Fundamentals of synoptic meteorology

Lecture 15

Sahraei

Physics Department

Razi University

http://www.razi.ac.ir/sahraei

3D frontogenesis function

$$F=\frac{1}{|\nabla\theta|}\Bigg(\frac{\partial\theta}{\partial x}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial y}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial z}\left\{\left(\frac{p_0^{\kappa}}{C_p}\right)\left[\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right)\right\}\Bigg)$$

Terms 1, 5, 9: Diabatic Terms

Terms 2, 3, 6, 7: Horizontal Deformation Terms

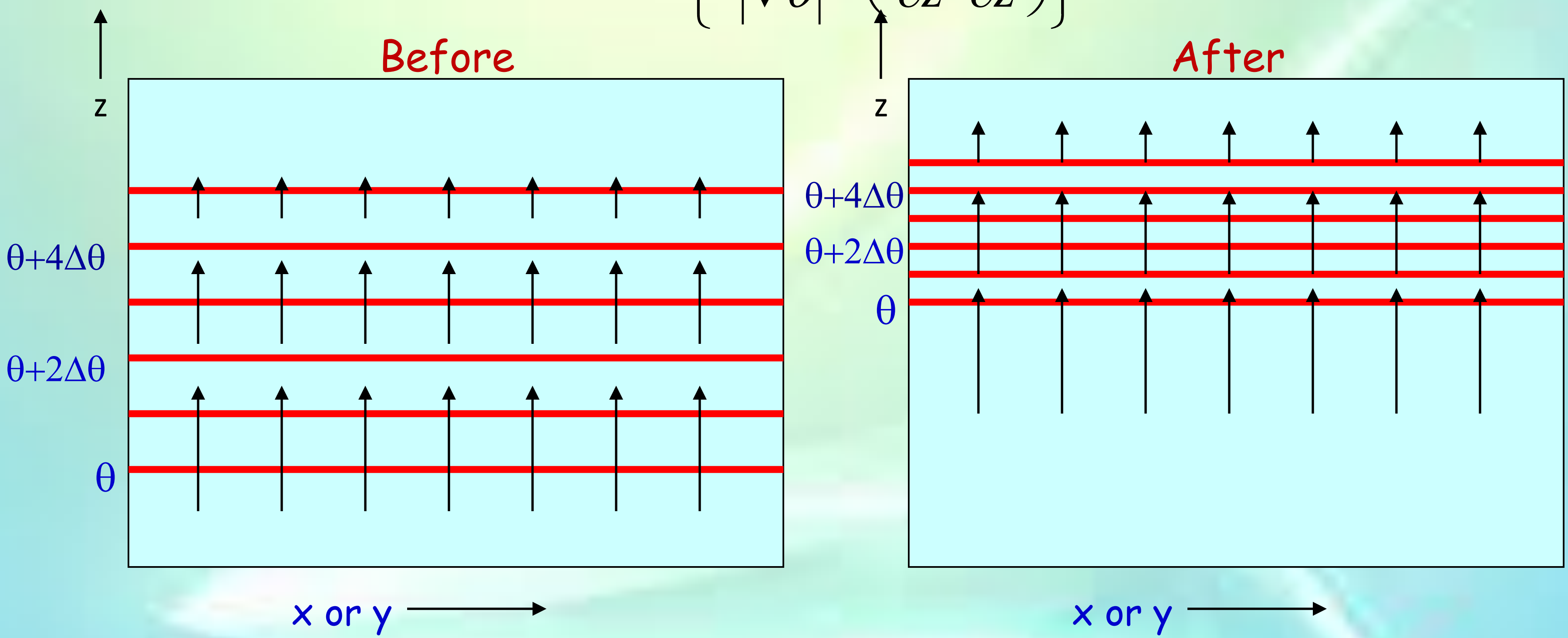
Terms 10 and 11: Vertical Deformation Terms

Terms 4 and 8: Tilting Terms

Term 12: Vertical Divergence Terms

Differential vertical motion

$$F = -\left\{ \frac{\partial\theta/\partial z}{|\nabla\theta|} \left(\frac{\partial w}{\partial z} \frac{\partial\theta}{\partial z} \right) \right\}$$



Vertical Divergence
the contribution
to frontogenesis due to divergence.

