دانشگاه رازی

Dynamic Meteorology 1

Lecture 14

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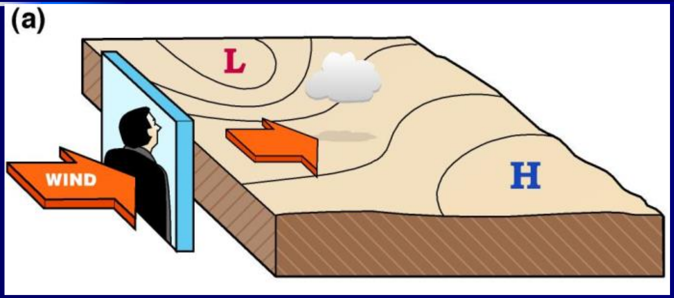
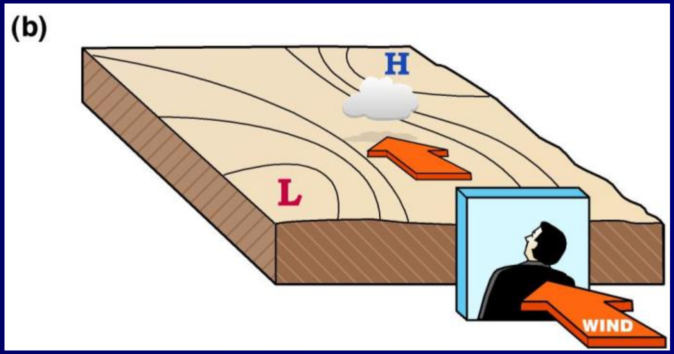
Buys-Ballot rule (Northern Hemisphere)



"If the wind blows into your back, the Low will be to your Left (and the high will be to your right)."



This rule works well if the wind is above the earth's boundary layer, (a geostrophic wind) not channeled by topography, etc.

This is because wind travels counterclockwise around low pressure zones in the Northern Hemisphere.

معادلات مولفه ای شارش در سیستم مختصات طبیعی

$$\begin{cases} dv/dt = -\partial\phi/\partial s \\ v^2/R + fv + \partial\phi/\partial n = 0 \end{cases}$$

$$V_g = -\frac{1}{f}\frac{\partial\Phi}{\partial n}$$

$$\frac{V^2}{R} + fV - fV_g = 0$$

V= Gradient wind speed

V_g= Geostrophic wind speed

$$\frac{V_g}{V} = 1 + \frac{V}{fR}$$

For normal cyclonic flow

$$\frac{V_g}{V} = 1 + \frac{V}{fR}$$

$$Rf > 0$$

$$1 + \frac{V}{fR} > 1$$

$$\Rightarrow V_g > V$$

the gradient wind around the low pressure system is less than the geostrophic wind if the pressure gradient force is constant.

For normal anti-cyclonic flow

$$\frac{V_g}{V} = 1 + \frac{V}{fR} \qquad if \quad fR < 0$$

$$1 + \frac{V}{fR} < 1 \qquad \Rightarrow \frac{V_g}{V} < 1 \quad \Rightarrow V_g < V$$

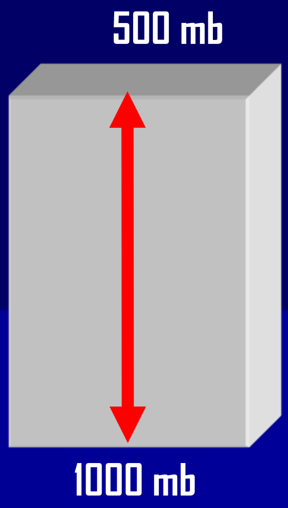
Gradient flow around a high pressure system will be faster than the geostrophic flow if the pressure gradient force is constant.

Therefore, the geostrophic wind is an overestimate of balanced wind in a region of cyclonic curvature and an understimate in a region of anticyclonic curvature.

For midlatitude synoptic system, the difference Between the gradient and geostrophic wind speeds Generally does not exceed 10-20%.

(The magnitude of $V/(fR)$ is just Rossby No.)

Thickness…



Start with a column of air.

This column has some thickness: it is some distance between 1000 mb and 500 mb.

If we heat the column of air, it will expand, warm air is less dense

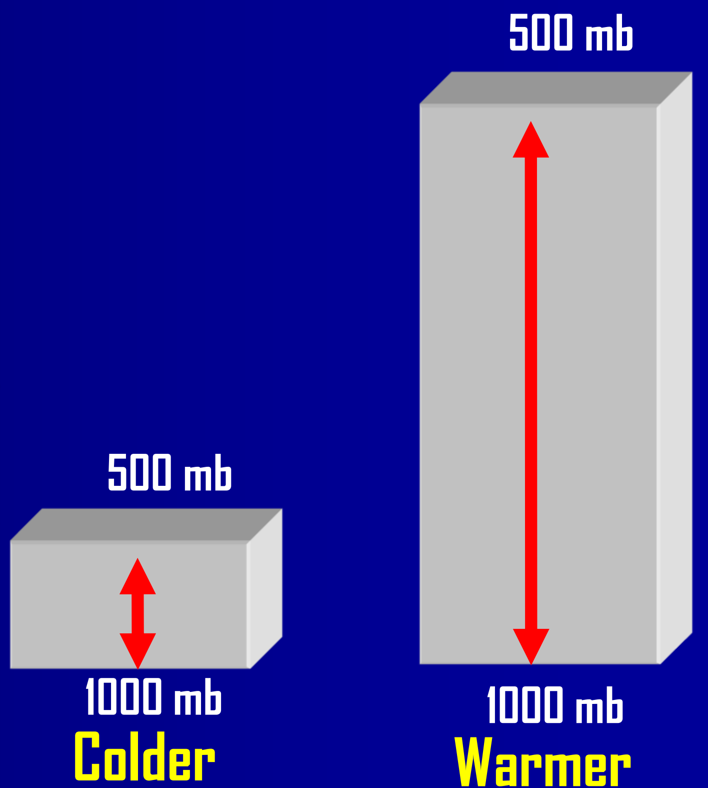
The thickness of the column will increase

500mb is now *farther from the ground*

If we cool the column of air, it will shrink, cool air is more dense

The thickness of the column will decrease

500mb is now *closer to the ground*



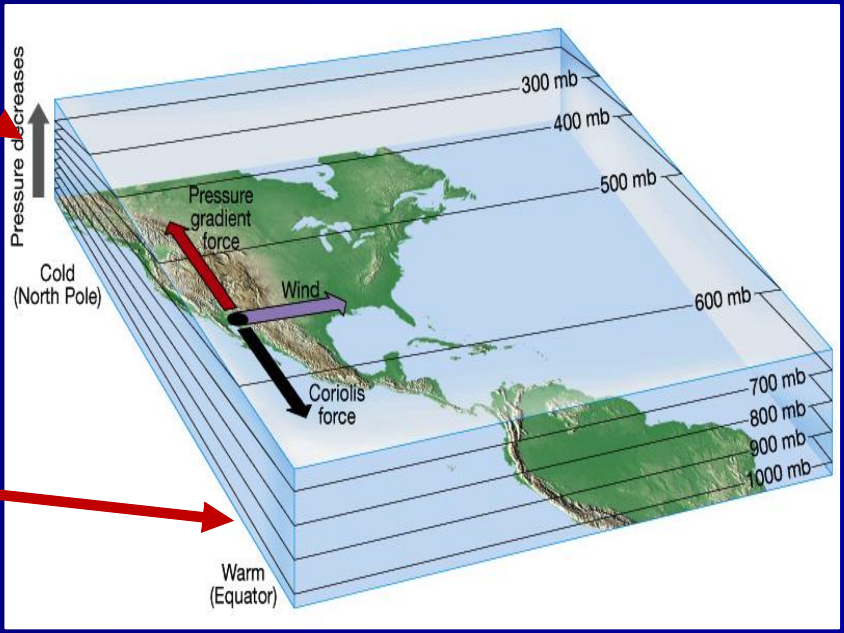
In fact, temperature is the **ONLY** factor in the atmosphere that determines the thickness of a layer.

At the poles, 700 mb is quite low to the ground

These layers are not very "thick"

