Dynamic Meteorology 1

Lecture 10

Sahraei
Department of Physics,
Razi University

http://www.razi.ac.ir/sahraei

Continuity Equation

$$\frac{\partial \rho}{\partial t} = -\nabla.(\rho\vec{V})$$

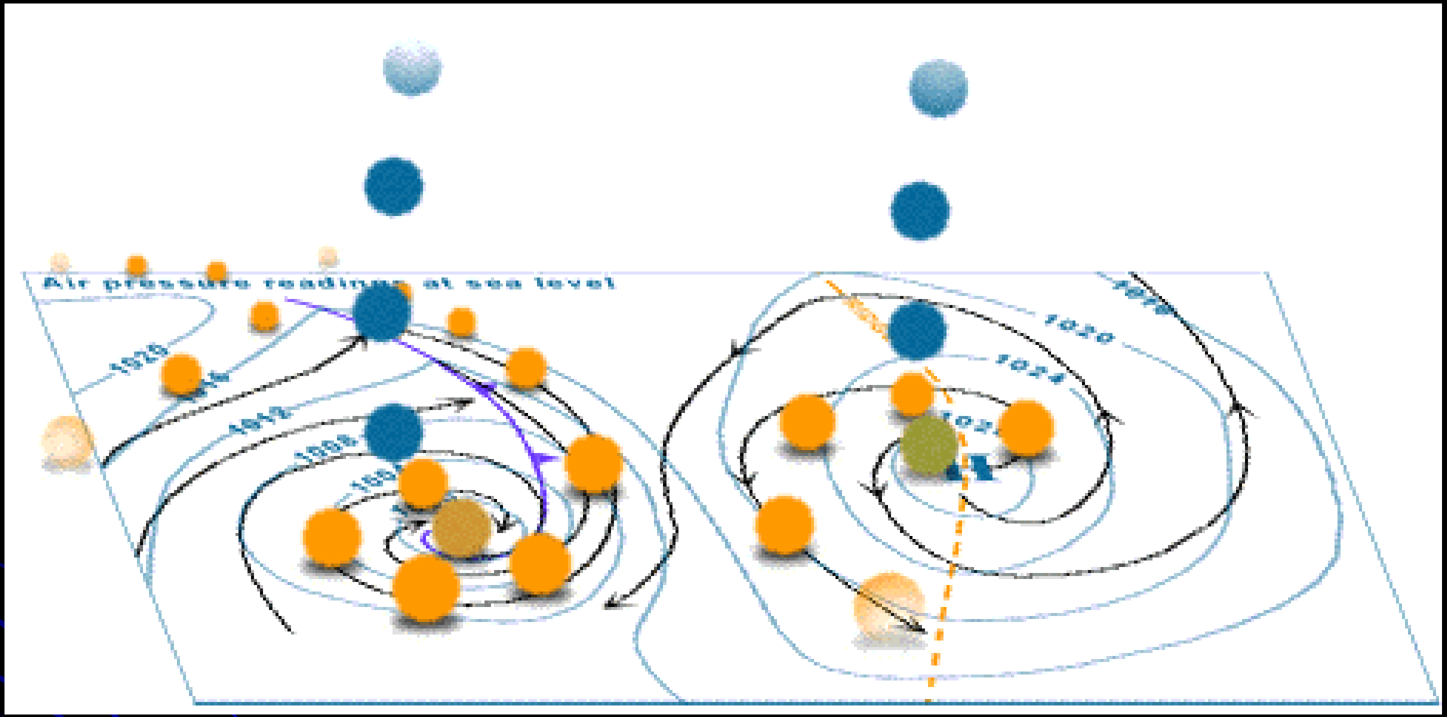
A physical interpretation of this form of the continuity equation is that the change in density at a fixed point in space is dependent upon the divergence of the mass flux.

If there is divergence of the mass flux then

$\nabla.(\rho\vec{V}) > 0$ and density will decrease $\frac{\partial \rho}{\partial t} < 0$

If there is convergence of the mass flux then

$\nabla.(\rho\vec{V}) < 0$ and density will increase $\frac{\partial \rho}{\partial t} > 0$


Air pressure readings at sea level
1020
1020
1024

An alternative form of the continuity equation is obtained by applying the vector identity:

$$\nabla.(\rho\vec{V}) \equiv \rho\nabla.\vec{V} + \vec{V}.\nabla\rho$$

$$\rho\nabla.\vec{V} + \vec{V}.\nabla\rho = -\frac{\partial\rho}{\partial t}$$

$$\rho\nabla.\vec{V} = -\frac{\partial\rho}{\partial t} - \vec{V}.\nabla\rho$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{V}.\nabla$$

$$\frac{1}{\rho}\frac{d\rho}{dt}+\nabla.\vec{V}=0$$

The velocity divergence form of the continuity equation

$$\frac{1}{\rho}\frac{d\rho}{dt}=-\nabla.\vec{V}$$

آهنگ تغییر نسبی چگالی که با همگرایی سرعت برابر است.