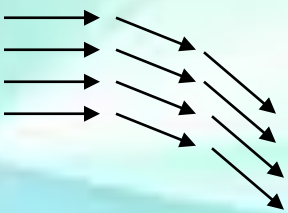
The Curl of a Vector Field

This is a lot harder to visualize than the divergence, but not impossible. Suppose you are in a boat in a huge river (or Pass) where the current flows mainly in the *x* direction but where the speed of the current (flux of water) varies with *y*.

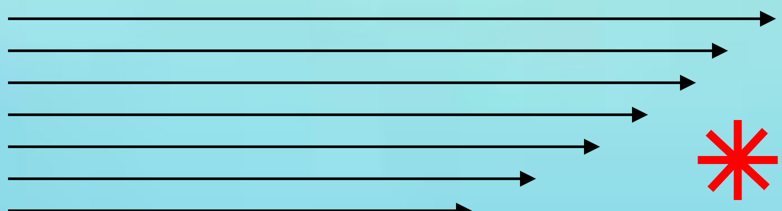
Vorticity

Vorticity is the microscopic measure of spin and rotation in a fluid.

Vorticity is defined as the curl of the velocity: $\nabla \times \vec{V}$



Wind direction varies → clockwise spin



Wind speed varies → clockwise spin

Expansion of relative vorticity into Cartesian components:

$$\nabla\times\vec{V}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ u & v & w\end{vmatrix}$$

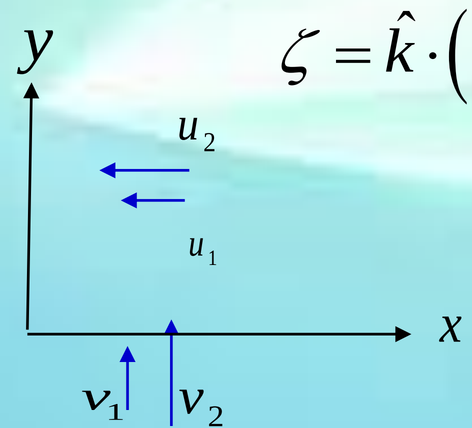
$$\nabla\times\vec{V}=\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)\hat{i}+\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)\hat{j}+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\hat{k}=\hat{i}\xi+\hat{j}\eta+\hat{k}\zeta$$

For large scale dynamics, the vertical component of vorticity is most important.

$$\xi,\eta \ll \zeta \Rightarrow \vec{\nabla}\times\vec{V}=\vec{\zeta}$$

$$\zeta=\hat{k}\cdot\left(\nabla\times\vec{V}\right)$$

relative vorticity



$$\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

$$\omega_1=\frac{\partial v}{\partial x}$$

$$\omega_2=-\frac{\partial u}{\partial y}$$

Another view of the 2D frontogenesis function

For many, but noy all, tupes of frontal development it is sufficient to consider the 2-D equation in which the tilting terms are neglected. The resulting expression:

$$F_{2D} = \frac{1}{|\nabla \theta|}\left[\left(-\frac{\partial \theta}{\partial x}\right)\left(\frac{\partial u}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial \theta}{\partial y}\right) - \left(\frac{\partial \theta}{\partial y}\right)\left(\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial \theta}{\partial y}\right)\right]$$

Recall the kinematic quantities:

Divergence (D) $$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

Vorticity (ζ) $$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

stretching deformation (F_1) $$F_1 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$$

shearing deformation (F2) $$F_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

and note that:

$$\frac{\partial u}{\partial x}=\frac{D+F_1}{2} \qquad \frac{\partial v}{\partial x}=\frac{\zeta+F_2}{2} \qquad \frac{\partial u}{\partial y}=\frac{F_2-\zeta}{2} \qquad \frac{\partial v}{\partial y}=\frac{D-F_1}{2}$$

Substituting:

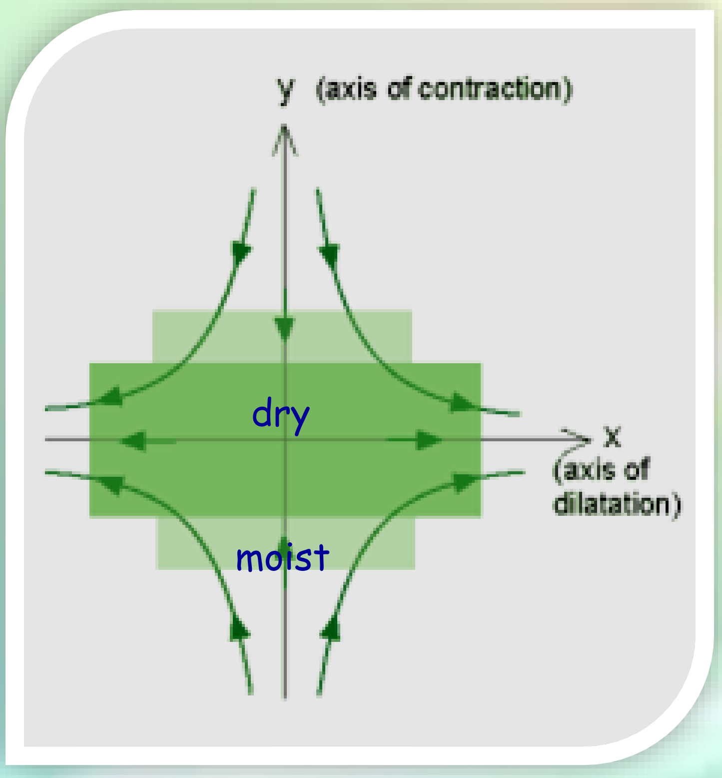
$$F_{2D}=\frac{1}{|\nabla\theta|}\left\{-\left(\frac{\partial\theta}{\partial x}\right)\left[\left(\frac{D+F_1}{2}\right)\frac{\partial\theta}{\partial x}+\left(\frac{\zeta+F_2}{2}\right)\frac{\partial\theta}{\partial y}\right]-\left(\frac{\partial\theta}{\partial y}\right)\left[\left(\frac{F_2-\zeta}{2}\right)\frac{\partial\theta}{\partial x}+\left(\frac{D-F_1}{2}\right)\frac{\partial\theta}{\partial y}\right]\right\}$$

This expression can be reduced to:

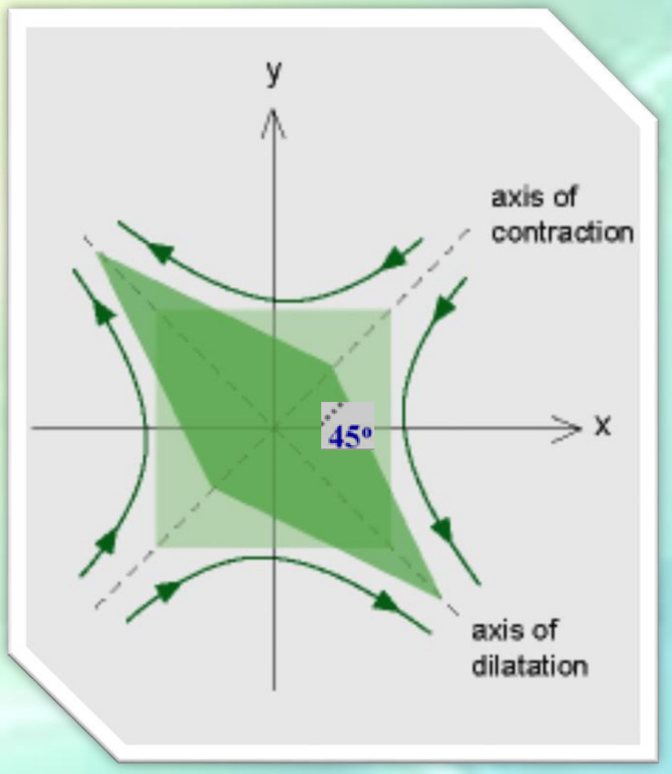
$$F_{2D}=\frac{-1}{2|\nabla\theta|}\left[D\left((\frac{\partial\theta}{\partial x})^2+(\frac{\partial\theta}{\partial y})^2\right)+F_1\left((\frac{\partial\theta}{\partial x})^2-(\frac{\partial\theta}{\partial y})^2\right)+2F_2\left(\frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial y}\right)\right]$$

Thus, horizontal frontogenesis is induced by divergence, streteching and shearing deformation.
Note that the vorcity terms cancel, which implies that the frontogensis is independent of vorticity.
Even thought vorticity is produced in the frontal zone, it playes no direct role in frontogensis.