In the tropics, 700mb is much higher above the ground

See how "thick" these layers are

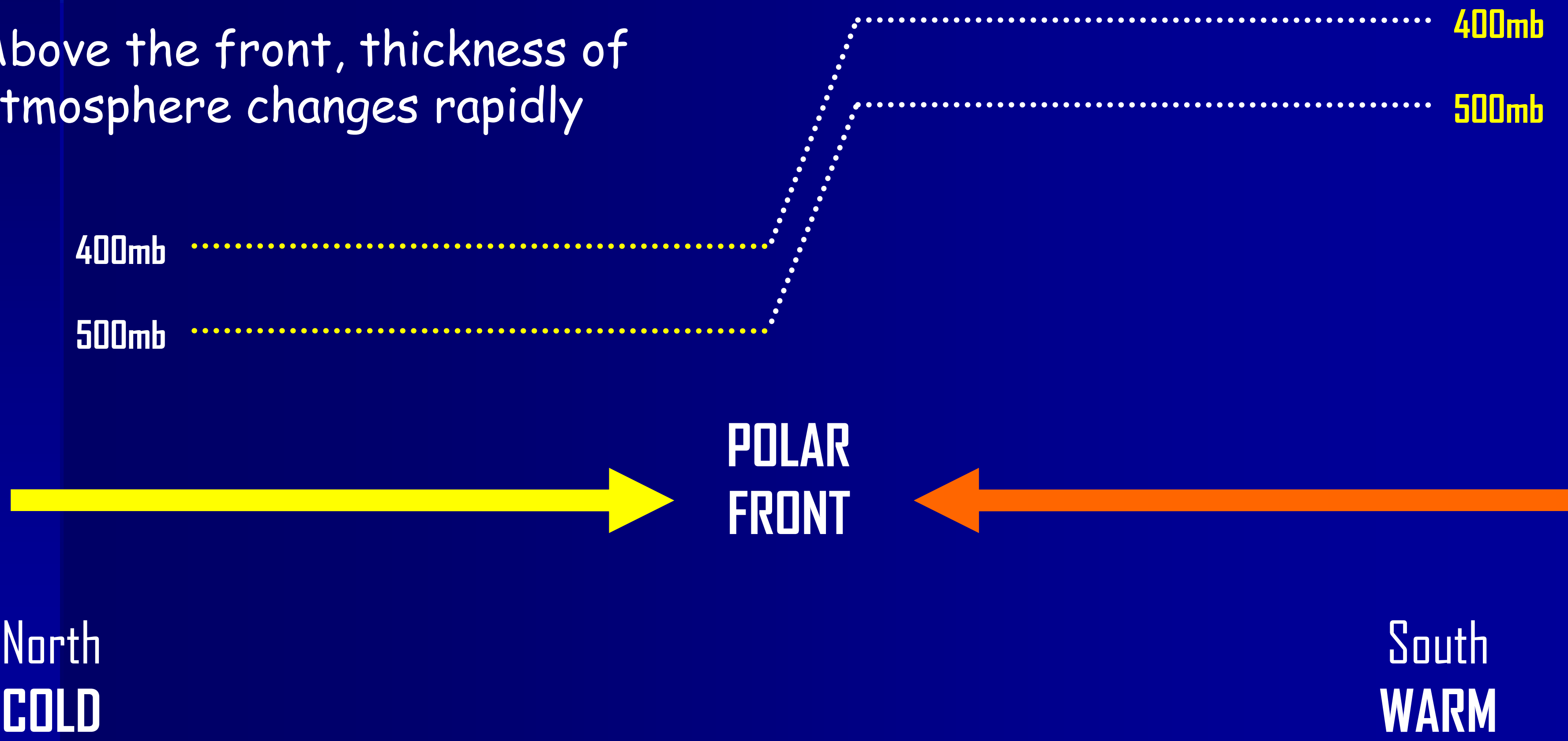


This is a cross section of the atmosphere

These winds meet at the polar front (a strong temperature gradient)

Now, think about what we just learned about how temperature controls the THICKNESS of the atmosphere

Above the front, thickness of atmosphere changes rapidly

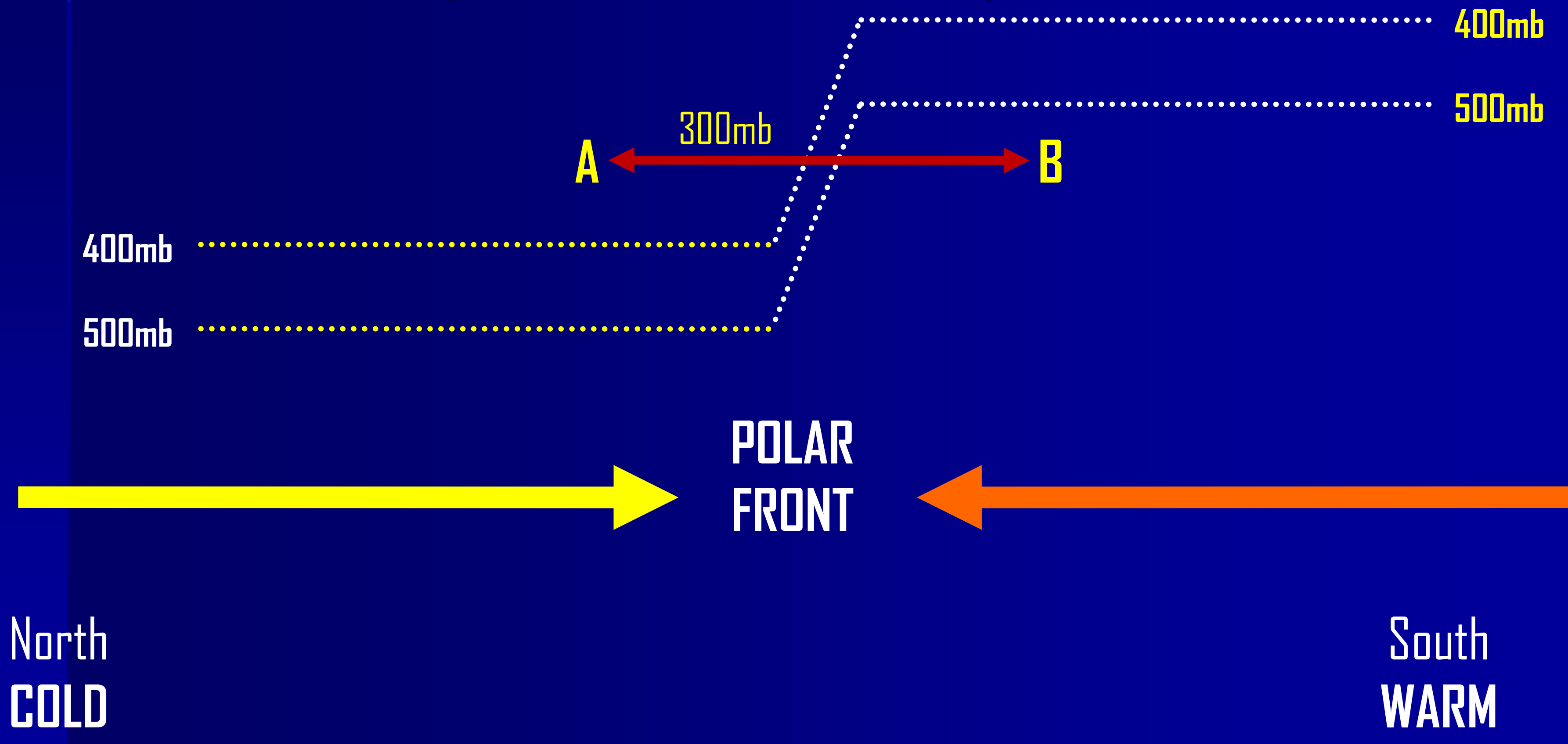


Now, what about the PGF above the front?

Let's draw a line between the cold side of the front and the warm side

What is the pressure at point A?

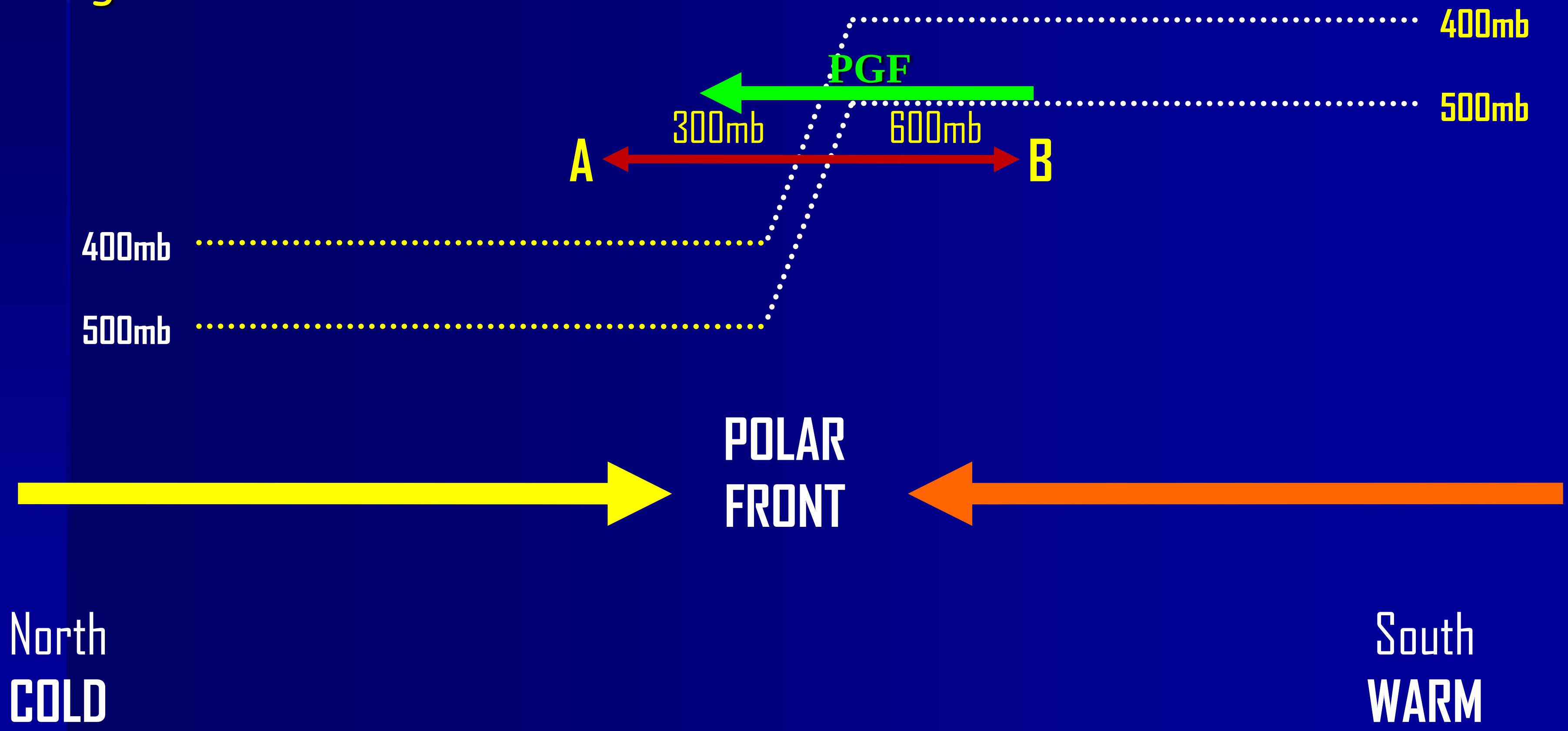
The pressure at point A is less than 400mb, since it is higher than the 400mb isobar on this plot. Let's estimate the pressure as 300mb.



