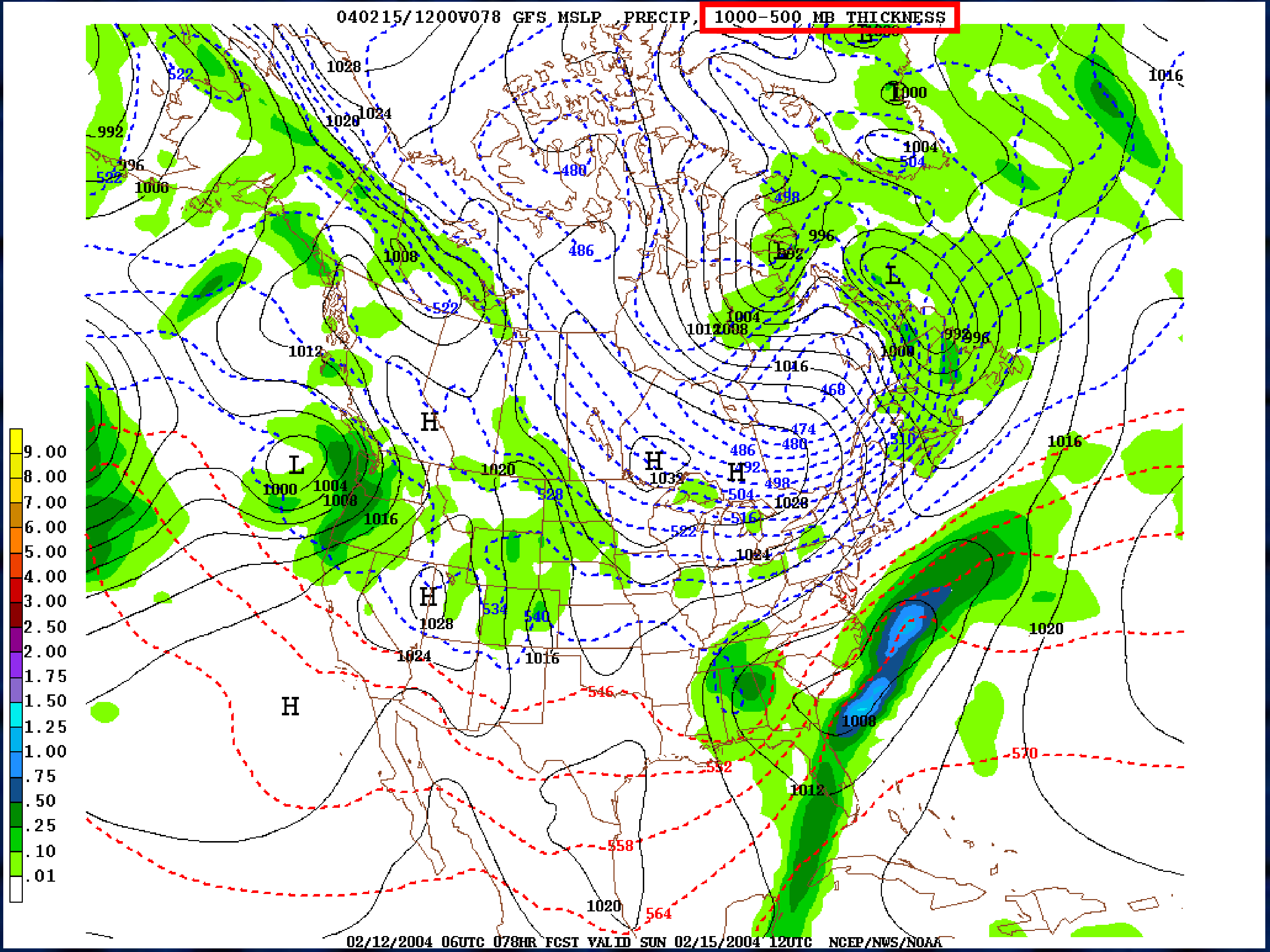
$$1 = (g_0 / g)(dZ / dz) \qquad dZ = (g / g_0)\, dz$$

$$Z = (g / 980)\ z$$

It is roughly the height of a pressure surface in the atmosphere above mean sea level.