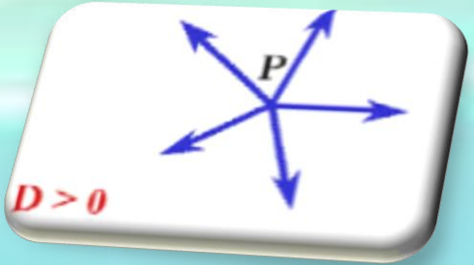
Stretching and Shearing deformation
"look alike" with axes rotated



Stretching



Shearing



However, vorticity can still play an indirect role by rotating isotherms into alignment with the axis of dilatation.

Stretching and Shearing deformation and horizontal divergence

$$F_{2D} = \frac{1}{|\nabla\theta|}\left[\left(-\frac{\partial\theta}{\partial x}\right)\left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}+\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial\theta}{\partial y}\right)\left(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right)\right]$$

$$F_{2D} = \frac{1}{|\nabla\theta|}\left[-\left(\frac{\partial\theta}{\partial x}\right)^2\frac{\partial u}{\partial x}-\frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial y}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)-\left(\frac{\partial\theta}{\partial y}\right)^2\frac{\partial v}{\partial y}\right]$$

$$\frac{\partial u}{\partial x} = \frac{D+F_1}{2} \qquad F_2 = \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y} \qquad \frac{\partial v}{\partial y} = \frac{D-F_1}{2}$$

$$F_{2D} = \frac{1}{|\nabla\theta|}\left[-\left(\frac{\partial\theta}{\partial x}\right)^2\left(\frac{1}{2}(D+F_1)\right)-\frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial y}F_2-\left(\frac{\partial\theta}{\partial y}\right)^2\left(\frac{1}{2}(D-F_1)\right)\right]$$

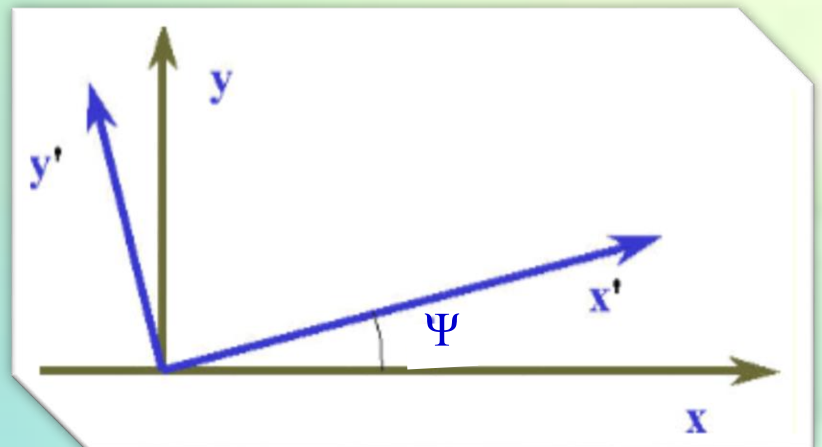
$$F_{2D} = \frac{1}{|\nabla\theta|}\left[-\left(\frac{\partial\theta}{\partial x}\right)^2\left(\frac{1}{2}(D+F_1)\right)-\frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial y}F_2-\left(\frac{\partial\theta}{\partial y}\right)^2\left(\frac{1}{2}(D-F_1)\right)\right]$$

