ANALYTICAL MECHANICS 1

Lecture 16

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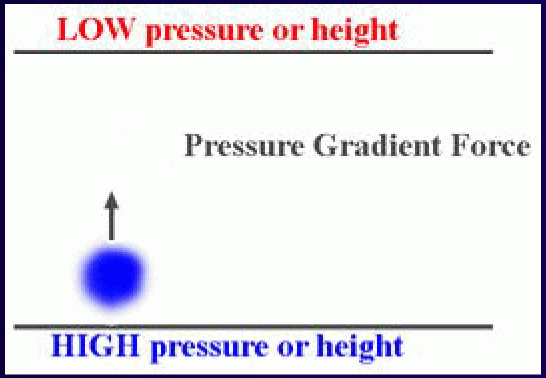
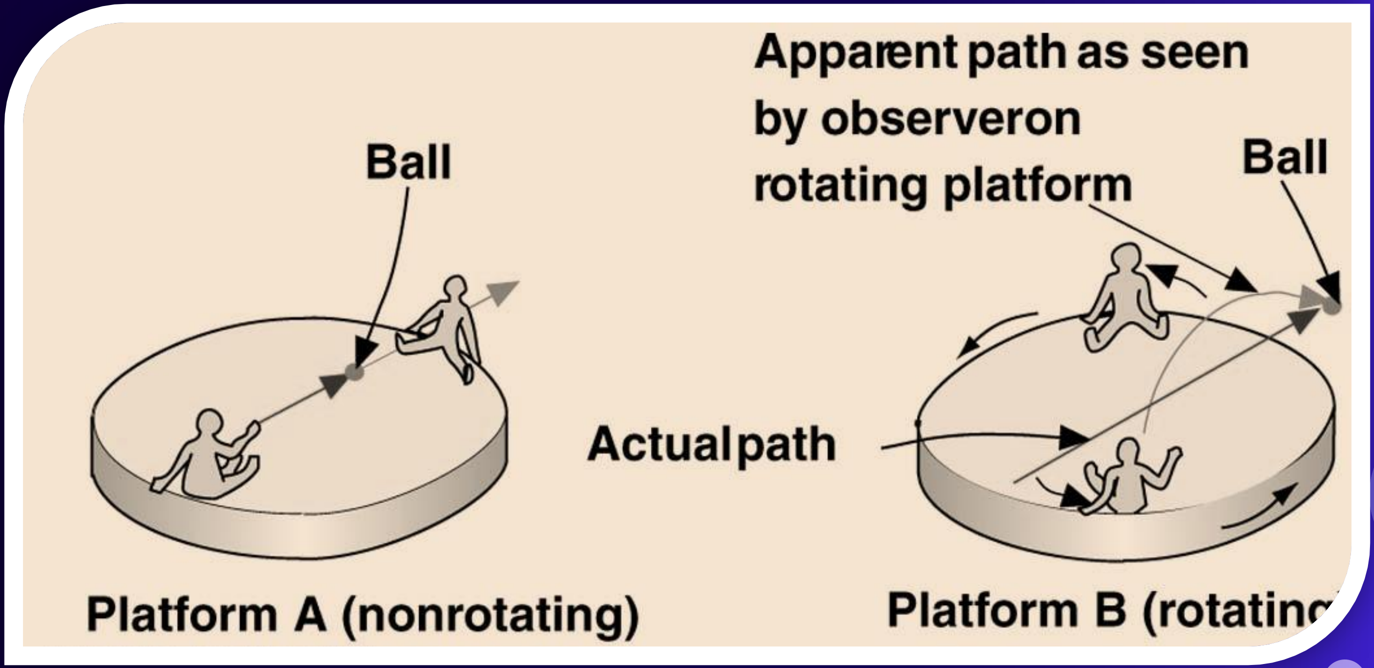
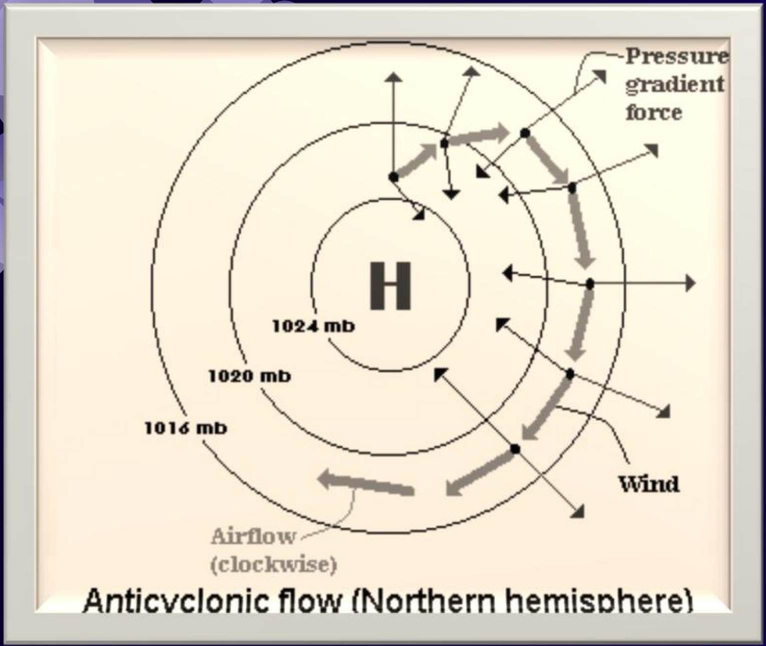