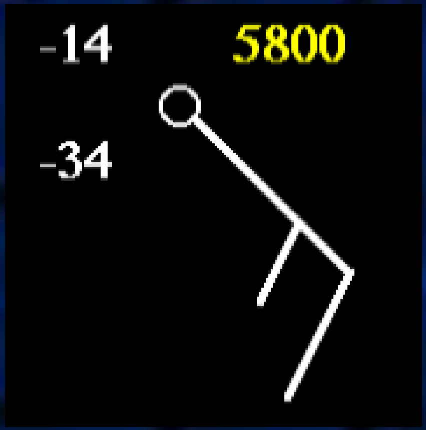
Thus when gravity g has its near average value of 980 cm/sec^2, heights in geopotential meters and and geometric meters are the same;
for $g < 980$ cm/sec^2 the height in geopotential meters is the smaller,
for $g > 980$ cm/sec^2 it is bigger.

Geopotential (from the dynamic point of view) is a better measure of height in the atmosphere than is geometric height since energy is in general lost or gained when air moves along a geometrically level surface but not when it moves along an equigeopotential surface.


040215/1200V078 GFS MSLP ,PRECIP, 1000-500 MB THICKNESS
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.50
2.00
1.75
1.50
1.25
1.00
.75
.50
.25
.10
.01
02/12/2004 06UTC 078HR FCST VALID SUN 02/15/2004 12UTC NCEP/NWS/NOAA

Geopotential Height

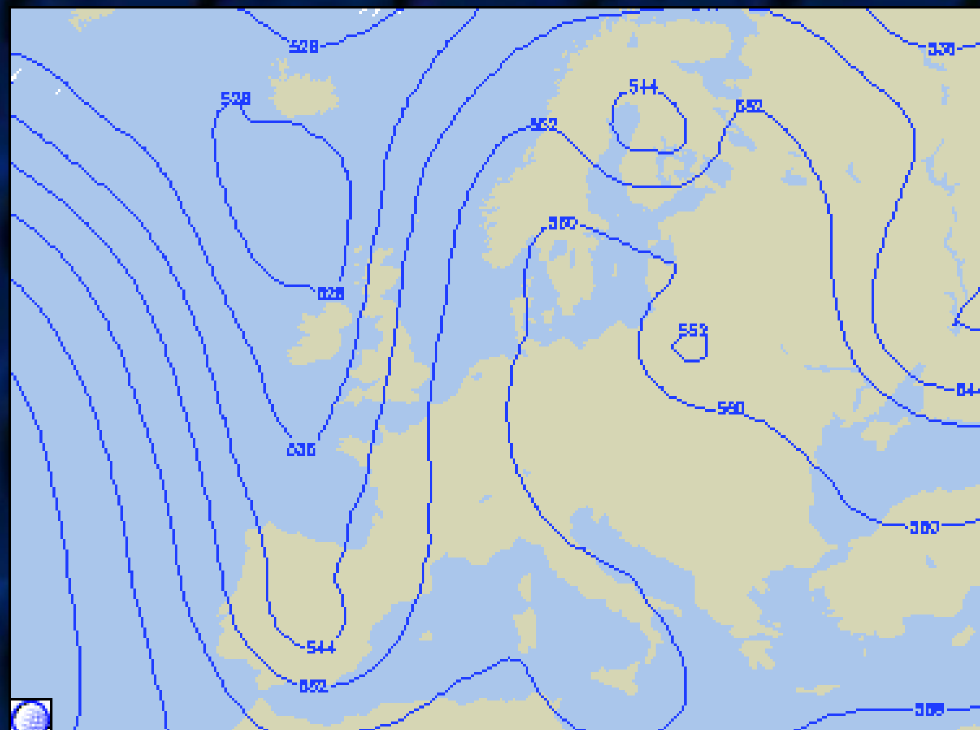
upper air station reports



The value highlighted in yellow located in the upper right corner (in the diagram above) represents the geopotential height of a given pressure surface in meters (as reported by weather balloons). Geopotential Height approximates the actual height of a pressure surface above mean sea-level. Therefore, for the example given above, the height of the pressure surface on which the observation was taken is 5800 meters.