$$\frac{d\rho}{dt}=-\rho\nabla.\vec{V}$$

آهنگ تغییرات تام چگالی

ب) دیدگاه لاگرانژی

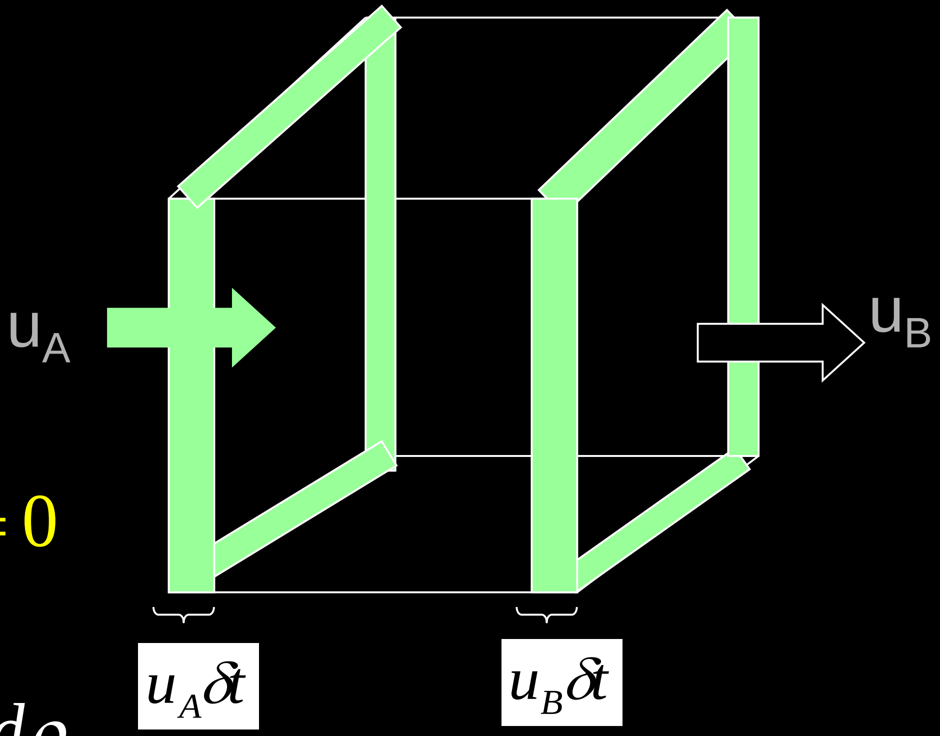
The physical meaning of divergence can be illustrated by the following method.

$$\delta V = \delta x \delta y \delta z$$

$$\delta M = \rho \delta V = \rho \delta x \delta y \delta z$$

$$\frac{d}{dt}(\delta M) = \rho \frac{d}{dt}(\delta V) + \delta V \frac{d\rho}{dt} = 0$$

$$\frac{1}{\delta M}\frac{d}{dt}(\delta M) = \frac{1}{\delta V}\frac{d}{dt}(\delta V) + \frac{1}{\rho}\frac{d\rho}{dt} = 0$$



$$\frac{1}{\delta V}\frac{d}{dt}(\delta V)=\frac{1}{\delta x}\frac{d}{dt}(\delta x)+\frac{1}{\delta y}\frac{d}{dt}(\delta y)+\frac{1}{\delta z}\frac{d}{dt}(\delta z)$$

δx ,δy ,δz =?

$$u_A=\frac{dx}{dt} \qquad u_B=\frac{d(x+\delta x)}{dt}$$

$$\delta u=u_B-u_A=\frac{d(x+\delta x)}{dt}-\frac{dx}{dt}=\frac{d(\delta x)}{dt}$$

$$\delta v=\frac{d(\delta y)}{dt} \qquad \delta w=\frac{d(\delta z)}{dt}$$

$$\frac{1}{\delta V}\frac{d(\delta V)}{dt}=\frac{1}{\delta x}\delta u+\frac{1}{\delta y}\delta v+\frac{1}{\delta z}\delta w$$

$$\lim_{\delta x\delta y\delta z\to 0}\left[\frac{1}{\delta V}\frac{d}{dt}(\delta V)\right]=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\nabla.\vec{V}$$

تغییرات نسبی حجم بسته هوا در واحد زمان برابر است با واگرایی سرعت .

$$\frac{1}{\rho}\frac{d\rho}{dt}+\nabla.\vec{V}=0$$

آهنگ تغییرات نسبی چگالی برابر است با همگرایی سرعت .

Continuity equation can be written in one of the two forms:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{V}) = 0$$

شکل واگرایی جرم از معادله پیوستگی
آهنگ تغییر محلی چگالی با منفی واگرایی جرم برابر است.

The velocity divergence form of the continuity equation

$$\frac{1}{\rho}\frac{d\rho}{dt} + \nabla \cdot \vec{V} = 0$$

آهنگ تغییر نسبی چگالی که با همگرایی سرعت برابر است.