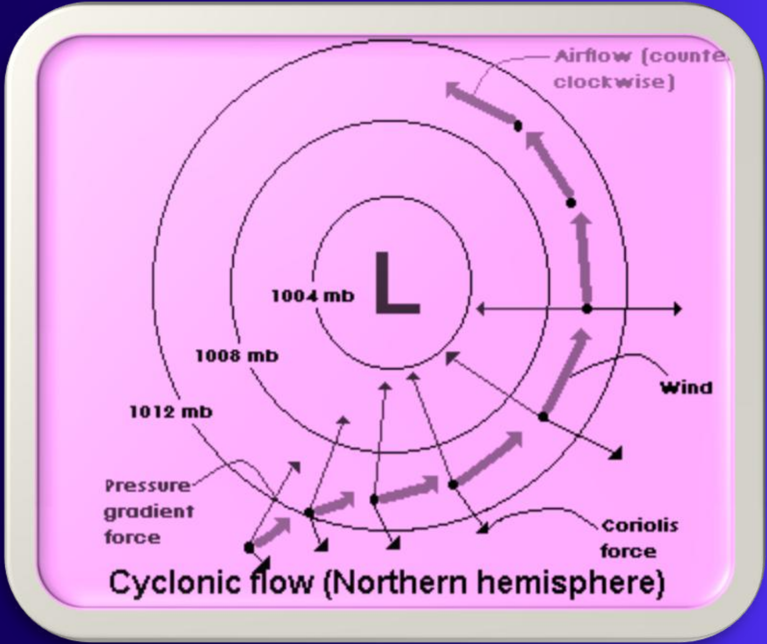
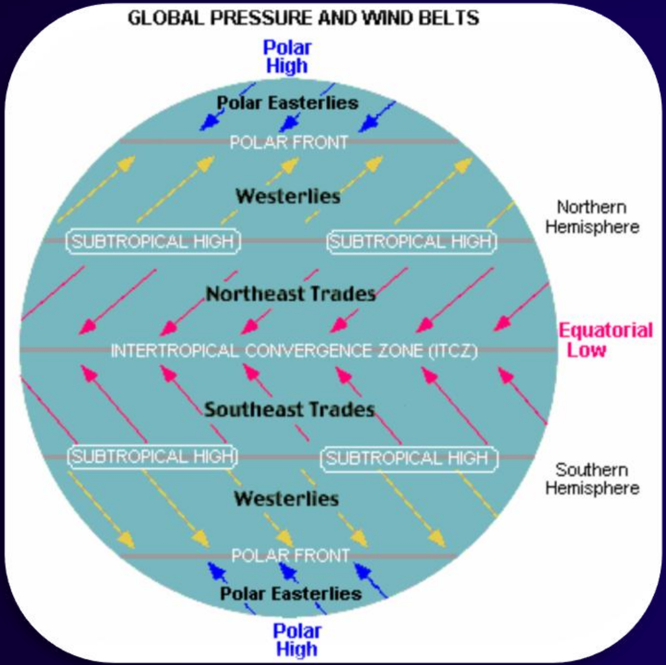
$$\vec{F} - m\vec{A}_0 - 2m\vec{\omega}\times\vec{v}' - m\dot{\vec{\omega}}\times\vec{r}' - m\vec{\omega}\times(\vec{\omega}\times\vec{r}') = m\vec{a}'$$