When a collection of geopotential height reports are contoured on a given pressure surface, we are able to identify upper air troughs and ridges, which are very important influences on surface weather conditions.

Geopotential height 500 hPa [gpdm]
Fri 10 Apr, 16:00 BST (15:00 UTC)

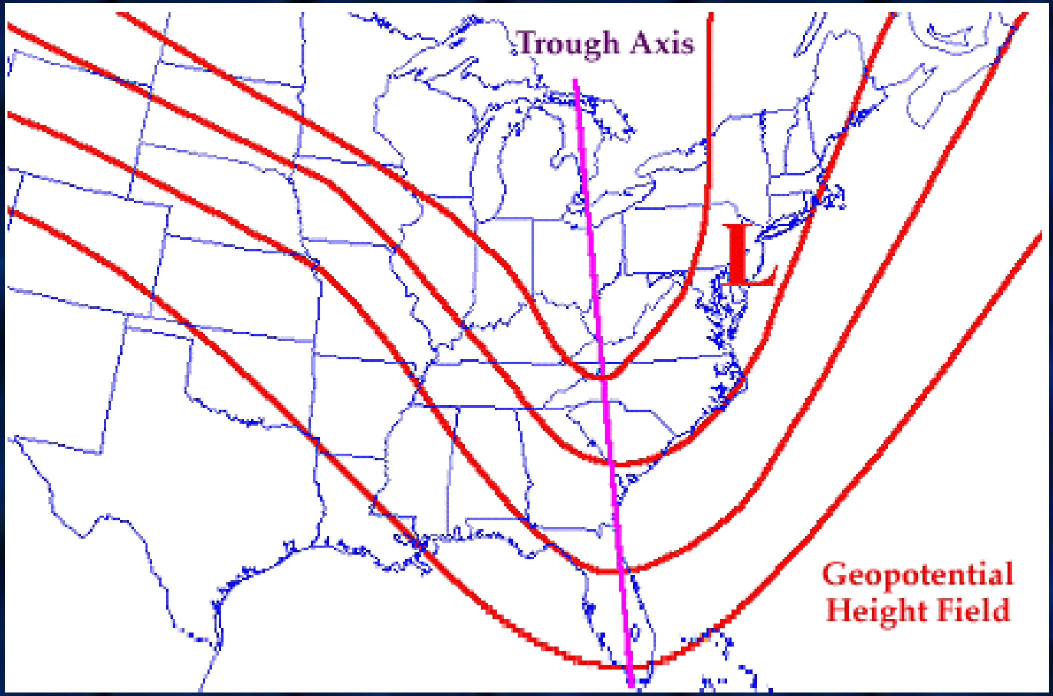


http://www.weatheronline.co.uk/

Troughs

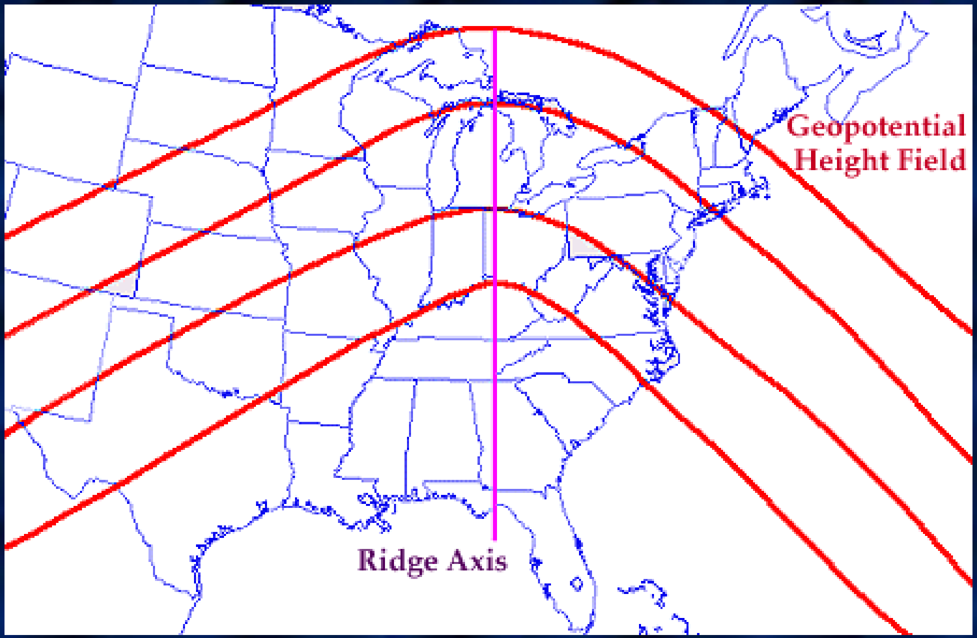
upper level lows

When the height contours bend strongly to the south, (as in the diagram below), it is called a TROUGH. Strong troughs are typically preceded by stormy weather and colder air at the surface. Below is an example of a trough in an upper-level height field (red contours). The trough axis is denoted by the purple line.



Ridges
upper level highs

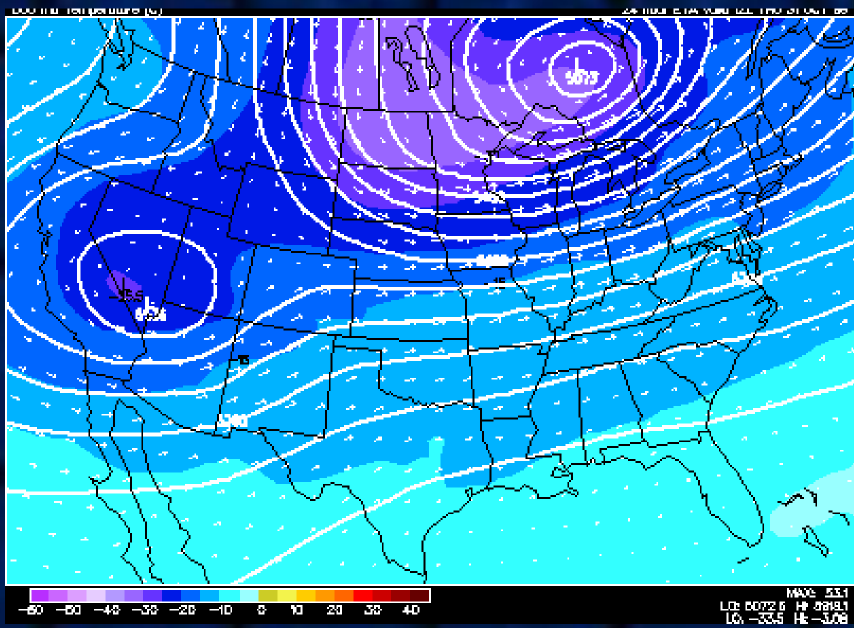
When the height contours bend strongly to the north (as in the diagram below), this is known as a RIDGE. Strong ridges are accompanied by warm and dry weather conditions at the surface. Below is an example of a ridge in an upper-level height field (red contours). The purple line denotes the ridge axis.



Geopotential Height

height of a given pressure Geopotential height approximates the actual height of a pressure surface above mean sea-level.
Therefore, a geopotential height observation represents the height of the pressure surface on which the observation was taken. A line drawn on a weather map connecting points of equal height (in meters) is called a height contour. That means, at every point along a given contour, the values of geopotential height are the same. An image depicting the geopotential height field is given below.

