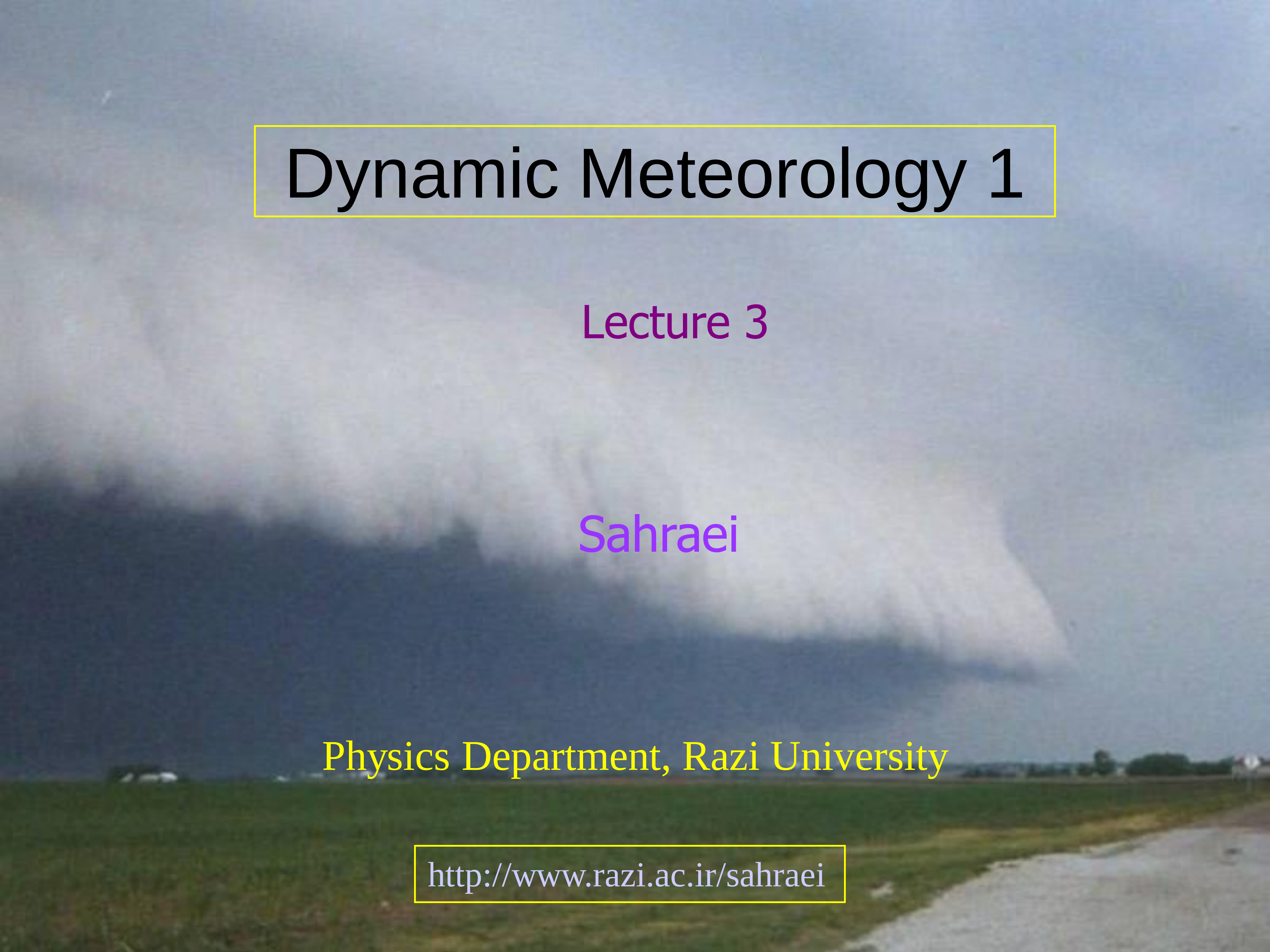

Dynamic Meteorology 1

Lecture 3

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The gradient

$$grad\,(f) = \nabla f = \partial f/\partial x,\ \partial f/\partial y, \ldots \qquad \text{(a vector)}$$

measures the maximum rate of change with direction (Fig 1)

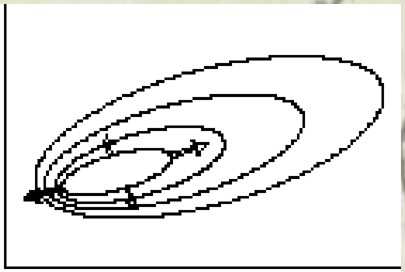


Figure 1

The divergence

$$div\,(f) = \nabla\cdot f = \partial f/\partial x + \partial f/\partial y + \ldots \qquad \text{(a scalar product)}$$

measures the net outflow of *f* in a unit of volume (fig. 2).