What is the pressure at point B?

The pressure at point B is more than 500mb, since it is lower than the 500mb isobar on this plot. Let's estimate the pressure as 600mb.

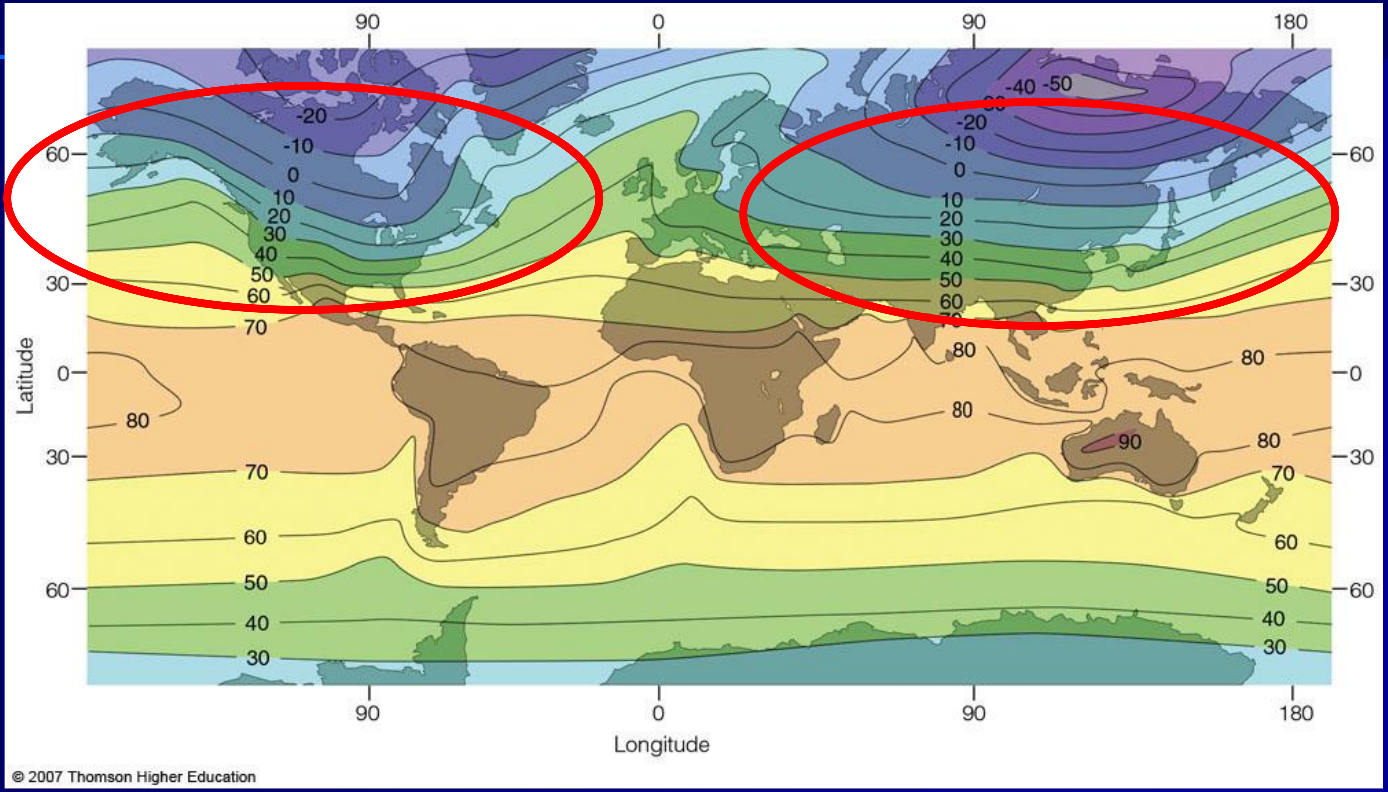
The pressure gradient force between point B & A is HUGE Therefore, all along the polar front, there will be a strong pressure gradient force aloft, pushing northward



Strong PGF is:

Aloft (above the surface)

Above the Polar Front (strong temperature gradient!)



PGF pushes to the north (in the Northern Hemisphere)

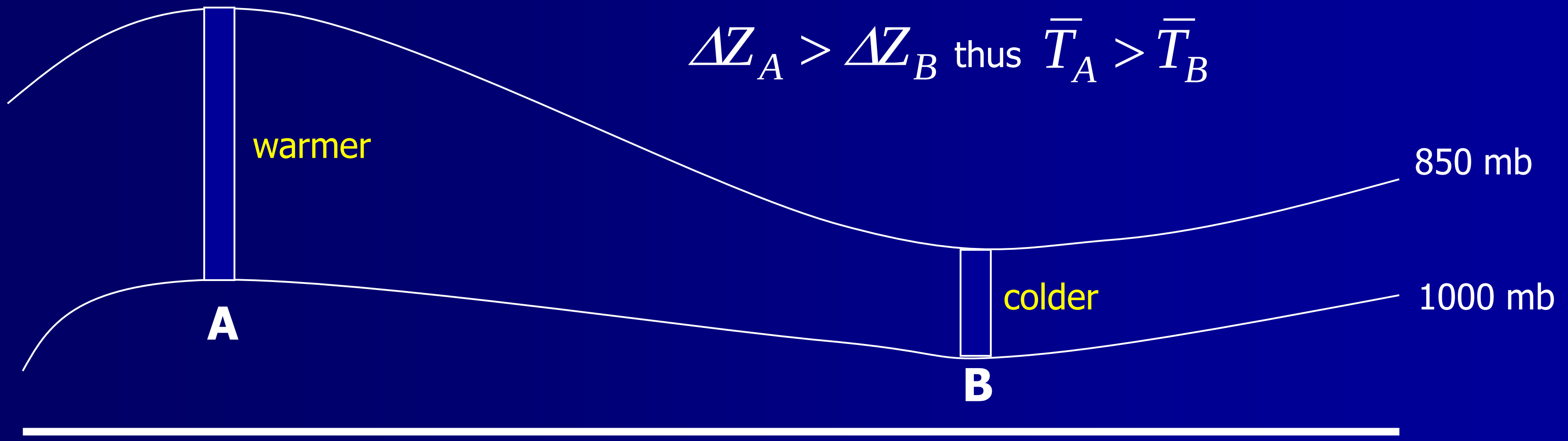
How does this cause the midlatitude jet stream?

Hyposmetric Equation and Thickness

$$\Delta Z \equiv Z_2 - Z_1 = \frac{R}{g}\int_{p_2}^{p_1} T\, d(\ln p)$$

The hypsometric equation relates the thickness, or vertical distance between two pressure levels. The thickness, ΔZ, is proportional to the mean temperature of the layer.

$\Delta Z_A > \Delta Z_B$ thus $\bar{T}_A > \bar{T}_B$



Thermal Wind Equation

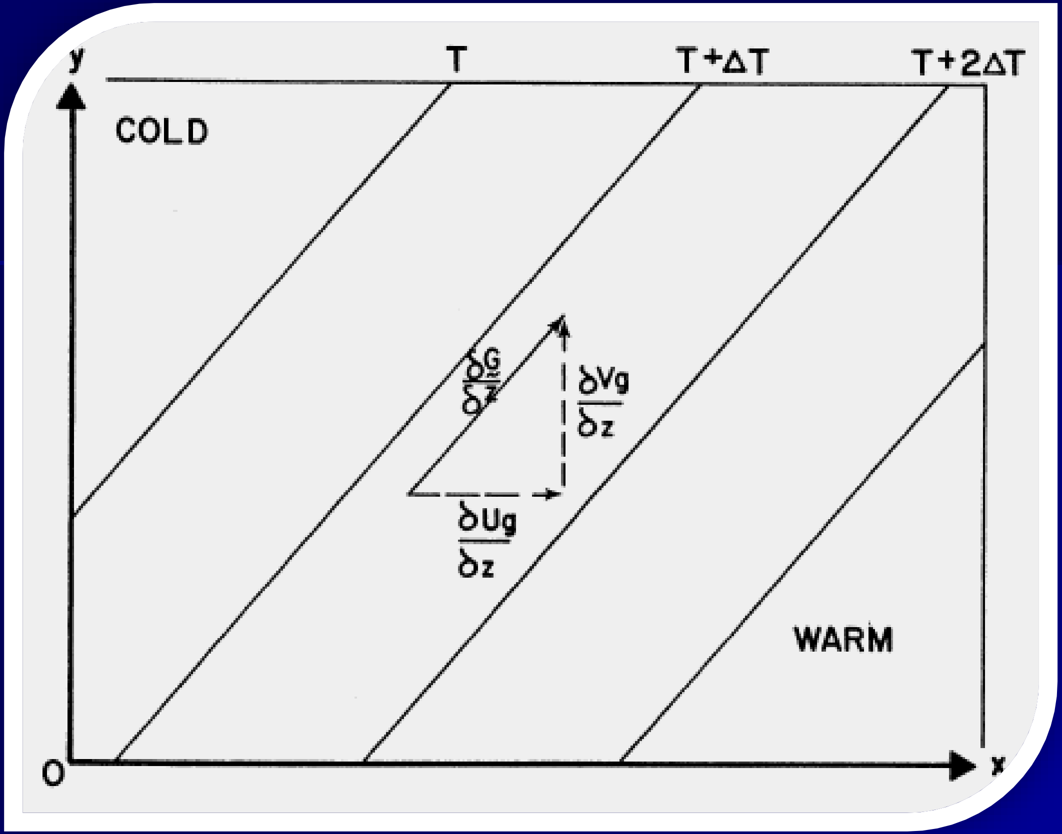
$$u_g = -\frac{1}{\rho f}\frac{\partial p}{\partial y} \qquad v_g = \frac{1}{\rho f}\frac{\partial p}{\partial x}$$

$$p = \rho g z$$

$$u_g = -\frac{g}{f}\frac{\partial z}{\partial y} \qquad v_g = \frac{g}{f}\frac{\partial z}{\partial x}$$

$$p = \rho R T$$

$$\frac{\partial u_g}{\partial z} = -\frac{g}{fT}\frac{\partial T}{\partial y} \qquad \frac{\partial v_g}{\partial z} = \frac{g}{fT}\frac{\partial T}{\partial x}$$