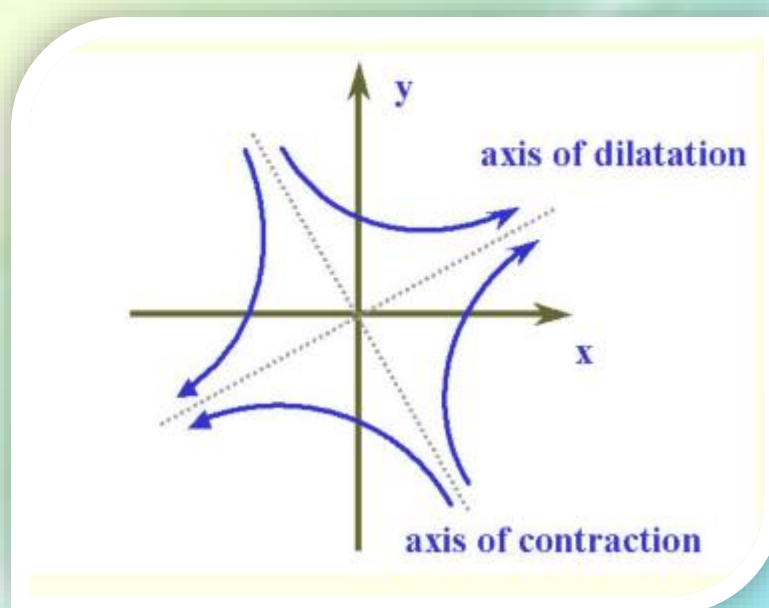
The advantage of this equ. is that it explicitly expresses the effects of confluence and diffluence in terms of the contributions to each from both horizontal divergence and deformation.

The total deformation is equal to the magnitude of the stretching and shearing deformation, i.e.,

$$F = (F_1^2 + F_2^2)^{1/2} = \left[\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]^{1/2}$$

We can simplify the 2D frontogenesis equation by rotating our coordinate axes to align with the axis of dilatation of the flow (x')

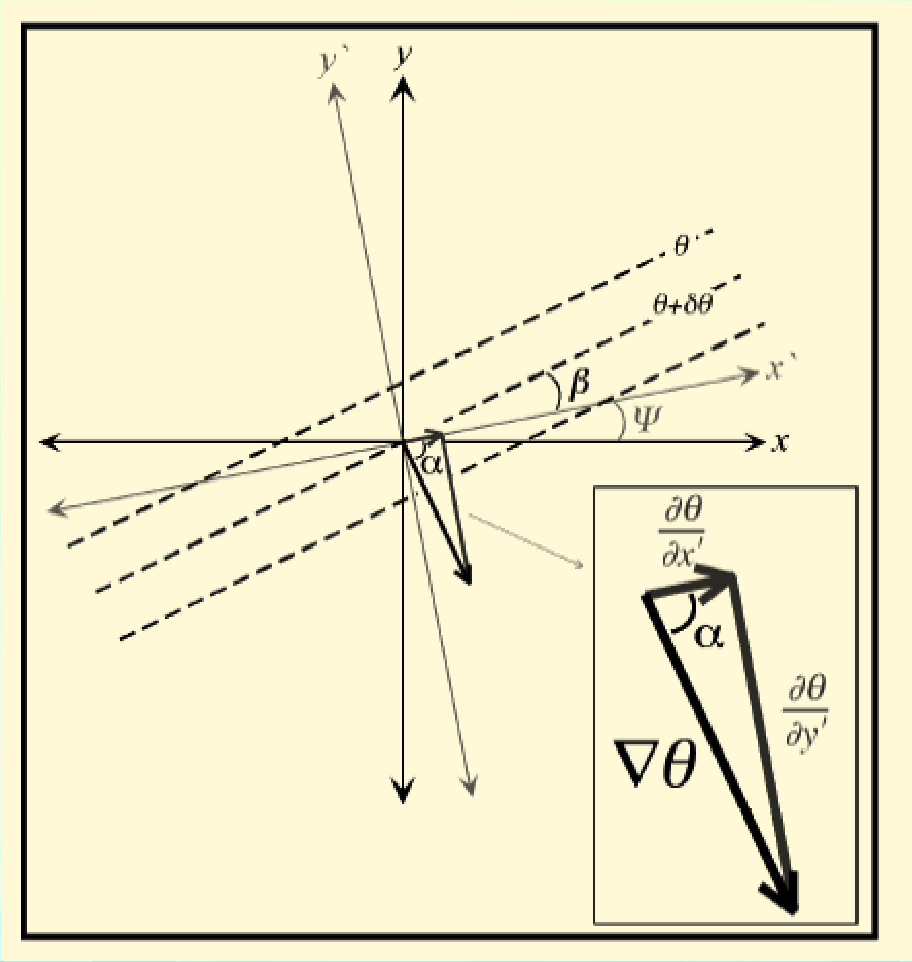




The coordinate system is rotated such that the x'-axis lies along either the *axis of dilatation or the axis of contraction.*

If the x-axis is rotated so as to lie along the axis of dilatation, the y-axis lies along the axis of contraction, and vice versa.