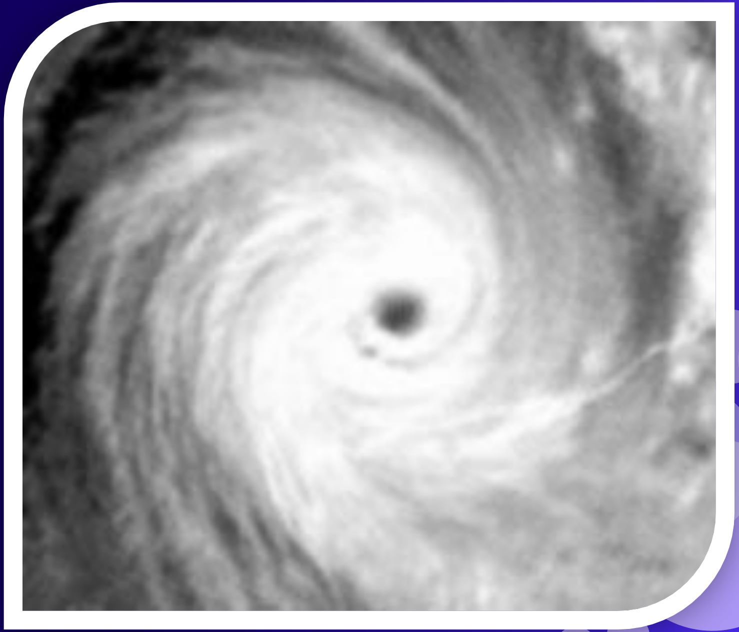





Pressure gradient force
H
1024 mb
1020 mb
1016 mb
Wind
Airflow (clockwise)
Anticyclonic flow (Northern hemisphere)


Airflow (counte clockwise)
L
1004 mb
1008 mb
1012 mb
Wind
Pressure gradient force
Coriolis force
Cyclonic flow (Northern hemisphere)


GLOBAL PRESSURE AND WIND BELTS
Polar High
Polar Easterlies
POLAR FRONT
Westerlies
SUBTROPICAL HIGH
SUBTROPICAL HIGH
Northern Hemisphere
Northeast Trades
Equatorial Low
INTERTROPICAL CONVERGENCE ZONE (ITCZ)
Southeast Trades
SUBTROPICAL HIGH
SUBTROPICAL HIGH
Southern Hemisphere
Westerlies
POLAR FRONT
Polar Easterlies
Polar High



Example 1:

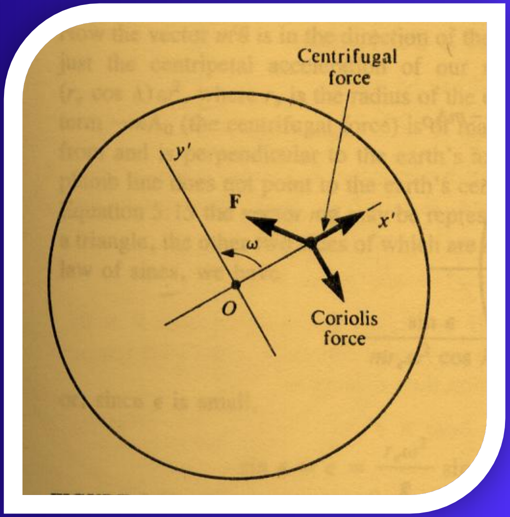
$$\vec{r}' = x'\hat{i}' \qquad \dot{\vec{r}}' = \dot{x}'\hat{i}' = v\hat{i}' \qquad \ddot{\vec{r}}' = 0$$