Now we compute the difference in geostrophic wind between the two levels:

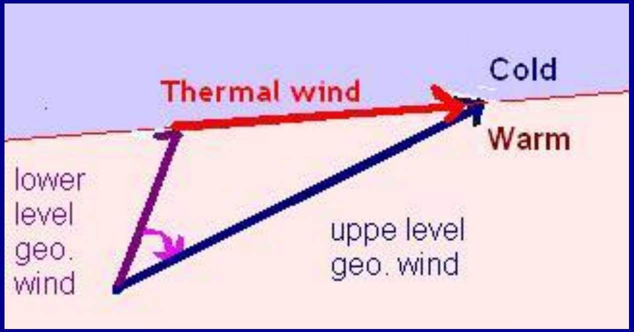
We begin by writing the vector form of the geostrophic wind equation in isobaric coordinates for two levels:

Thermal wind and temperature advection

Direction of the thermal wind determines the thermal structure of the atmosphere.
In NH, the thermal wind always points parallel to lines of constant thickness (parallel to isotherms) with lower thicknesses to the left. Therefore, the thermal wind always has the colder air to the left.

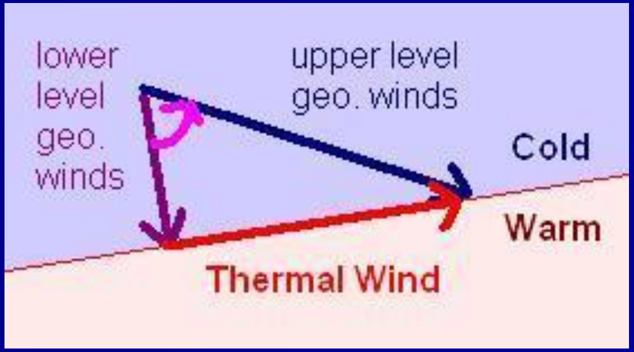
Veering winds (clockwise rotation with height):

warm air advection



Backing winds (counterclockwise rotation with height):

cold air advection



Wind backing and veering allow an estimation of the horizontal temperature gradient with data from an atmospheric sounding.

Thermal Wind Equation

We begin by writing the vector form of the geostrophic wind equation in isobaric coordinates for two levels:

$$\vec{V}_g(p_1) = \frac{1}{f}\hat{k} \times \nabla_p \Phi_1$$

$$\vec{V}_g(p_2) = \frac{1}{f}\hat{k} \times \nabla_p \Phi_2$$

Now we compute the difference in geostrophic wind between the two levels:

$$\vec{V}_T \equiv \vec{V}_g(p_2) - \vec{V}_g(p_1)$$

$$\vec{V}_T = \frac{1}{f}\hat{k} \times \nabla_p (\Phi_2 - \Phi_1)$$

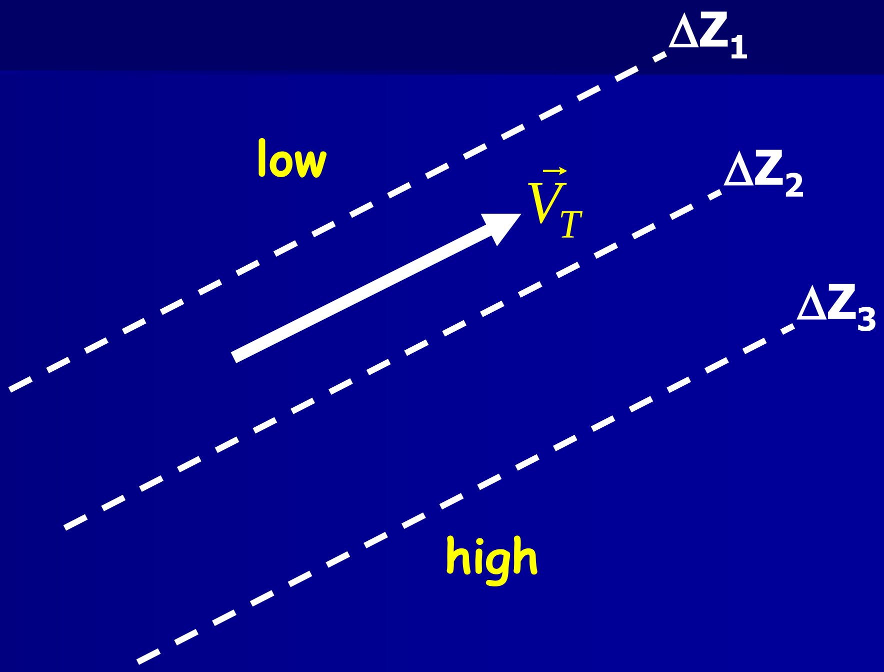
thermal wind equation

Thermal Wind

The thermal wind is the vertical shear of the geostrophic wind between two levels.

The thermal wind is a vector that is oriented parallel to the thickness isolines with lower values to the left (in the N. Hemisphere).

Its magnitude is proportional to the thickness gradient.

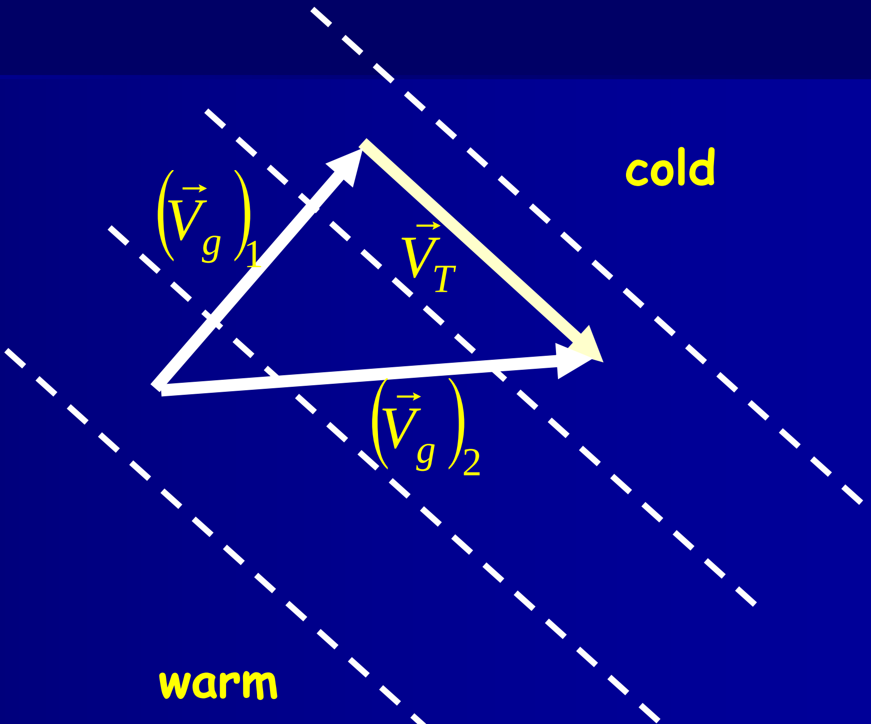


Geostrophic Wind Shear and Thermal Advection

Case 1: Geostrophic wind veers (i.e., turns clockwise) with height.

Lower level wind is from SW.
Upper level wind is from W.

Since colder air must lie to the left of the thermal wind, the layer average wind blows from warm to cold, which implies warm advection.



$$(\vec{V}_g)_1 + \vec{V}_T = (\vec{V}_g)_2$$

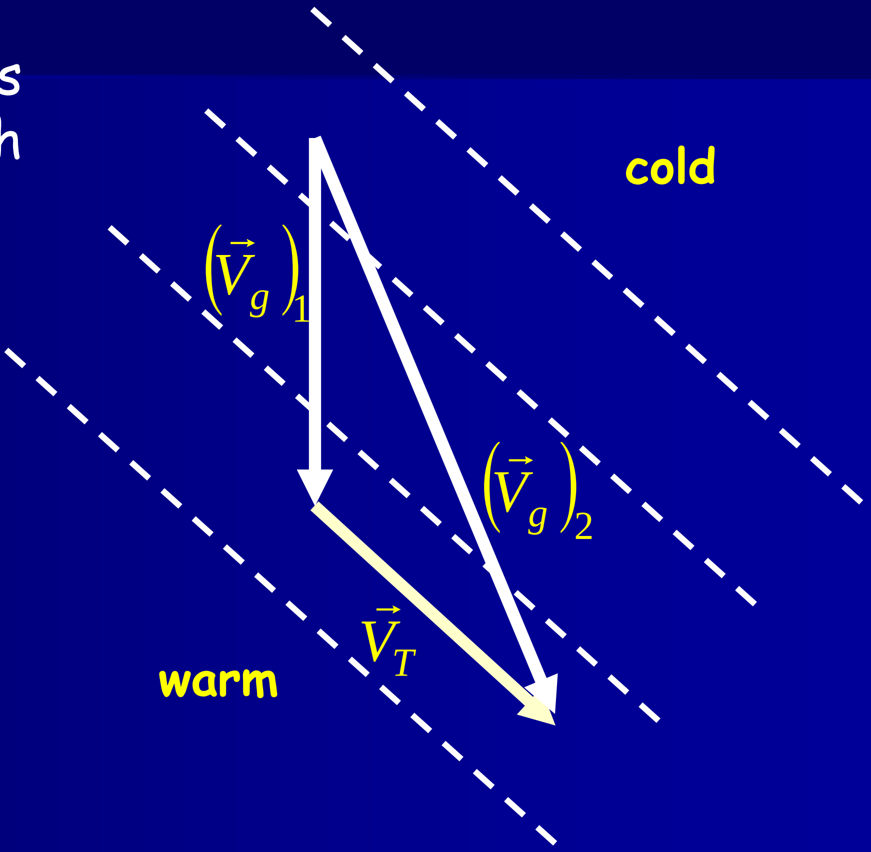
$$\vec{V}_T = (\vec{V}_g)_2 - (\vec{V}_g)_1$$

