ANALYTICAL MECHANICS 1

Lecture 21

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Limits of the Radial Motion. Effective Potential

$$\frac{1}{2}mv^2 + V(r) = E$$

$$\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + V(r) = E$$

$$h = r^2\dot{\theta}$$

$$\frac{1}{2}m(\dot{r}^2 + \frac{h^2}{r^2}) + V(r) = E$$

$$\frac{1}{2}m\dot{r}^2 + \frac{mh^2}{2r^2} + V(r) = E$$

$$\frac{1}{2}m\dot{r}^2+\frac{mh^2}{2r^2}+V(r)=E$$

$$U_{eff}(r)=\frac{mh^2}{2r^2}+V(r)$$

$$\frac{1}{2}m\dot{r}^2+U_{eff}(r)=E$$

$$\dot{r}=0\rightarrow U_{eff}(r)-E=0$$

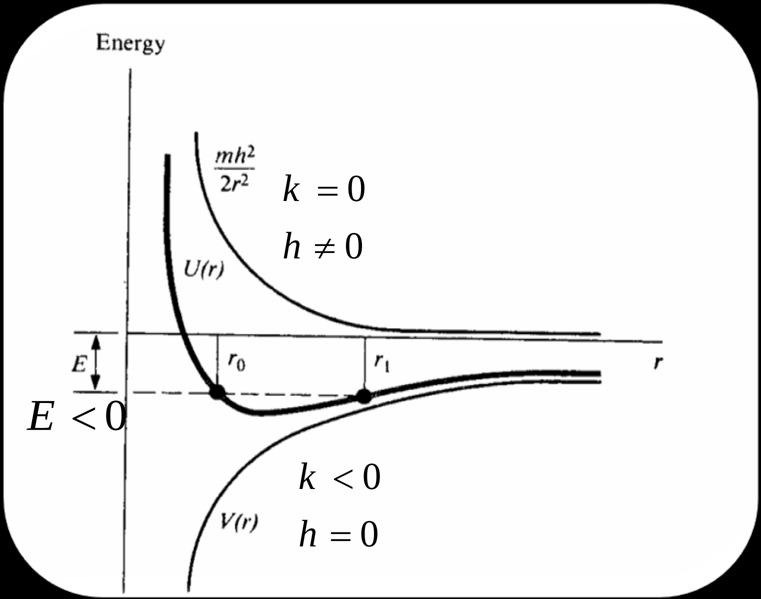
$$\frac{mh^2}{2r^2}+V(r)-E=0$$

$$U_{eff}(r)\leq E$$

$$\dot{r}^2\geq 0$$

$$U_{eff}(r)=\frac{mh^2}{2r^2}-\frac{k}{r}$$

$$U_{eff}(r) = \frac{mh^2}{2r^2} - \frac{k}{r}$$



$$f(r) = -\frac{dU_{eff}(r)}{dr}$$

$$\frac{dU_{eff}(r)}{dr} = 0$$

$$-\frac{mh^2}{r^3} + \frac{k}{r^2} = 0 \qquad r = \frac{mh^2}{k}$$

Circle orbit

$$r_\circ = \frac{mh^2}{k(1+e)}$$

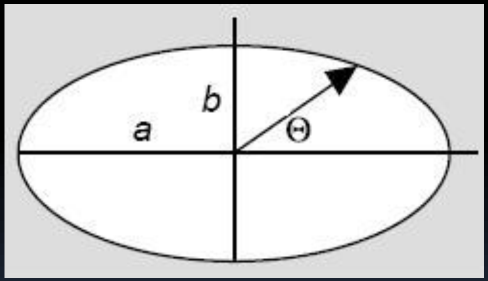
$$if \quad e = 0 \rightarrow r = r_\circ$$

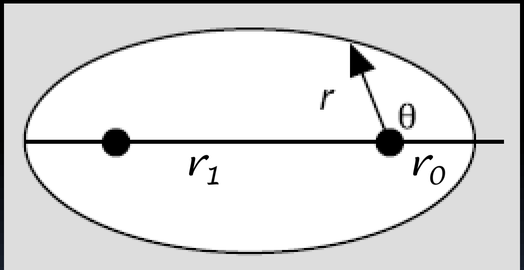
$$U_{eff}(r) - E = 0 \qquad \frac{mh^2}{2r^2} - \frac{k}{r} - E = 0$$

$$-2Er^2 - 2kr + mh^2 = 0$$

$$r_{1,\circ} = \frac{k \pm (k^2 + 2Emh^2)^{1/2}}{-2E}$$

$$2a = r_1 + r_\circ = \frac{k}{-E}$$





$$a = -\frac{k}{2E} > 0$$

$$e^2 = 1 - \frac{b^2}{a^2}, \quad a > b$$

example 8:

$$a = \frac{k}{-2E} \qquad \frac{1}{2}mv^2 - \frac{GMm}{r} = E$$

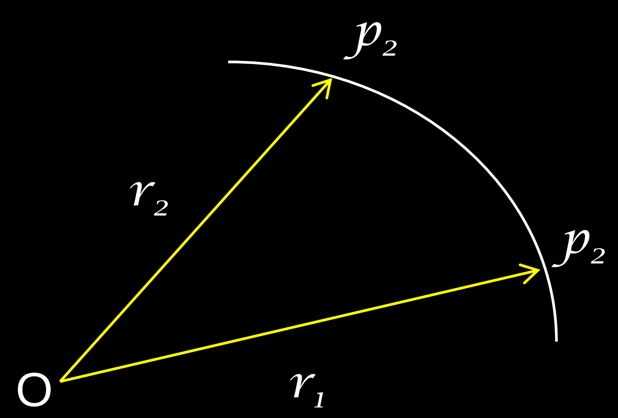
$$a = \frac{GMm}{-2\left(\frac{mv^2_{com}}{2} - \frac{GMm}{r_{com}}\right)} \qquad V = \frac{v_{com}}{v_e} \qquad R = \frac{r_{com}}{a_e}$$

$$GM = a_e v_e^2 \qquad a = \frac{a_e}{\frac{2}{R} - V^2} \qquad R = 4 \quad V = 0.5$$

$$a = \frac{a_e}{\left[0.5 - (0.5)^2\right]} = 4a_e$$

Periodic Time of Orbital Motion

$$\frac{dA}{dt} = \dot{A} = \frac{h}{2} = cte$$



$$|h| = \frac{L}{m} = |\vec{r} \times \vec{v}|$$

$$\int dA = \int_{t_1}^{t_2} \dot{A}\,dt \rightarrow A_{12} = \dot{A}t_{12}$$

$$t_{12} = \frac{A_{12}}{\dot{A}} = A_{12}\frac{2}{|h|}$$

$$\tau = \frac{2\pi ab}{h}$$

$$\frac{b}{a} = \sqrt{1-e^2}$$

$$\tau = \frac{2\pi a^2}{h}\sqrt{1-e^2}$$

$$r_\circ = \frac{mh^2}{k(1+e)}$$

$$r_1 = r_\circ \frac{1+e}{1-e}$$

$$2a = r_0 + r_1 = \frac{mh^2}{k}\left(\frac{1}{1+e} + \frac{1}{1-e}\right) = \frac{2mh^2}{k(1-e^2)}$$

$$2a = \frac{2mh^2}{k(1-e^2)} \qquad a = \frac{mh^2}{k(1-e^2)} \qquad 1-e^2 = \frac{mh^2}{ka}$$

$$\tau = \frac{2\pi a^2}{h}\sqrt{1-e^2} \qquad \tau = \frac{2\pi a^2}{h}\left(\frac{mh^2}{ka}\right)^{1/2}$$

$$\tau^2 = \frac{4\pi^2 m}{k} a^3 \qquad k = GMm$$

Problem 5

Projection of the second Newton's law $m\ddot{r} = F$ into the radial direction has a form

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v^2 = \frac{GM}{r} \qquad \tau = \frac{2\pi r}{v}$$

$$\tau = \frac{2\pi r\sqrt{r}}{\sqrt{GM}} \qquad \tau^2 = \frac{4\pi^2 r^3}{GM}$$

$$c = \frac{4\pi^2}{GM} \qquad \tau^2 = cr^3$$

Motion in an Inverse-Square Repulsive Field.
Scattering of Atomic Particles

$$f(r)=\frac{Qq}{r^2}$$

$$\frac{d^2u}{d\theta^2}+u=-\frac{1}{mh^2u^2}f(u^{-1})$$

$$\frac{d^2u}{d\theta}+u=-\frac{Qq}{mh^2}$$

$$u=u_g+u_p$$

$$u^{-1}=r=\frac{1}{A\cos(\theta-\theta_0)-Qq/mh^2}$$

$$r = -\frac{mh^2k^{-1}}{1+(1+2Emh^2k^{-2})^{1/2}\cos\theta}$$

$$r = \frac{mh^2Q^{-1}q^{-1}}{-1+(1+2Emh^2Q^{-2}q^{-2})^{1/2}\cos(\theta-\theta_0)}$$

$$k = -Qq \qquad E = \frac{1}{2}mv^2 + \frac{Qq}{r}$$

$$E > 0 \rightarrow e > 1 \rightarrow hyperbola$$