Both these names are very descriptive.

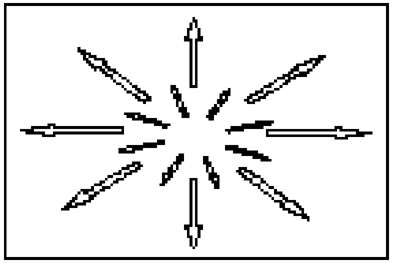


Figure 2

The Laplacian is

$$grad\,(f) = \nabla\cdot\nabla f = \nabla^2 f = \partial^2 f/\partial x^2 + \partial^2 f/\partial y^2$$

It is a divergence, and so a scalar product and a measure of flow. It may also be thought of as a measure of the difference between *f* at a point R, and the *average* of *f* in a region around R. This is possibly easier to understand for the discrete Laplacian, which we consider later (fig. 3).

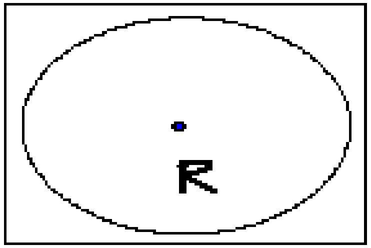


Figure 3

Physical meaning of ∇^2

The Laplacian gives the smoothness of a function.It measures the difference between the value of Ψ at a point and its mean value at surrounding points.

A little to the left of x

$$\Psi(x-a) = \Psi(x) - a\frac{\partial \Psi}{\partial x} + \frac{a^2}{2}\frac{\partial^2 \Psi}{\partial x^2} + \ldots$$

A little to the right of x

$$\Psi(x+a) = \Psi(x) + a\frac{\partial \Psi}{\partial x} + \frac{a^2}{2}\frac{\partial^2 \Psi}{\partial x^2} + \ldots$$

On taking the average

$$\overline{\Psi} = \frac{1}{2}\left[\Psi(x-a) + \Psi(x+a)\right] = \Psi(x) + \frac{a^2}{2}\frac{\partial^2 \Psi}{\partial x^2}$$

$$\overline{\Psi} - \Psi(x) = \frac{a^2}{2}\frac{\partial^2 \Psi}{\partial x^2}$$

The deviation from the value of Ψ at a point and its mean value in the surrounding region is proportional to Laplacian Ψ

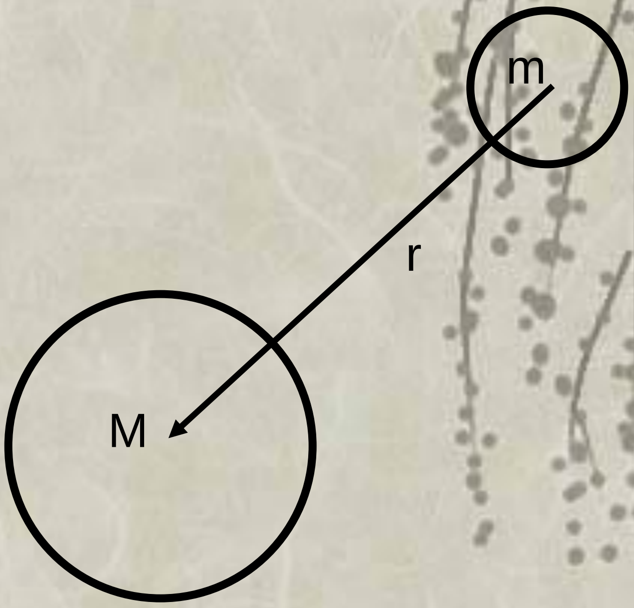
gravitational force

the force of attraction between all masses in the universe; especially the attraction of the earth's mass for bodies near its surface; "the more remote the body the less the gravity"; "the gravitation between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance between them"