A pure deformation flow



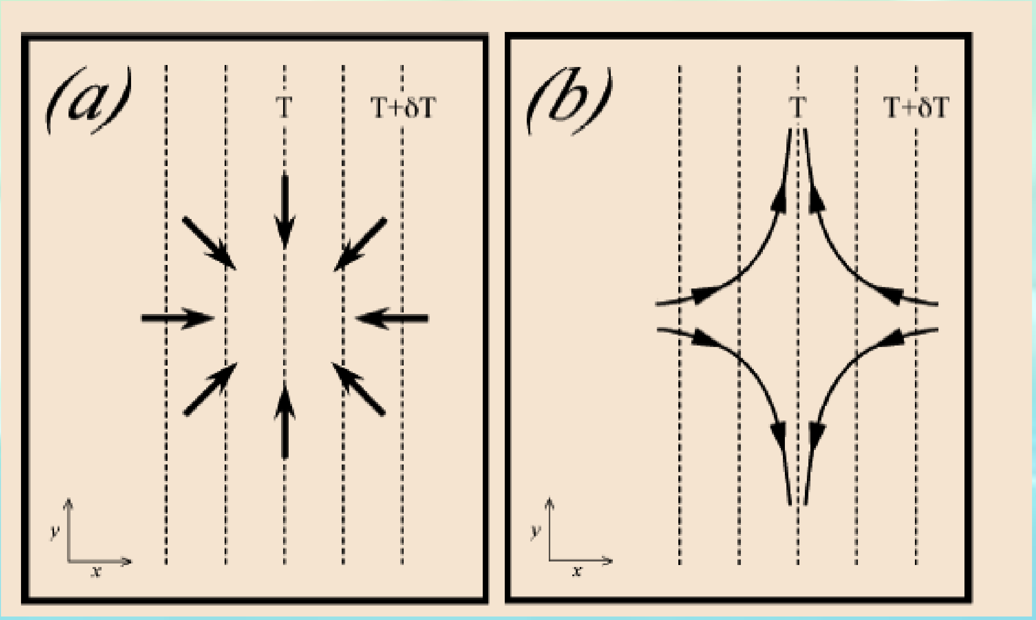
β= angle between the isentropes and the axis of dilatation

β= -90° - α, where a is the angle between the horizontal potential temperature gradient $|\nabla\theta|$ and the x-axis.

In this coordinate system, the equ. can be re-written as:

$$F_{2D} = \frac{1}{2|\nabla\theta|}\left\{D\left(|\nabla\theta|^2\right) + F\left[\left(\frac{\partial\theta}{\partial x'}\right)^2 - \left(\frac{\partial\theta}{\partial y'}\right)^2\right]\right\}$$

This equation illustrates that horizontal frontogenesis is only associated with divergence and deformation, but not vorticity



$$F_{2D} = \frac{|\nabla\theta|}{2}\left\{D + \frac{F}{|\nabla\theta|^2}\left[\left(\frac{\partial\theta}{\partial x'}\right)^2 - \left(\frac{\partial\theta}{\partial y'}\right)^2\right]\right\}$$

Note that

$$\frac{F}{|\nabla\theta|^2}\left[\left(\frac{\partial\theta}{\partial x'}\right)^2 - \left(\frac{\partial\theta}{\partial y'}\right)^2\right] = F\cos 2\alpha = -F\cos 2\beta$$

$$F_{2D} = \frac{|\nabla\theta|}{2}(F\cos 2\beta - D)$$

Where F is the total deformation of the flow, β is the angle between the isentropes and the dilatation axis of the total deformation field, and D is divergence (D <0 for convergence)

$$F_{2D} = \frac{|\nabla\theta|}{2}(F\cos 2\beta - D)$$

F is the resultant deformation in the new coordinate system.