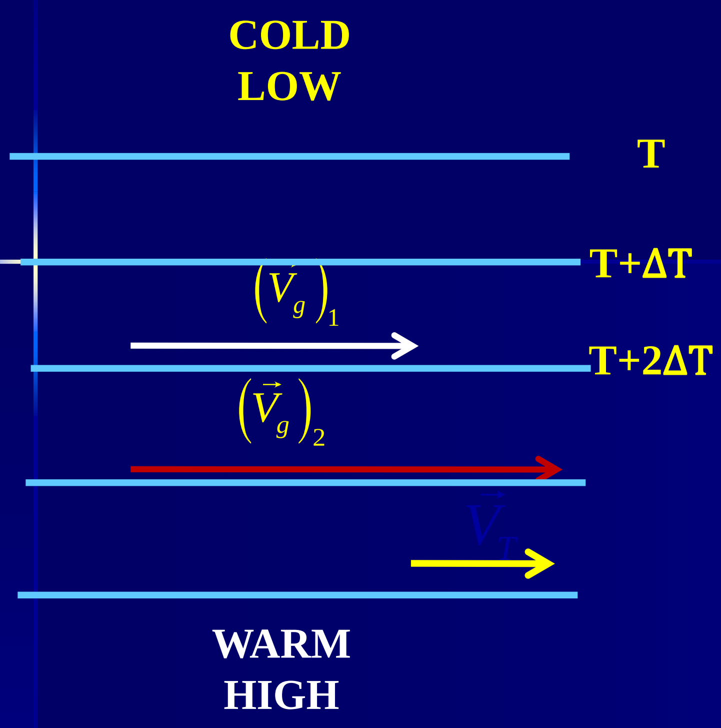
Geostrophic Wind Shear and Thermal Advection

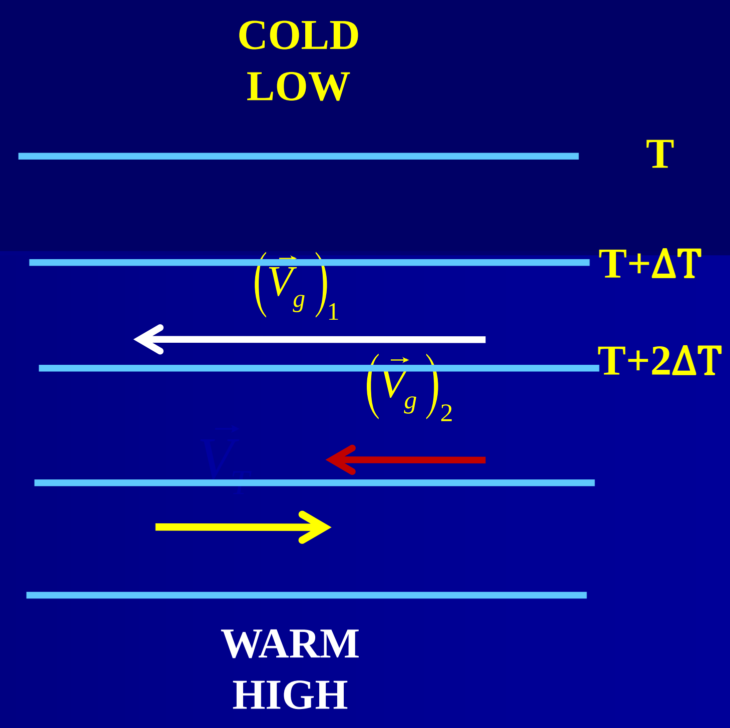
Case 2: Geostrophic wind backs (i.e., turns counterclockwise) with height.

Lower level wind is from N.
Upper level wind is from NW.

Since colder air must lie to the left of the thermal wind, the layer average wind blows from cold to warm, which implies cold advection.




COLD
LOW
T
T+ΔT
$(\vec{V}_g)_1$
T+2ΔT
$(\vec{V}_g)_2$
WARM
HIGH


COLD
LOW
T
T+ΔT
$(\vec{V}_g)_1$
T+2ΔT
$(\vec{V}_g)_2$
WARM
HIGH

Baroclinic vs. Barotropic

Barotropic	Baroclinic
$\rho=\rho(p)$ only **Implications:** 1) isobaric and isothermal surfaces coincide 2) no vertical wind shear (thermal wind = 0) 3) no tilt of pressure systems with height	$\rho=\rho(p,T)$ **Implications:** 1) isobaric and isothermal surfaces intersect 2) vertical wind shear (thermal wind ≠ 0) 3) pressure systems tilt with height

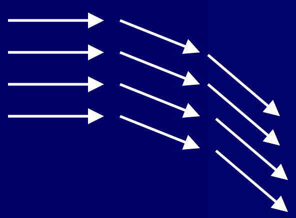
Seasons: Atmosphere is most baroclinic in winter.
Atmosphere is least baroclinic in summer.

Geographic: *Atmosphere is most baroclinic in midlatitudes*
Atmosphere is least baroclinic in the Tropics

Vorticity

Vorticity is the microscopic measure of spin and rotation in a fluid.

Vorticity is defined as the curl of the velocity: $\nabla \times \vec{V}$



Wind direction varies → clockwise spin

Wind speed varies → clockwise spin

Absolute vorticity (inertial reference frame): $\vec{\omega}_a \equiv \nabla \times \vec{V}_a$

Relative vorticity (relative to rotating earth): $\vec{\omega} \equiv \nabla \times \vec{V}$

Expansion of relative vorticity into Cartesian components:

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\nabla \times \vec{V} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)\hat{k}$$

For large scale dynamics, the vertical component of vorticity is most important. The vertical components of absolute and relative vorticity in vector notation are:

$$\zeta = \hat{k} \cdot \left(\nabla \times \vec{V} \right)$$ relative vorticity

$$\eta = \hat{k} \cdot \left(\nabla \times \vec{V}_a \right)$$ absolute vorticity

From now on, vorticity implies the vertical component (unless otherwise stated.)

The absolute vorticity is equal to the relative vorticity plus the earth's vorticity. Since the earth's vorticity is

$$\hat{k}\cdot\left(\nabla\times\vec{V}_e\right)=2\Omega\sin\phi=f$$

$$\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

and

$$\eta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}+f=\zeta+f$$

For large scale circulations, a typical magnitude for vorticity is

$$\zeta\approx\frac{U}{L}=10^{-5}\ s^{-1}$$

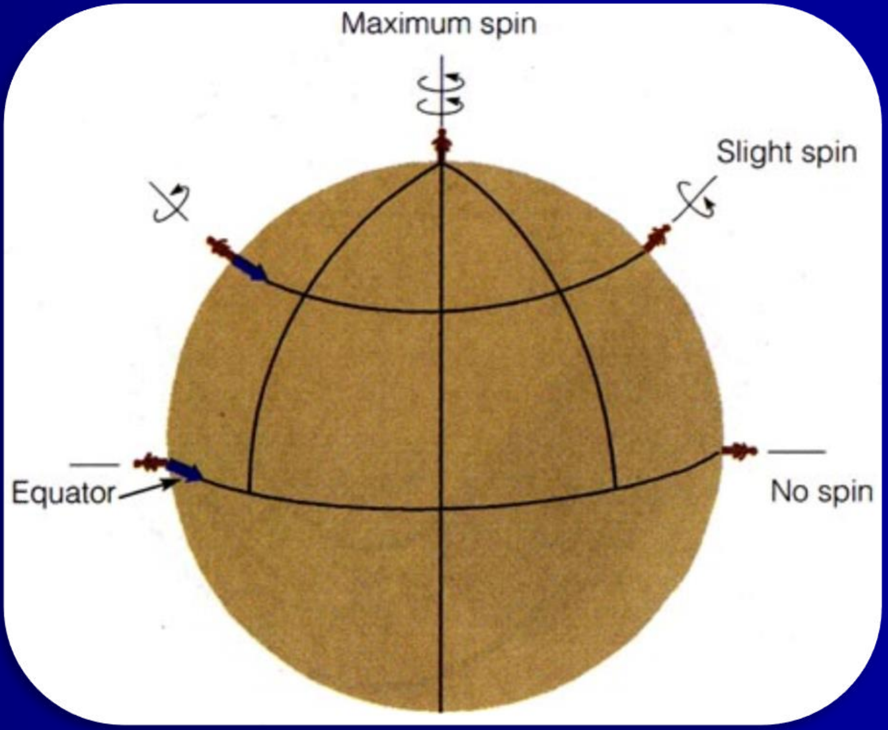
$$\eta = \zeta + f$$

Planetary Vorticity is spin produced by earth's rotation

$$f = 2\Omega \sin \phi$$

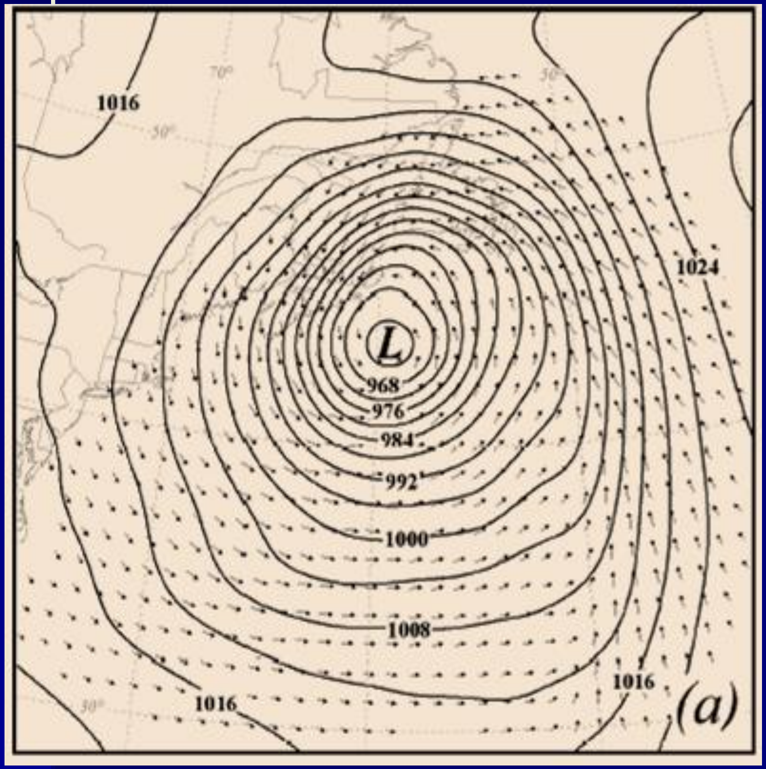
Component of earth's rotation oriented around local vertical