Gravitational Force

Newton's law of universal gravitation states that gravitational force exerted by mass M on mass m is:

$$\vec{F}_g = -\frac{GMm}{r^2}\left(\frac{\vec{r}}{r}\right)$$



If the earth is taken to be the mass M and m is taken to be the mass of a fluid parcel or volume element, then we can write the force perunit mass exerted on the fluid by the earth as

$$\frac{\vec{F}_g}{m} \equiv \vec{g}^* = -\frac{GM}{r^2}\left(\frac{\vec{r}}{r}\right)$$

$$g^* = -\frac{GM}{(a+z)^2} = -\frac{GM}{a^2\left(1+\frac{z}{a}\right)^2}$$

If a is the radius of the earth and z is the distance above sea level, then

$$\vec{g}^* = \frac{\vec{g}_0^*}{(1+z/a)^2},$$

where

$$\vec{g}_0^* = -\left(\frac{GM}{a^2}\right)\left(\frac{\vec{r}}{r}\right)$$

Because the depths of the atmosphere and ocean are small compared to the radius of the earth $(z << a)$ we can treat the gravitational force per unit mass as a constant.

$$\vec{g}^* = \vec{g}_0^*$$

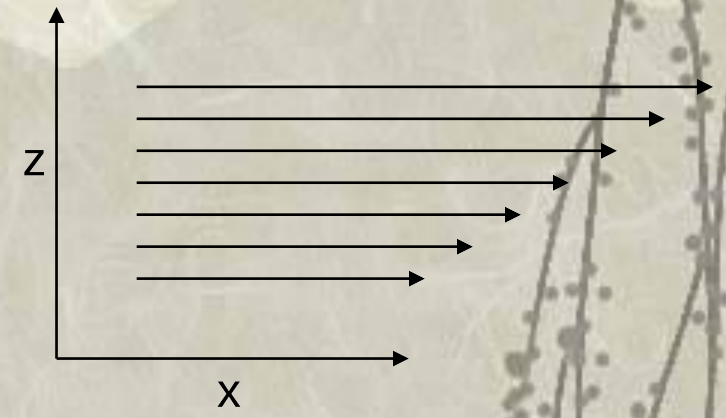
- The surface of the Earth exerts a frictional drag on the air blowing just above it. This friction can act to change the wind's direction and slow it down -- keeping it from blowing as fast as the wind aloft. Actually, the difference in terrain conditions directly affects how much friction is exerted. For example, a calm ocean surface is pretty smooth, so the wind blowing over it does not move up, down. By contrast, hills and forests force the wind to slow down and/or change direction much more.

As we move higher, surface features affect the wind less until the wind is indeed *geostrophic*.This level is considered the top of the *boundary (or friction) layer*. The height of the boundary layer can vary depending on the type of terrain, wind, and vertical temperature profile.

The time of day and season of the year also affect the height of the boundary layer. However, usually the boundary layer exists from the surface to about 1-2 km above it.

Viscous Force

- If the wind velocity varies with height, random molecular motions will cause momentum to be transferred vertically.
- In other words, there is a drag exerted by the layers above and below the level of interest.