let us first consider the effects of deformation upon frontogenesis, i.e.,

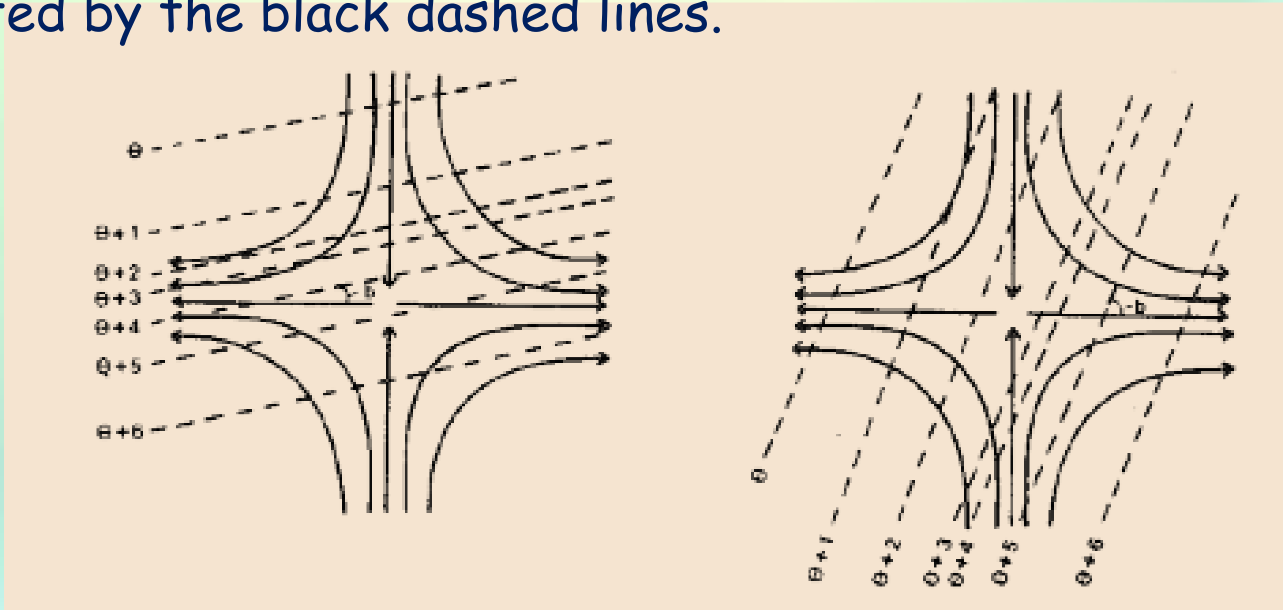
$$F_{2D} \approx \frac{|\nabla\theta|}{2}(F\cos 2\beta)$$

When the axis of dilatation lies within a 45° angle of the horizontal potential temperature gradient, deformation is a frontogenetical process.

When the axis of dilatation lies between a 45° and 90° angle of the horizontal potential temperature gradient, deformation is a frontolytic process.

Pure deformation flow (black streamlines) with the x-axis aligned along the axis of dilatation (e.g., along the stretching axis) and the y-axis aligned along the axis of contraction.

Isentropes representing the horizontal potential temperature gradient are depicted by the black dashed lines.



In the left panel, the angle between the horizontal potential temperature gradient and the axis of dilatation is less than 45°, a frontogenetic situation.

In the right panel, the angle between the horizontal potential temperature gradient and the axis of dilatation is greater than 45°, a frontolytic situation.

This can be confirmed by visually interpolating how the flow will cause the horizontal potential temperature gradient to evolve with time.