$$\vec{\omega} = \omega\hat{k}'$$

$$\vec{F}_{cor} = -2m\vec{\omega}\times\dot{\vec{r}}' = -2m\omega v'(\hat{k}'\times\hat{i}')$$

$$= -2m\omega v\hat{j}'$$

$$\vec{F}_{trans} = -m\dot{\vec{\omega}}\times\vec{r}' = 0 \;\; (\vec{\omega} = const)$$



$$\vec{F}_{cent} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r}')$$

$$= -m\omega^2[\hat{k}'\times(\hat{k}'\times x\hat{i}')]$$

$$= -m\omega^2(\hat{k}'\times\hat{j}'x') = m\omega^2 x\hat{i}'$$

$$\vec{F} - m\vec{A}_0 - 2m\vec{\omega}\times\vec{v}' - m\dot{\vec{\omega}}\times\vec{r}' - m\vec{\omega}\times(\vec{\omega}\times\vec{r}') = m\vec{a}'$$

$$\vec{F} - 2m\omega v\hat{j}' + m\omega^2 x\hat{i}' = 0$$

Example 2:

$$\left|\vec{F}\right| = \mu_s m g$$

$$\vec{F} - 2m\omega v'\hat{j}' + m\omega^2 x'\hat{i}' = 0$$

$$[(2m\omega v')^2 + (m\omega^2 x')^2]^{1/2} = \mu_s m g$$

$$x' = \frac{[\mu_s{}^2 g^2 - 4\omega^2 (v')^2]^{1/2}}{\omega^2}$$

Effects of the Earth`s Rotation

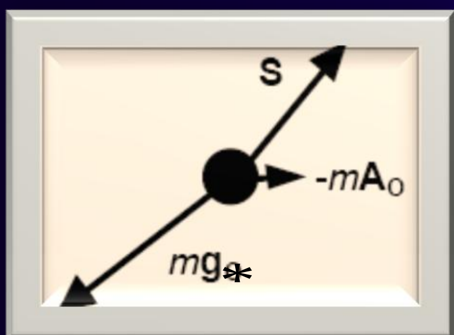
$$\omega = \frac{2\pi rad}{1 day} = 7.27 \times 10^{-5} rad\ / s$$