$$F = \mu \frac{Au_0}{l}$$

$$F = \mu \frac{A\delta u}{\delta z}$$

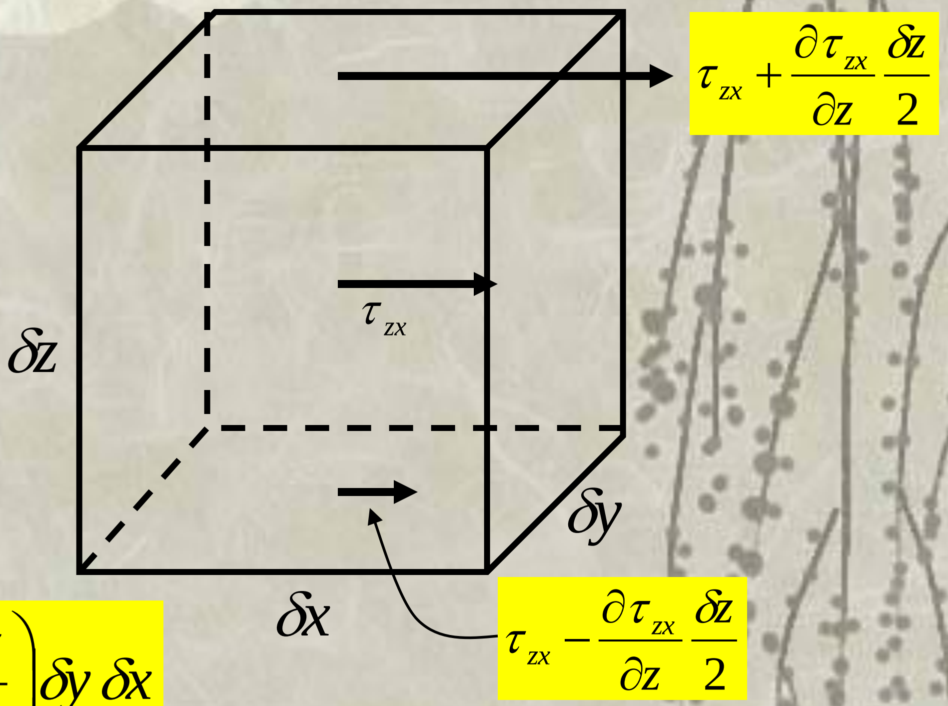
$$\delta u = u_0 \frac{\delta z}{l}$$

$$\tau_{zx} = \lim_{\delta z \to 0} \mu \frac{\delta u}{\delta z} = \mu \frac{\partial u}{\partial z}$$

The stress due to the velocity shear is given by

$$\tau_{zx} = \mu \frac{\partial u}{\partial z}$$

where μ is the dynamic viscosity coefficient.



Using Taylor series expansion to express the net viscous force:

$$\left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2} \right) \delta y \, \delta x - \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2} \right) \delta y \, \delta x$$

$$= \frac{\partial \tau_{zx}}{\partial z} \delta z \, \delta y \, \delta x$$

Net viscous force $= \dfrac{\partial \tau_{zx}}{\partial z}\delta z\,\delta y\,\delta x$

Dividing the above expression by the mass $\rho\ \delta x\ \delta y\ \delta z$ yields the viscous force per unit mass:

$$\frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) = \nu\frac{\partial^2 u}{\partial z^2}$$

$\nu = \mu/\rho =$ kinematic viscosity coefficient = 1.46 x 10^{-5} m^2 s^{-1}

Molecular viscosity is too small to be important except very close (cm) to the Earth's surface and above 100 km. Other sources of momentum transfer are more important in the lower atmosphere, and these will be discussed later in the semester.

$$F_x = F_{xx} + F_{yx} + F_{zx}$$

$$F_x = \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)$$

$$F_{yx} = \frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial y},\ F_{xx} = \frac{1}{\rho}\frac{\partial \tau_{xx}}{\partial x}$$

$$F_x = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$F_x = \nu \nabla^2 u$$

$$\Rightarrow F_x \cong \nu \frac{\partial^2 u}{\partial z^2} = F_{zx}$$

$$F_y \cong \nu \frac{\partial^2 V}{\partial z^2} = F_{zy}$$

$$F_z \cong \nu \frac{\partial^2 w}{\partial z^2} = F_{zz}$$

$$\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2} \langle\langle \frac{\partial^2 u}{\partial z^2}$$

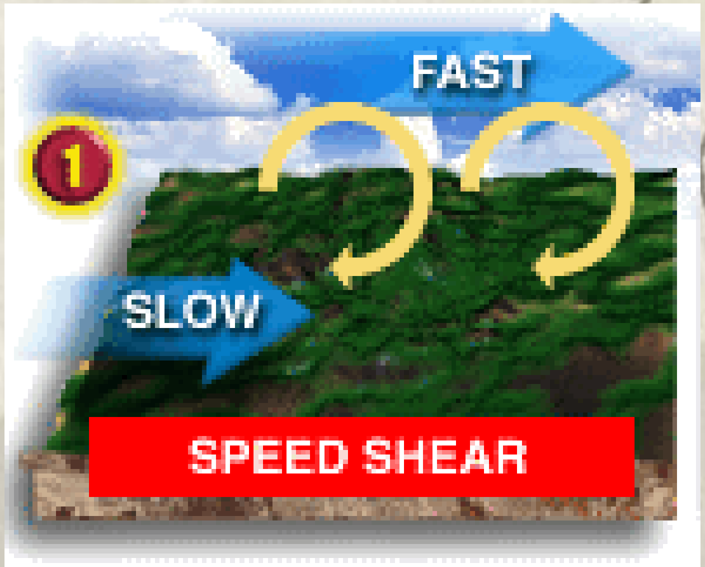
$$\vec{F} = \hat{i} F_x + \hat{j} F_y + \hat{k} F_z$$

