Differential Equations

Lecture 9

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Review of lecture 8

Linear Equations

$$\frac{dy}{dx} + p(x)y = Q(x) \quad *$$

$$y = e^{-\int p\,dx}\left(\int e^{\int p\,dx} Q(x)\,dx + c\right)$$

is the general solution of *

As we have seen, the general second order differential equation has the form:

$$F(x, y, y', y'') = 0$$

Solving 2nd Order Linear Equation

Try reducing to first order equations.
This works for equations of the form:

Case 1-Dependent variable missing $f(x, y', y'') = 0$

$$y' = p \ , \ y'' = \frac{dp}{dx} \qquad f(x, p, \frac{dp}{dx}) = 0$$

Case 2-Independent variable missing $g(y, y', y'') = 0$

$$y' = p \ , \ y'' = \frac{dp}{dx} = \frac{dp}{dy}\frac{dy}{dx} = p\frac{dp}{dy} \qquad g(y, p, p\frac{dp}{dy})$$

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$$yy'' + (y')^2 = 0$$

$$f(y, y', y'') = 0$$

$$y' = \frac{dy}{dx} = p$$

$$y'' = \frac{dp}{dx} = \frac{dp}{dy}\frac{dy}{dx} = p\frac{dp}{dy}$$

$$yp\frac{dp}{dy} + p^2 = 0 \rightarrow y\frac{dp}{dy} + p = 0$$

$$\frac{dp}{p} = -\frac{dy}{y}$$

$$\ln p = -\ln y + c'$$

$$p = c'/y$$

$$p = \frac{dy}{dx} = \frac{c'}{y} \qquad ydy = c'dx$$

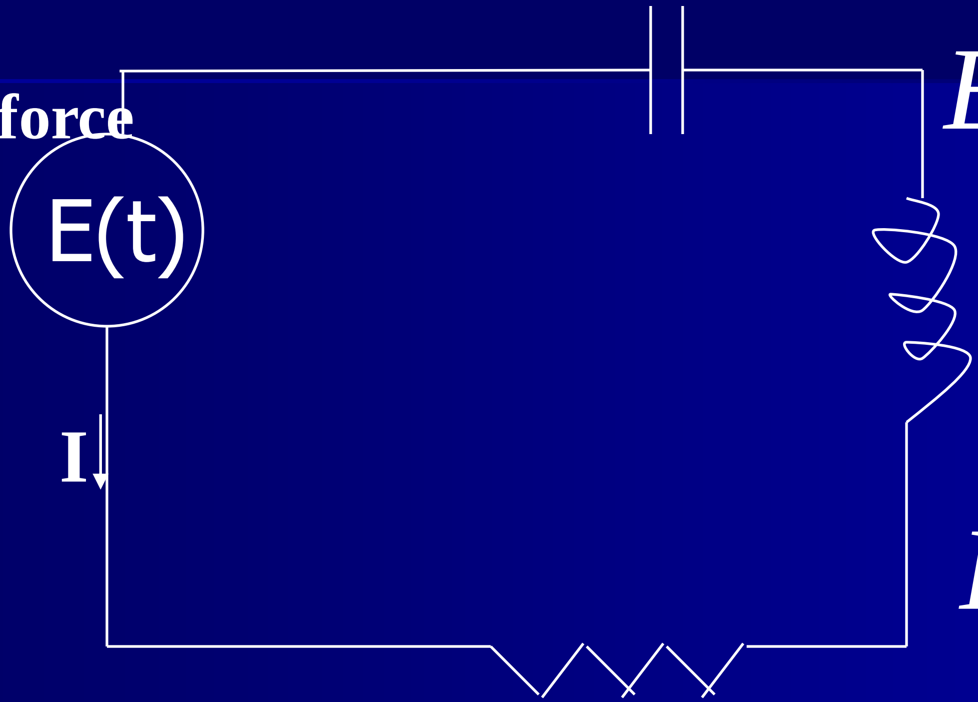
$$\frac{1}{2} y^2 = c'x + c'' \qquad y^2 = c_1 x + c_2$$

LRC Circuits

C for capacitance

$$E_c = \frac{1}{C} Q$$

Electromotive force
(battery)



L
for inductance

$$E_L = L \frac{dI}{dt}$$

R
for resistance

$$E_R = RI$$

Kirchoff’s Law

“The sum of the voltage drops across the passive elements in the circuit equals the applied voltage.”

Passive elements: **inductor, resistor, capacitor**

Applied voltage: **what the battery supplies**

The Model

$$E(t) = L\frac{dI}{dt} + RI + \frac{1}{C}Q$$

Independent variable ***t***

Dependent variable ***I,Q***

Two dependent variables are o.k. for a partial d.e. or for a system. This model should only have one dependent variable.

$$I = dQ/dt$$

$$E(t) = L\frac{dI}{dt} + RI + \frac{1}{C}Q$$

$$\frac{dI}{dt} = \frac{d^2Q}{dt^2} \qquad I = \frac{dQ}{dt}$$

$$E(t) = L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q$$

Example1: RL-circuit

(Kirchhoff's voltage law)

$$E_L + E_R = E_0$$

$$L\frac{dI}{dt} + RI = E_0$$

$$\frac{dI}{E_0 - RI} = \frac{1}{L}dt$$

$$t = 0, \quad I = I_0$$

$$\ln(E_0 - RI) = -\frac{R}{L}t + \ln(E_0 - RI_0)$$

$$I = \frac{E_0}{R} + (I_0 - \frac{E_0}{R})e^{-Rt/L}$$

$$if \quad t \to \infty \Rightarrow E_0 = RI$$

$$if \quad I_0 = 0 \to I = \frac{E_0}{R}(1 - e^{-Rt/L})$$

$$if \quad E_0 = 0 \to I = I_0 e^{-Rt/L}$$

For Your Attention