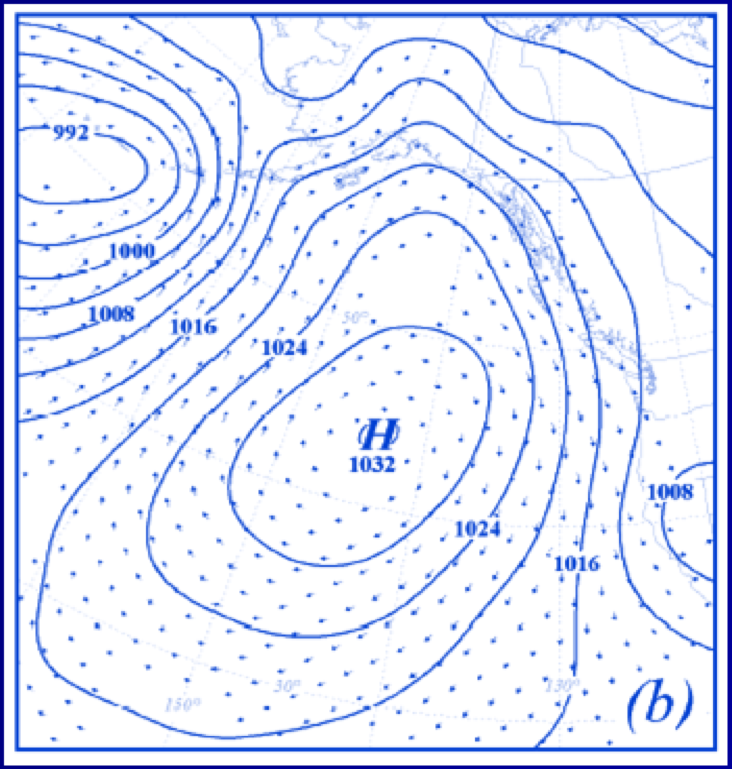
Always positive in Northern Hemisphere 0 at equator, increases northward



FLUID ROTATION

Circulation and Vorticity

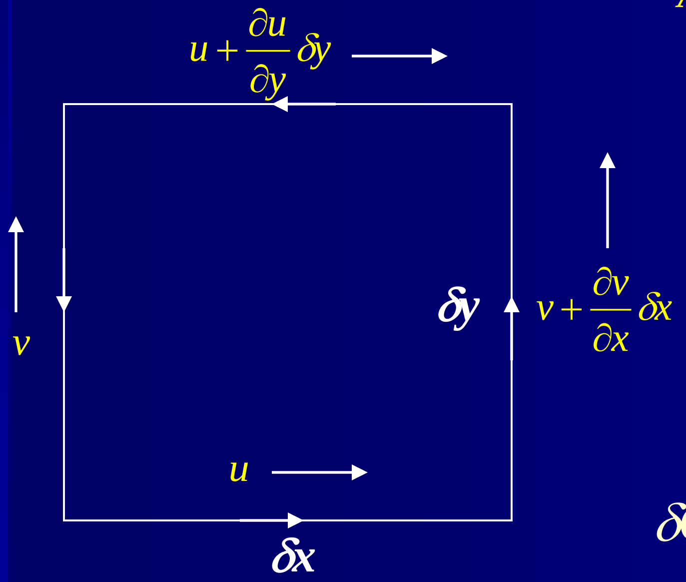




Circulation and Vorticity

The relationship between relative vorticity z and circulation can be seen by considering the following expression, in which we will define the relative vorticity as the circulation about a closed contour in the horizontal plane divided by the area enclosed by that contour, in the limit as the area approaches zero.

$$\zeta = \lim_{A \to 0} \left(\oint \vec{V} \cdot d\vec{l} \right) A^{-1}$$



Evaluating $\vec{V} \cdot d\vec{l}$ for each side of the rectangle yields the circulation:

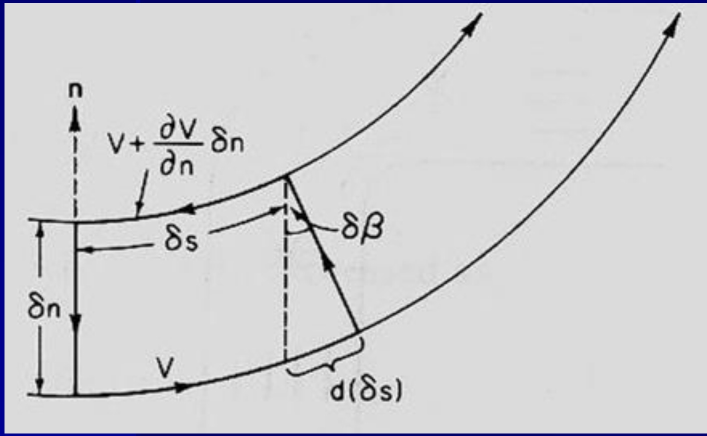
$$\delta C = u\,\delta x + \left(v + \frac{\partial v}{\partial x}\delta x \right)\delta y - \left(u + \frac{\partial u}{\partial y}\delta y \right)\delta x - v\,\delta y$$

$$\delta C = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)\delta x \delta y \rightarrow \frac{\delta C}{\delta A} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \zeta$$

Vorticity in Natural Coordinates

Using natural coordinates can make it easier to physically interpret the relationship between relative vorticity and the flow.

To express vorticity in natural coordinates, we compute the circulation around the infinitesimal contour shown below.



$$\delta C = V[\delta s + d(\delta s)] - \left(V + \frac{\partial V}{\partial n}\delta n\right)\delta s$$

From the diagram, $d(\delta s) = \delta\beta\ \delta n$, where $\delta\beta$ is the angular change in wind direction in the distance δn.

$$\delta C = \left(-\frac{\partial V}{\partial n} + V\frac{\partial \beta}{\partial s}\right)\delta n \delta s$$

In the limit $\delta n \delta s \rightarrow 0$,

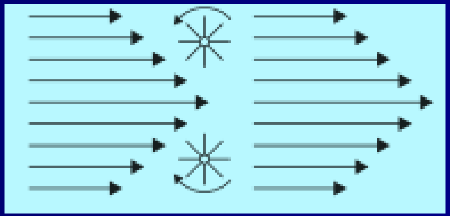
$$\zeta = \lim_{\delta n, \delta s \rightarrow 0} \frac{\delta C}{\delta n \delta s} = \boxed{-\frac{\partial V}{\partial n} + \frac{V}{R_s}}$$

Vorticity in natural coordinates:

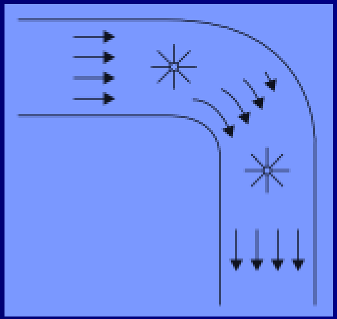
$$\zeta = -\frac{\partial V}{\partial n} + \frac{V}{R_s}$$