$$\vec{F} = \nu \nabla^2 \vec{U}$$

$$\vec{F} = \nu (\hat{i} \frac{\partial^2 u}{\partial z^2} + \hat{j} \frac{\partial^2 v}{\partial z^2} + \hat{k} \frac{\partial^2 w}{\partial z^2})$$

$$F \cong \nu \frac{\partial^2 \vec{U}}{\partial z^2}$$

- The major cause of the air turbulence that sometimes makes planes bounce up and down in flight is wind shear. The term *wind shear* refers to a change in wind speed or direction, or both, over a short distance. Such changes help create eddies, or swirls of air, that cause turbulence. Wind shear can be both vertical and horizontal and can cause anything from minor turbulence to tornadoes, depending on the scale of shear.



Wind shear is a sudden change of direction and/or speed of the airflow.

Wind shear is the difference between the wind in 2 points divided by the distance between them.

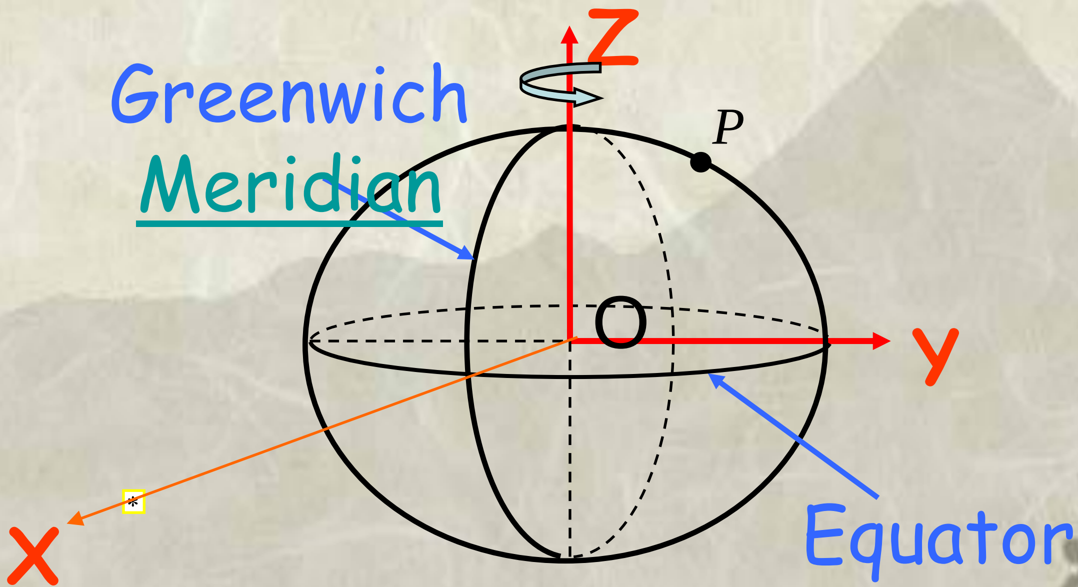
Frames of Reference

- Newton’s laws of motion are valid in a coordinate system that is fixed in space.
- A coordinate system fixed in space is known as an inertial (or absolute) frame of reference.
- A coordinate system that is not fixed in space, such as one defined with respect to the rotating earth, is a noninertial frame of reference.
- Because we are interested in atmospheric and oceanic motions from an earth-based perspective, we must formulate the laws of motion in the noninertial frame of reference defined with respect to the rotating earth.