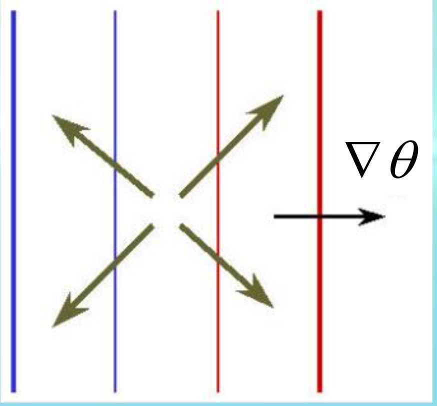
$$F_{2D} = \frac{|\nabla \theta|}{2}(F\cos 2\beta - D)$$

to consider the effects of divergence upon frontogenesis, i.e.,

$$F_{2D} = -\frac{1}{2}|\nabla \theta| D$$

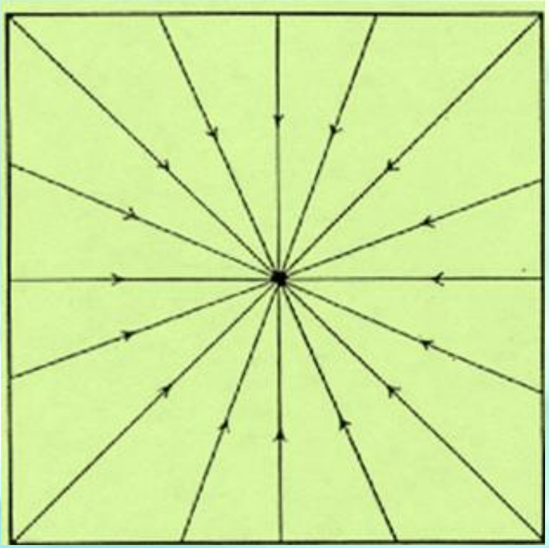
Since D > 0 for divergence and D < 0 for convergence, we find that convergent flow is a frontogenetic process while divergent flow is a frontolytic process.

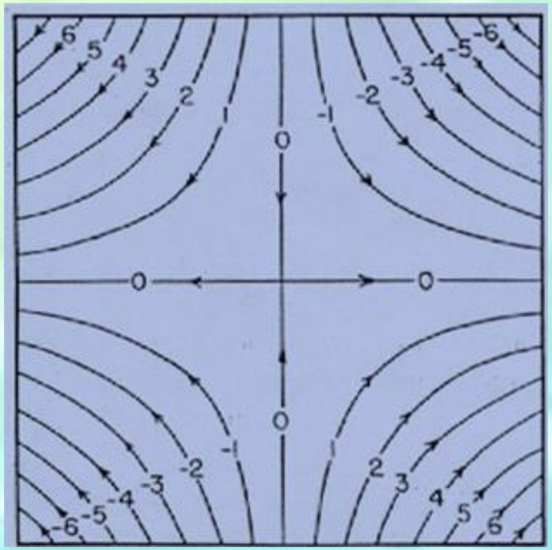
Note that this is true no matter the orientation of the horizontal potential temperature gradient.



In the above discussion, we considered the effects of deformation and horizontal divergence upon frontogenesis.

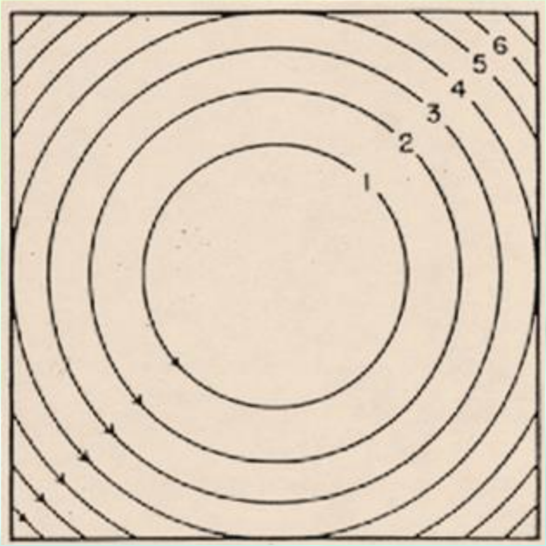
It may be natural, therefore, to ask whether vertical vorticity can result in frontogenesis or frontolysis.





Directly, rotational flows cannot; they can merely serve to rotate the horizontal potential temperature gradient.

However, such rotation can change the angle of the horizontal potential temperature gradient with respect to the axes of dilatation and contraction, thereby indirectly impacting frontogenesis and/or frontolysis.



Deformation itself can also change the angle of the horizontal potential temperature gradient with respect to the axes of dilatation and contraction, although this is perhaps not as easy to visualize.

Thank you for your attention

Question ?