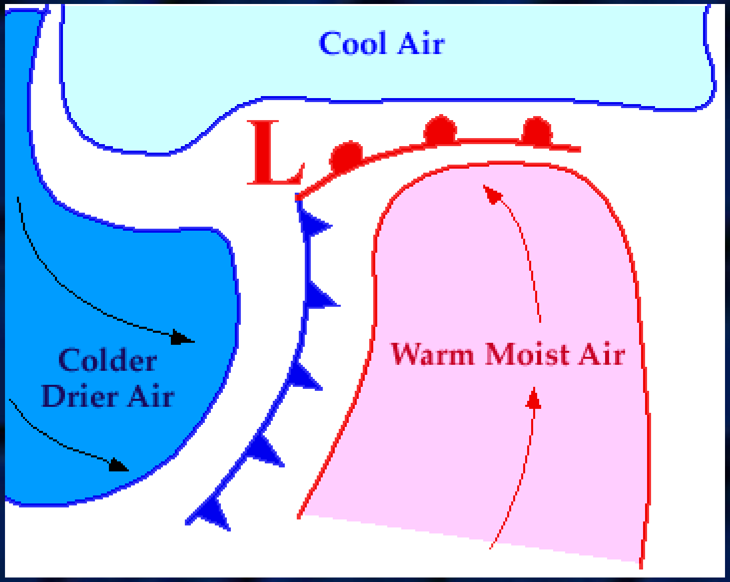
Height contours are represented by the solid lines. The small numbers along the contours are labels which identify the value of a particular height contour (for example 5640 meters, 5580 meters, etc.). This example depicts the 500 mb geopotential height field and temperatures (color filled regions). The height field is given in meters with an interval of 60 meters. Geopotential height is valuable for locating troughs and ridges which are the upper level counterparts of surface cyclones and anticyclones.



Cyclones

an idealized model

A cyclone is an area of low pressure around which the winds flow counterclockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere.