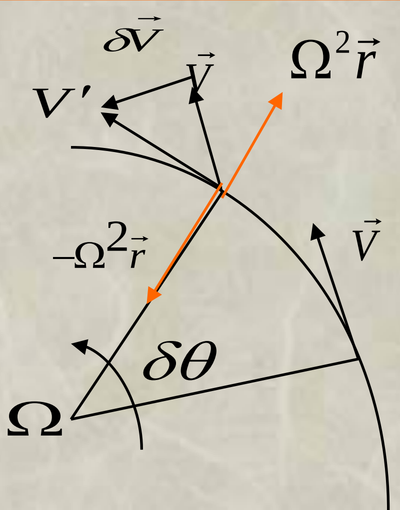
Apparent Forces

- In an inertial reference frame, a body at rest or in uniform motion has no net forces acting on it.
- A body at rest or in uniform motion relative to the rotating earth is not at rest or in uniform motion relative to a coordinate system fixed in space.
- To reconcile Newton’s laws with the noninertial reference frame of the rotating earth, two apparent forces must be introduced.
- These are the centrifugal force and the Coriolis force.

Noninertial Reference Frames and Apparent Forces
نیروهای ظاهری و دستگاه مختصات غیر لخت



The Centrifugal Force نیروی گریز از مرکز



Greenwich Meridian

$$\left|\delta\vec{V}\right| = \left|\vec{V}\right|\delta\theta$$

$$\lim_{\delta t\to 0}\frac{\left|\delta\vec{V}\right|}{\delta t} = \left|\vec{V}\right|\lim_{\delta t\to 0}\frac{\delta\theta}{\delta t}$$

$$\frac{d\vec{V}}{dt} = \left|\vec{V}\right|\frac{d\theta}{dt}\left(-\frac{\vec{r}}{r}\right) = (\Omega r)(\Omega)\left(-\frac{\vec{r}}{r}\right) = -\Omega^2\vec{r}$$