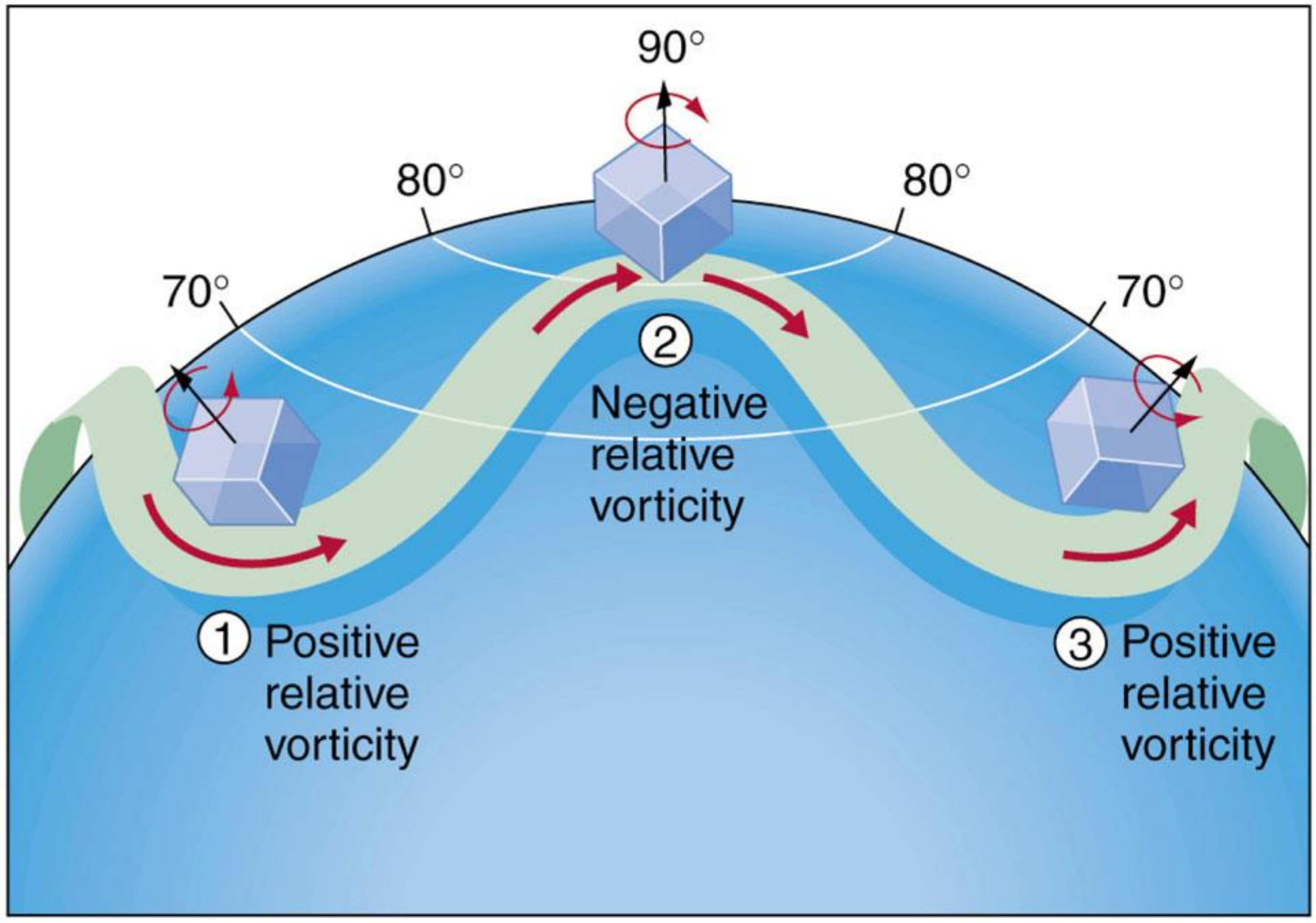
It is now apparent that the net vertical vorticity component is the result of the sum of two parts:

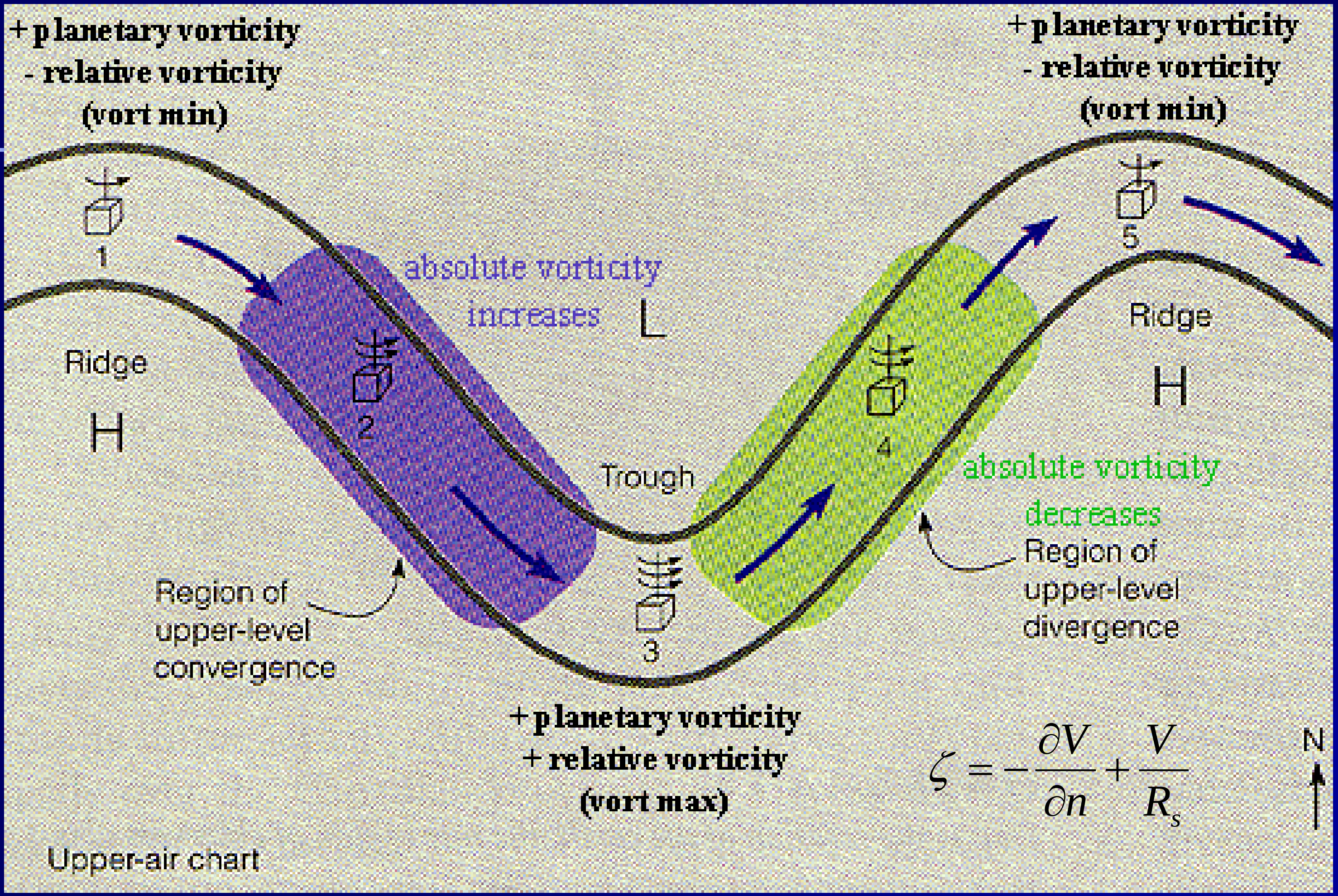
$-\frac{\partial V}{\partial n}$ The rate of change of wind speed normal to the direction of flow, which is called the shear vorticity.

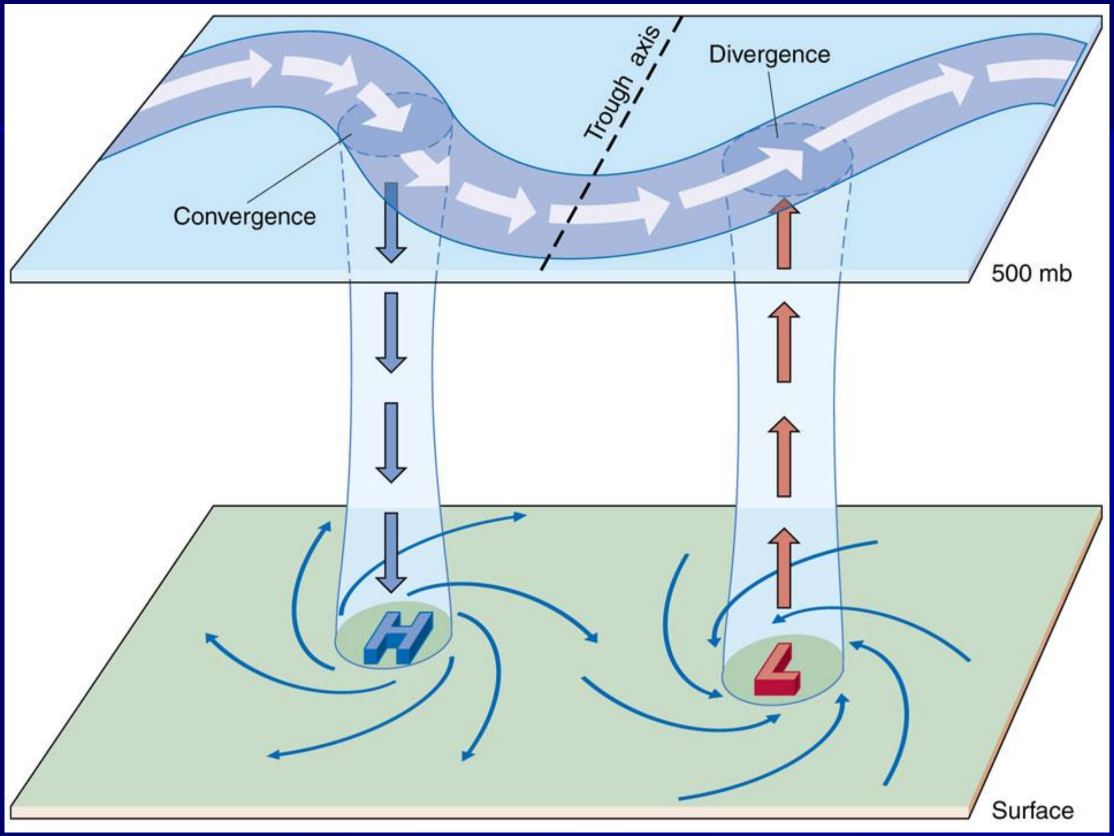


$\frac{V}{R_s}$ The turning of the wind along a streamline, which is called the curvature vorticity.