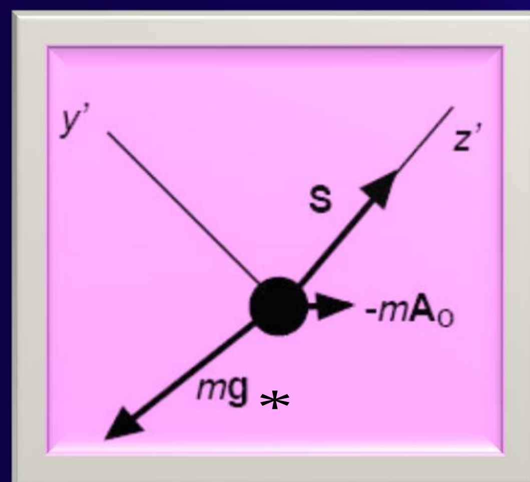
Static Effects. The Plumb Line

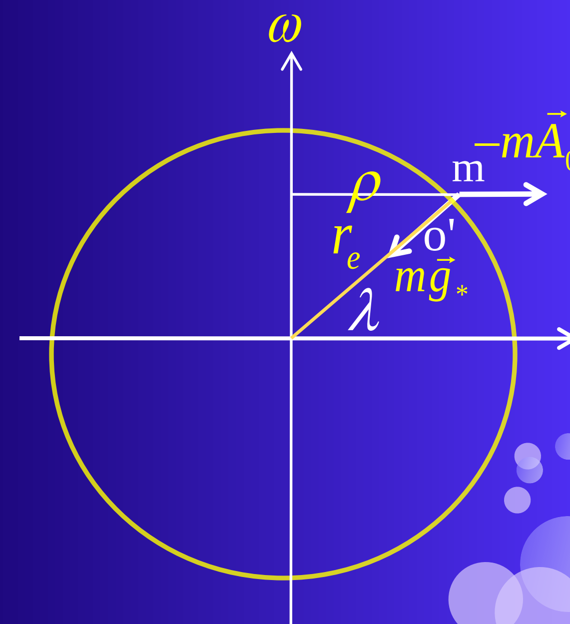
$$\vec{r}\,' = 0, \quad \vec{v}\,' = 0, \quad \vec{a}\,' = 0, \quad \dot{\vec{\omega}} = 0$$

$$\vec{F} - m\vec{A}_0 - 2m\vec{\omega}\times\vec{v}\,' - m\dot{\vec{\omega}}\times\vec{r}\,' - m\vec{\omega}\times(\vec{\omega}\times\vec{r}\,') = m\vec{a}\,'$$

$$\vec{F} - m\vec{A}_\circ = 0$$

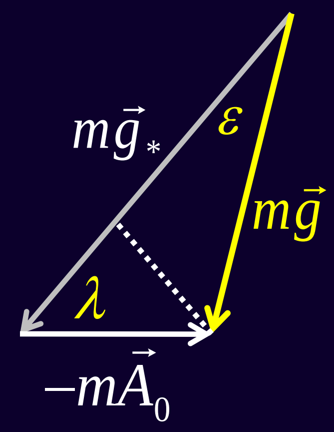






$$m\vec{g}_* - m\vec{A}_\circ = m\vec{g}$$

$$\vec{g} = \vec{g}_* - \vec{A}_\circ$$



$$mg \sin\varepsilon = \sin\lambda \left| m\vec{A}_0 \right|$$

$$\frac{\sin\varepsilon}{\left| m\vec{A}_0 \right|} = \frac{\sin\lambda}{mg} \quad \rightarrow \frac{\sin\varepsilon}{m r_e \omega^2 \cos\lambda} = \frac{\sin\lambda}{mg}$$

$$\left| \vec{A}_\circ \right| = \rho\omega^2 = (r_e \cos\lambda)\omega^2$$

$$at \;\; \text{Eq.} \rightarrow 6.4\times 10^6 \times (7.3\times 10^{-5})^2 = 0.034 m/s^2$$

$$\sin\varepsilon \approx \varepsilon = \frac{r_e\omega^2}{g}\sin\lambda\cos\lambda = \frac{r_e\omega^2}{2g}\sin 2\lambda$$

As expected, ε vanishes at the equator (λ = 0) and at poles (2λ = 180 degrees)

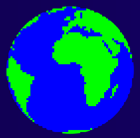
$$\lambda = 45^\circ \rightarrow \sin 90 = 1$$

$$\varepsilon_{\max} = \frac{r_e\omega^2}{2g} = \frac{6400\times10^3\,m\times(7.27\times10^{-5})^2}{2\times9.8}$$

$$= 1.7\times10^{-3}\,rad \simeq \frac{1}{10}\deg$$

Gravitation and Gravity

The terms gravity and gravitation are often used to explain the same thing, but there is a definite difference between the two.