حالت خاص

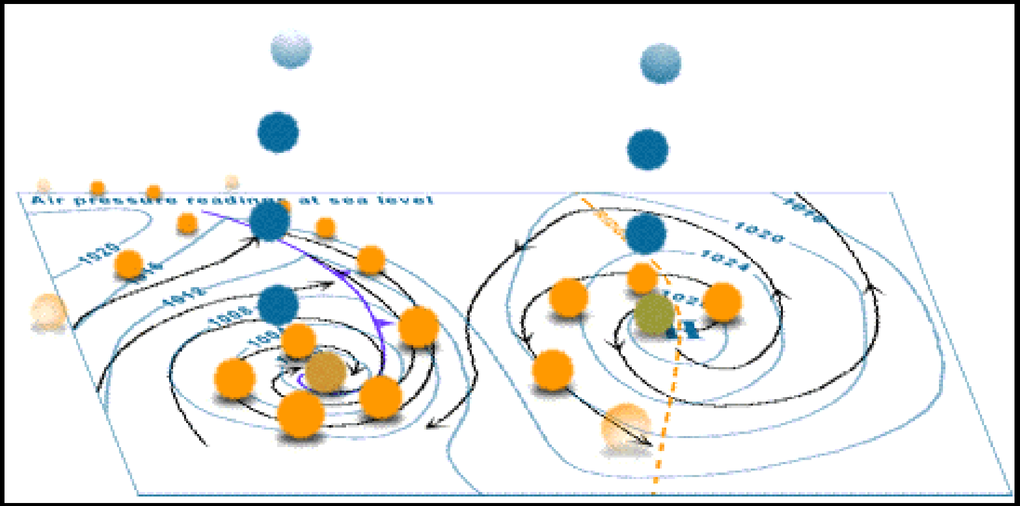
incompressible flow

اگر شارش را غیر قابل فشردن در نظر بگیریم بنابراین چگالی ثابت خواهد بود.

$$\rho = cte \rightarrow \nabla.\vec{V} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial z} = -\nabla_H .\vec{V}$$



Scale Analysis of Continuity Equation

When scaling the continuity equation, it is important to recognize that large variations in density are only relevant in the vertical.

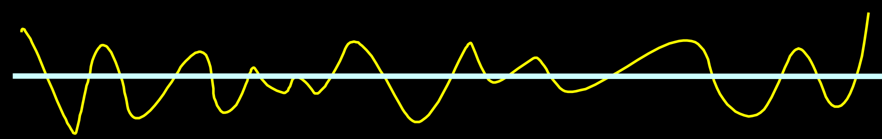
So we represent the density field as the sum of a horizontal reference value and a deviation from that value:

Substitute into velocity divergence form of the continuity equation:

$$\frac{1}{\rho}\frac{d\rho}{dt} + \nabla \cdot \vec{V} = 0$$

Next, we will assume that the density field may be expressed as a sum of a referenced density which is function only of height and a small perturbation, that is:

$$\rho = \rho_0(z) + \rho'(x, y, z, t)$$



$$\frac{1}{\rho_0}\left[\frac{\partial}{\partial t}\left(\rho_0 + \rho'\right) + \vec{V} \cdot \nabla\left(\rho_0 + \rho'\right)\right] + \nabla \cdot \vec{V} = 0$$

$$\frac{1}{\rho_0}\left[\left(\frac{\partial \rho_0}{\partial t}+\frac{\partial \rho'}{\partial t}\right)+u\left(\frac{\partial \rho_0}{\partial x}+\frac{\partial \rho'}{\partial x}\right)+v\left(\frac{\partial \rho_0}{\partial y}+\frac{\partial \rho'}{\partial y}\right)+w\left(\frac{\partial \rho_0}{\partial z}+\frac{\partial \rho'}{\partial z}\right)\right]+\nabla\cdot\vec{V}=0$$

$$\frac{1}{\rho_0}\left[\frac{\partial \rho'}{\partial t}+u\frac{\partial \rho'}{\partial x}+v\frac{\partial \rho'}{\partial y}+w\frac{d\rho_0}{dz}+w\frac{\partial \rho'}{\partial z}\right]+\nabla\cdot\vec{V}=0$$

$$\frac{1}{\rho_0}\left[\frac{\partial \rho'}{\partial t} + \vec{V}\cdot\nabla\rho'\right] + \frac{w}{\rho_0}\frac{d\rho_0}{dz} + \nabla\cdot\vec{V} = 0$$

A **B** **C**

A: $\frac{\rho'}{\rho_0}\frac{U}{L} \approx 10^{-7} s^{-1}$

B: $\frac{W}{H} \approx 10^{-6} s^{-1}$

C: $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \approx 10^{-1}\frac{U}{L} \approx 10^{-6} s^{-1}$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}+w\frac{d}{dz}\left(\ln\,\rho_0\right)=0$$

If only the first three terms in the above equation were present, then the atmosphere would be incompressible.

Compressible/Incompressible

Compressible fluid
Volume of an air or water parcel changes over time

$$\nabla.\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \neq 0$$

Incompressible fluid
Volume of an air or water parcel is constant over time

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Density of incompressible fluid constant along motion

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + (v.\nabla)\rho \;= -\rho(\nabla.v) = 0$$

Density of incompressible fluid changes at fixed point

$$\frac{\partial\rho}{\partial t} = -(\mathrm{v}.\nabla)\rho$$