90°
80°
80°
70°
70°
2
Negative
relative
vorticity
1
Positive
relative
vorticity
3
Positive
relative
vorticity

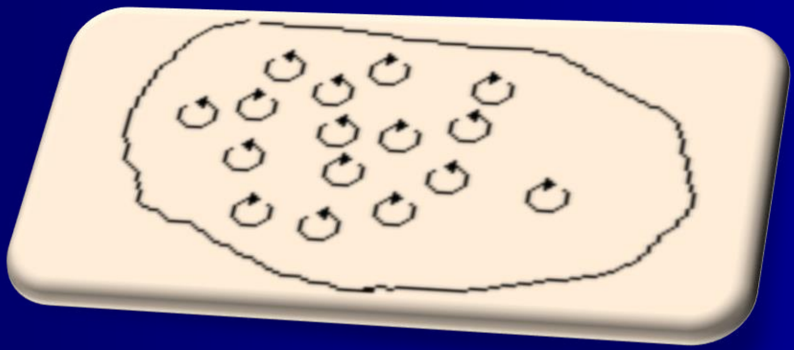

+ planetary vorticity
- relative vorticity
(vort min)
+ planetary vorticity
- relative vorticity
(vort min)
1
absolute vorticity
increases
L
5
Ridge
H
2
Ridge
H
4
Trough
absolute vorticity
decreases
Region of
upper-level
divergence
Region of
upper-level
convergence
3
+ planetary vorticity
+ relative vorticity
(vort max)
$\zeta = -\frac{\partial V}{\partial n} + \frac{V}{R_s}$
N
Upper-air chart


Trough axis
Divergence
Convergence
500 mb
Surface

Circulation, Vorticity, and Stokes Theorem

In more general terms the relationship between vorticity and circulation is given simply by Stokes's theorem applied to the velocity vector:

$$\oint \vec{V}.d\vec{l} = \iint_A (\nabla \times \vec{V}).\hat{n}dA$$



Here A is the area enclosed by the contour and n is a unit normal to the area element dA (positive in the right sense).

Thus, Stokes's theorem states that the circulation about any closed loop is equal to the integral of the normal component of vorticity over the area enclosed by the contour.

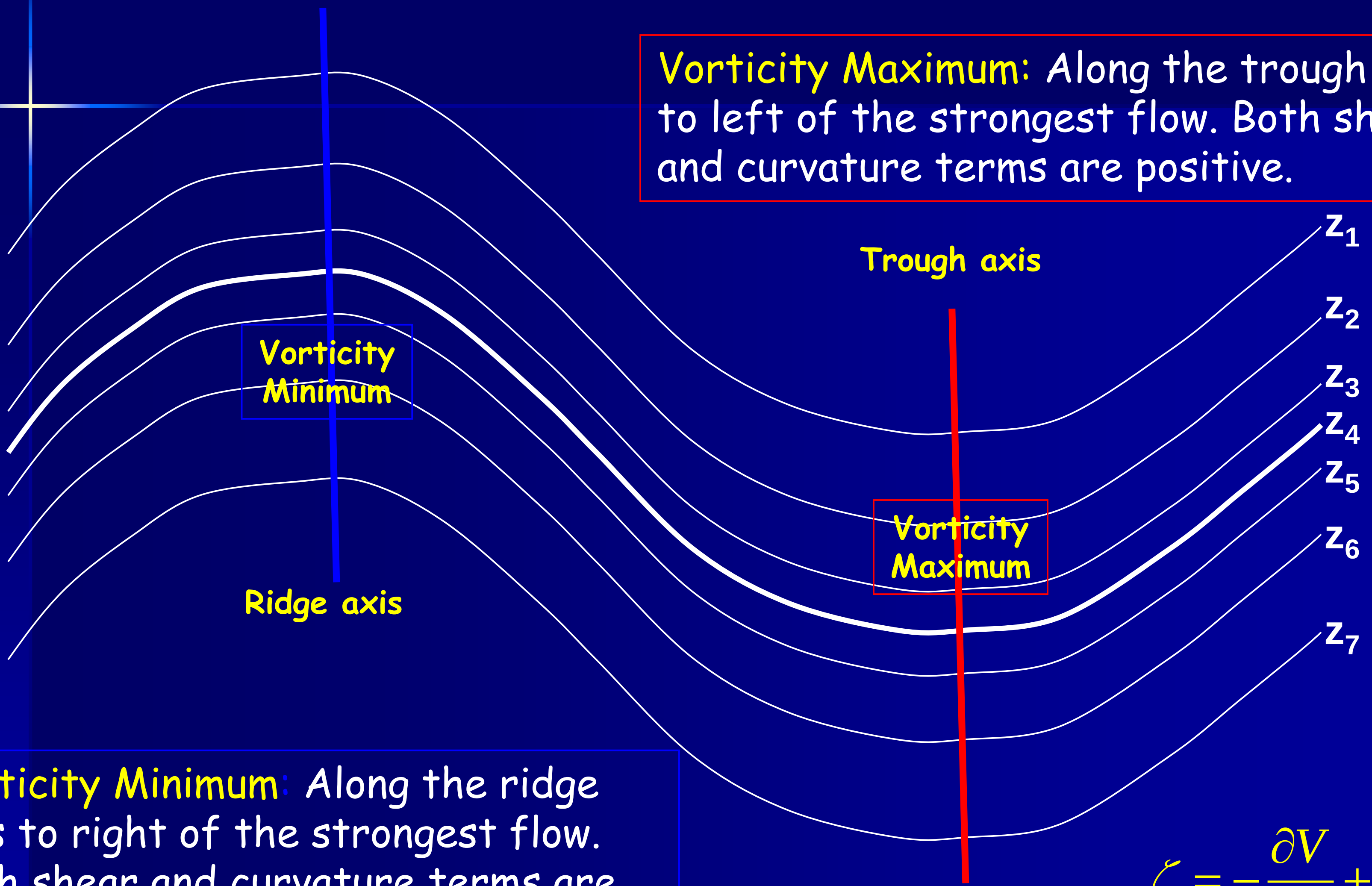
Hence, for a finite area, circulation divided by area gives the *average normal component of vorticity in* the region.

As a consequence, the vorticity of a fluid in solid-body rotation is just twice the angular velocity of rotation.

Vorticity may thus be regarded as a measure of the local angular velocity of the fluid.

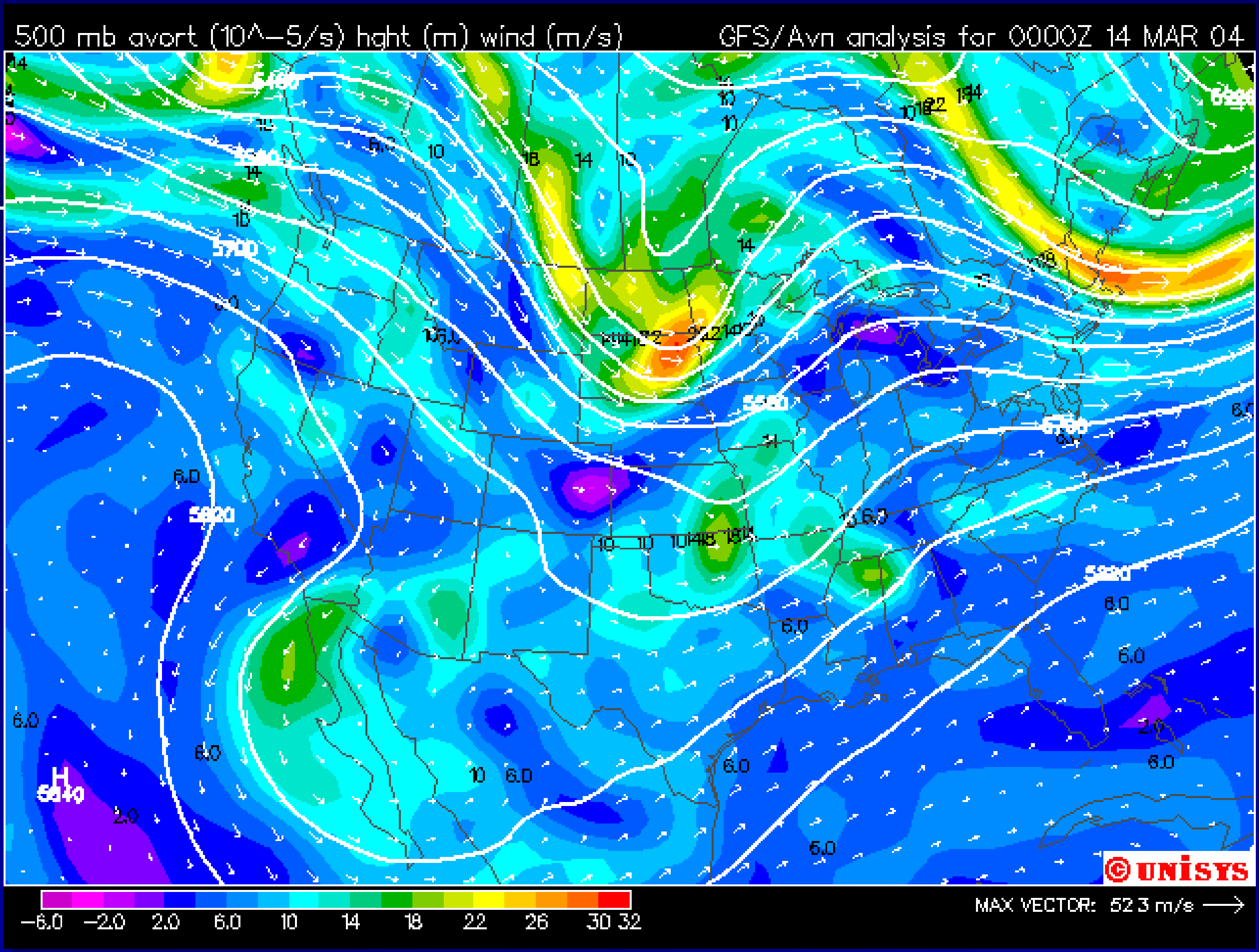
Vorticity On The Weather Map

Vorticity Maximum: Along the trough axis to left of the strongest flow. Both shear and curvature terms are positive.



Vorticity Minimum: Along the ridge axis to right of the strongest flow. Both shear and curvature terms are negative.

$$\zeta = -\frac{\partial V}{\partial n} + \frac{V}{R_s}$$



Absolute vorticity = $\zeta + f$

Thank you for your attention

Any Question ?