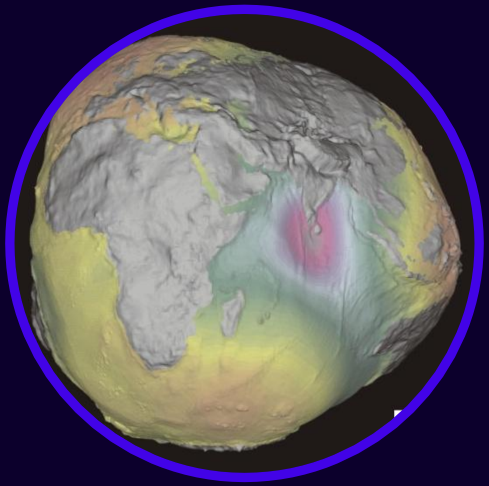


Gravitation is the attractive force existing between any two objects that have mass.
The force of gravitation pulls objects together.

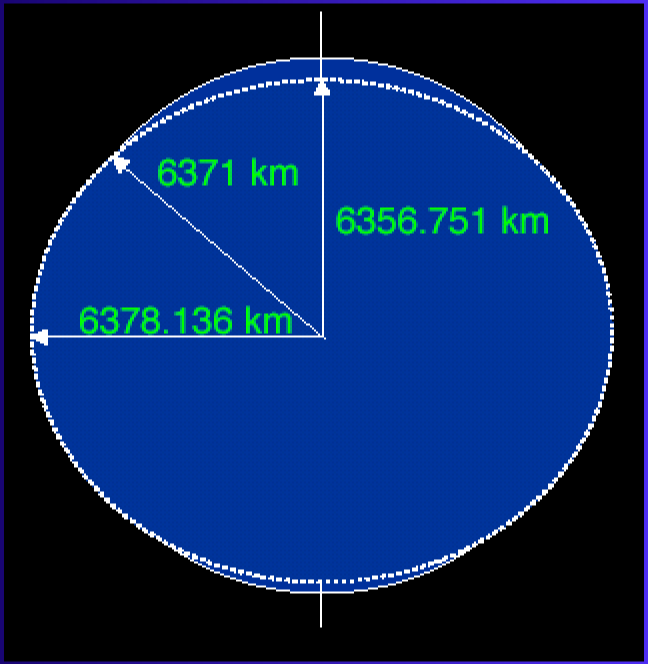
Gravity is the gravitational force that occurs between the earth and other bodies.
Gravity is the force acting to pull objects toward the earth.

The true physical equipotential surface is called the *geoid*.

A geoid is a close representation or physical model of the figure of the Earth



$$R_{pol} = 6356.7 \text{ km}, \quad R_{eq} = 6378.1 \text{ km}$$



$$\lambda = \pi/2 \rightarrow g_{pol} = 983.2 \text{ dyn/gr}$$

$$\lambda = \pi/4 \rightarrow g_{45} = 980.6 \text{ dyn/gr}$$

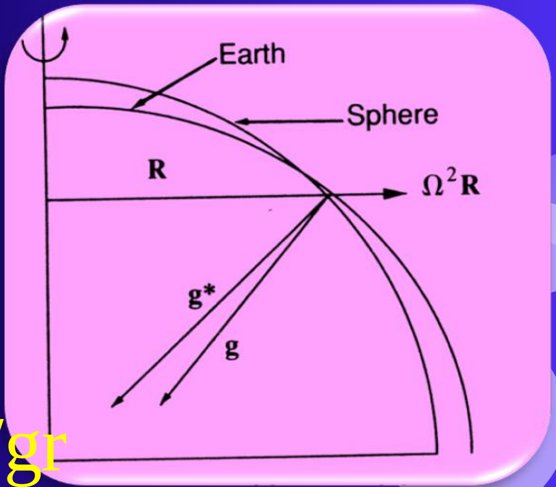
$$\lambda = 0 \rightarrow g_{Eq} = 978.4 \text{ dyn/gr}$$

$$\vec{g}_* = \vec{g}$$

در صورت عدم چرخش زمین

$$\vec{g}_* = \vec{g} \rightarrow pole$$

$$g = g_* - \omega^2 \rho \rightarrow Equator$$



$$g_x = 0, \quad g_y = 0, \quad g_z = -g$$

$$g_{*pol} = 983.2 \text{ dyn/gr}, \quad g_{*eq} = 981.4 \text{ dyn/gr}$$