The Gravity Forc نیروی گرانی

$$\vec{g}^* = \vec{g}$$

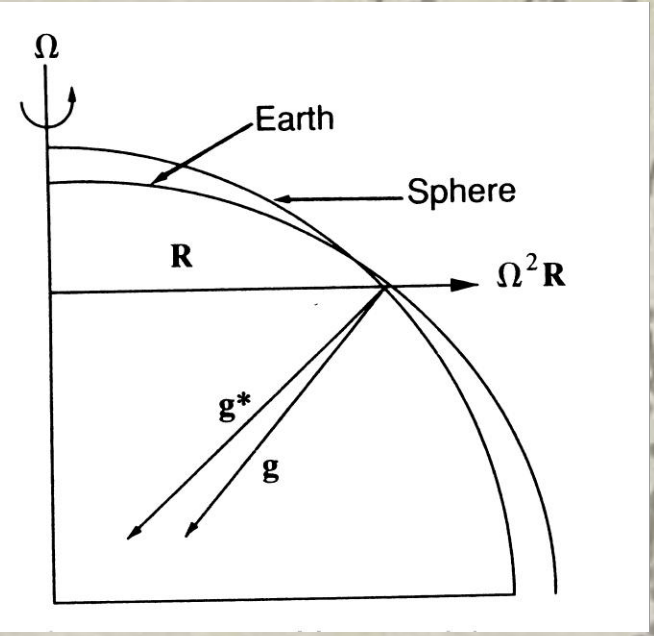
در صورت عدم چرخش زمین

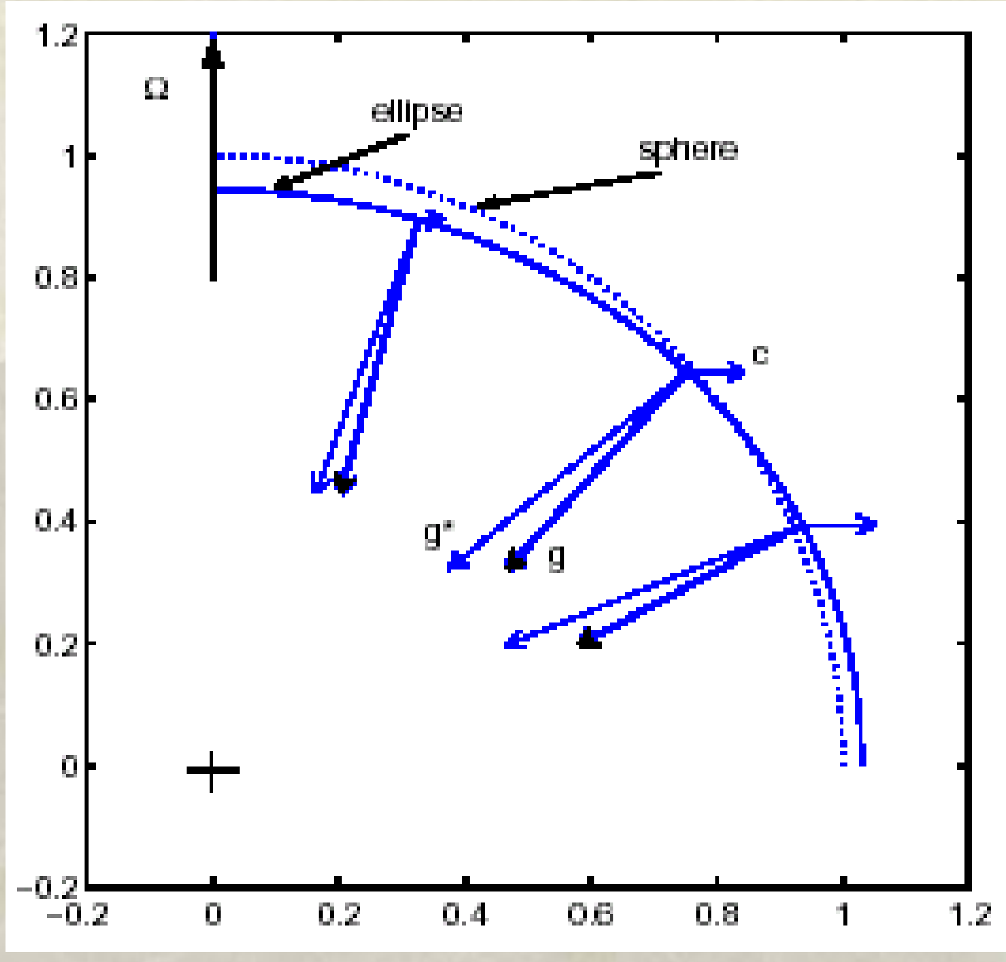
$$\vec{g}^* = \vec{g} \to pole$$

$$g = g^* - \Omega^2 R \to Equator$$

$$\Delta g = g_{pol} - g_{Eq} = 5.2 cm/s^2$$

$$g_x = 0, g_y = 0, g_z = -g$$





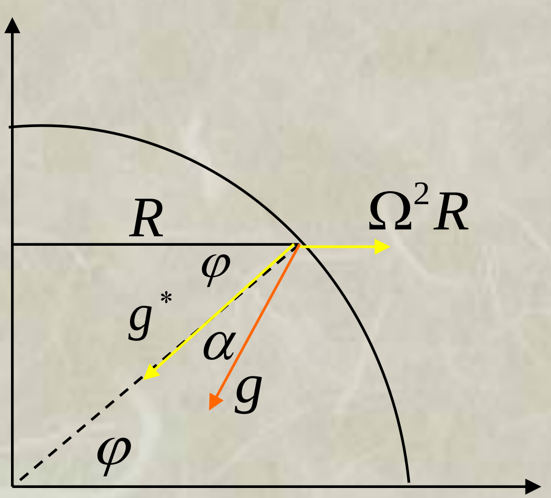
Ω
ellipse
sphere
c
g*
g
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.2
0
0.2
0.4
0.6
0.8
1
1.2

مثال: با صرفنظر کردن از تغییرات شعاع زمین در عرضهای متفاوت زاویه بین بردار گرانی و گرانشی را در سطح زمین بصورت تابعی از عرض جغرافیایی بدست آورید.

Problem 1:Neglecting the latitudinal variation in the radius of the earth, calculate the angle between the gravitational and gravity vectors at the surface of the earth as a function of latitude.

$$\vec{g}^* \times \vec{g} = \vec{g}^* \times (\vec{g}* + \Omega^2 \vec{R})$$

$$\left|\vec{g}^*\right|\left|\vec{g}\right| \sin\alpha = \left|\vec{g}^* \times \Omega^2 \vec{R}\right|$$



$$R = a\cos\varphi$$