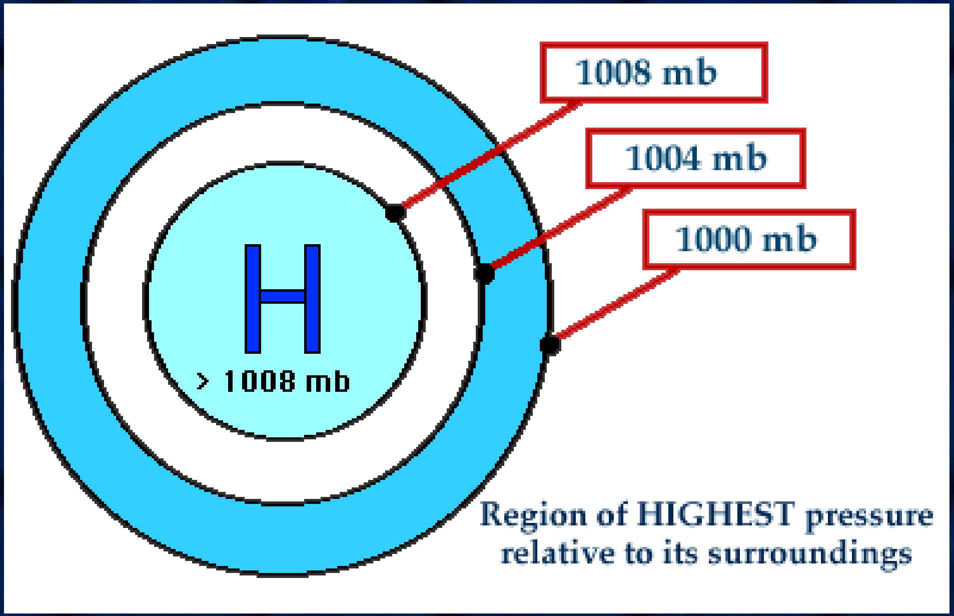


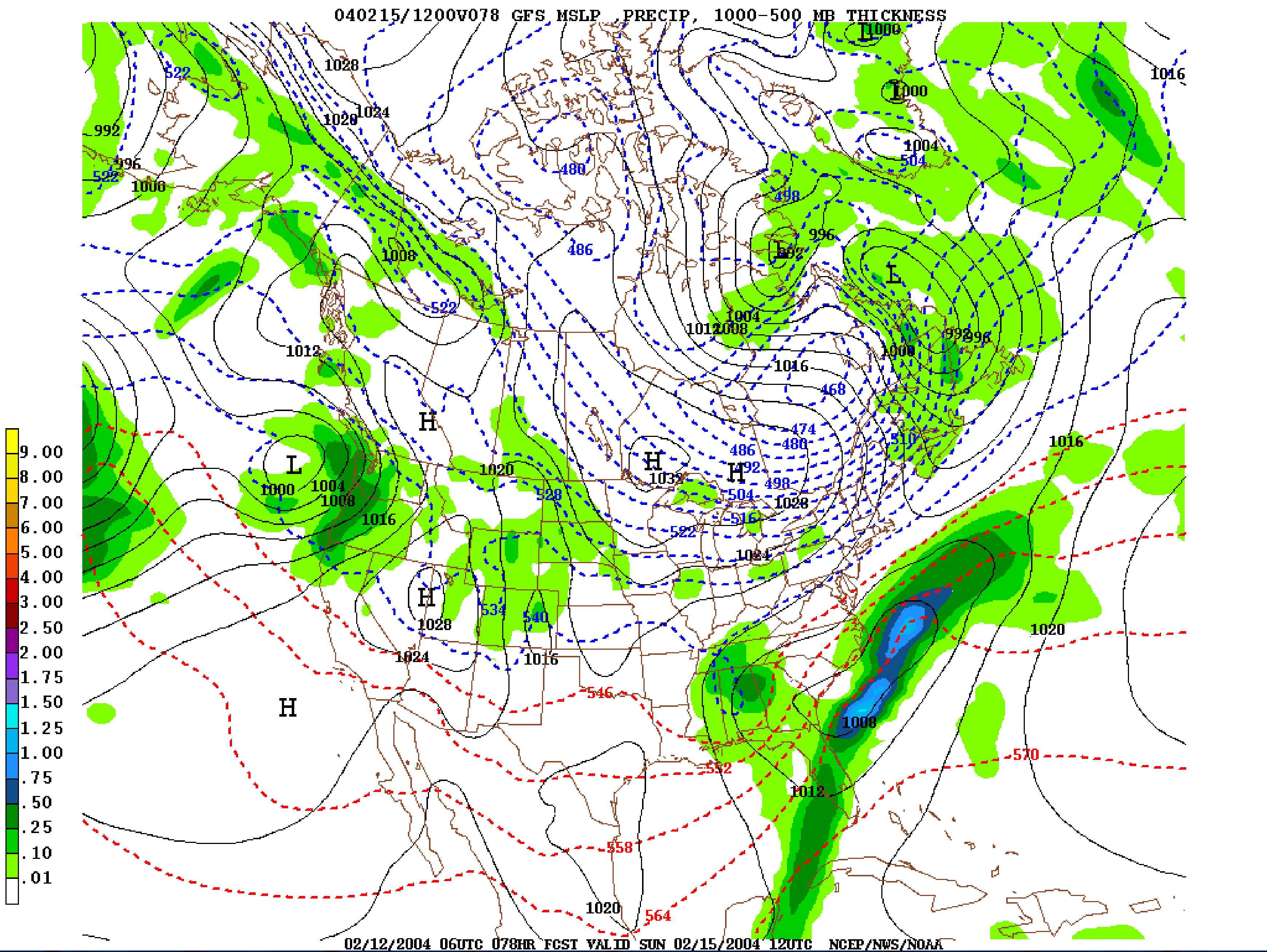
A developing cyclone is typically accompanied by a warm front pushing northward and a cold front pulling southward, marking the leading edges of air masses being wrapped around a center of low pressure, or the center of the cyclone.

High Pressure Centers

also known as anticyclones

A high pressure center is where the pressure has been measured to be the highest relative to its surroundings. That means, moving in any direction away from the "High" will result in a decrease in pressure. A high pressure center also represents the center of an anticyclone and is indicated on a weather map by a blue "H".




040215/1200V078 GFS MSLP ,PRECIP, 1000-500 MB THICKNESS
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.50
2.00
1.75
1.50
1.25
1.00
.75
.50
.25
.10
.01
02/12/2004 06UTC 078HR FCST VALID SUN 02/15/2004 12UTC NCEP/NWS/NOAA