Synoptic Meteorology 1

Lecture 14

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The solution

The Three-Dimensional Frontogenesis Function

$$F = \frac{d}{dt}\left|\nabla\theta\right| \qquad \frac{d\theta}{dt} = \left(\frac{p_0}{p}\right)^k \frac{1}{c_p}\frac{dQ}{dt}$$

becomes

$$F = \frac{1}{\left|\nabla\theta\right|}\Bigg(\frac{\partial\theta}{\partial x}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right] - \left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}\right) - \left(\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial y}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right] - \left(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}\right) - \left(\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right) - \left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial z}\left\{\left(\frac{p_0^{\kappa}}{C_p}\right)\left[\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)\right] - \left(\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}\right) - \left(\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}\right) - \left(\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right)\right\} \Bigg)$$

$$F = \frac{1}{|\nabla \theta|} \left(\frac{\partial \theta}{\partial x} \left\{ \frac{1}{C_p} \left(\frac{p_0}{p} \right)^{\kappa} \left[\frac{\partial}{\partial x} \left(\frac{dQ}{dt} \right) \right] - \left(\frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} \right) - \left(\frac{\partial v}{\partial x} \frac{\partial \theta}{\partial y} \right) - \left(\frac{\partial w}{\partial x} \frac{\partial \theta}{\partial z} \right) \right\} \right.$$

$$+ \frac{\partial \theta}{\partial y} \left\{ \frac{1}{C_p} \left(\frac{p_0}{p} \right)^{\kappa} \left[\frac{\partial}{\partial y} \left(\frac{dQ}{dt} \right) \right] - \left(\frac{\partial u}{\partial y} \frac{\partial \theta}{\partial x} \right) - \left(\frac{\partial v}{\partial y} \frac{\partial \theta}{\partial y} \right) - \left(\frac{\partial w}{\partial y} \frac{\partial \theta}{\partial z} \right) \right\}$$

$$\left. + \frac{\partial \theta}{\partial z} \left\{ \left(\frac{p_0^{\kappa}}{C_p} \right) \left[\frac{\partial}{\partial z} \left(p^{-\kappa} \frac{dQ}{dt} \right) \right] - \left(\frac{\partial u}{\partial z} \frac{\partial \theta}{\partial x} \right) - \left(\frac{\partial v}{\partial z} \frac{\partial \theta}{\partial y} \right) - \left(\frac{\partial w}{\partial z} \frac{\partial \theta}{\partial z} \right) \right\} \right)$$

Weighting Factors = $\dfrac{\text{Magnitude of } \theta \text{ gradient in one direction}}{\text{Magnitude of the total 3D, } \theta \text{ gradient}}$

$$F = \frac{1}{|\nabla \theta|} \Bigg(\frac{\partial \theta}{\partial x} \left\{ \frac{1}{C_p} \left(\frac{p_0}{p} \right)^{\kappa} \left[\frac{\partial}{\partial x} \left(\frac{dQ}{dt} \right) \right] - \left(\frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} \right) - \left(\frac{\partial v}{\partial x} \frac{\partial \theta}{\partial y} \right) - \left(\frac{\partial w}{\partial x} \frac{\partial \theta}{\partial z} \right) \right\}$$

$$+ \frac{\partial \theta}{\partial y} \left\{ \frac{1}{C_p} \left(\frac{p_0}{p} \right)^{\kappa} \left[\frac{\partial}{\partial y} \left(\frac{dQ}{dt} \right) \right] - \left(\frac{\partial u}{\partial y} \frac{\partial \theta}{\partial x} \right) - \left(\frac{\partial v}{\partial y} \frac{\partial \theta}{\partial y} \right) - \left(\frac{\partial w}{\partial y} \frac{\partial \theta}{\partial z} \right) \right\}$$

$$+ \frac{\partial \theta}{\partial z} \left\{ \left(\frac{p_0^{\kappa}}{C_p} \right) \left[\frac{\partial}{\partial z} \left(p^{-\kappa} \frac{dQ}{dt} \right) \right] - \left(\frac{\partial u}{\partial z} \frac{\partial \theta}{\partial x} \right) - \left(\frac{\partial v}{\partial z} \frac{\partial \theta}{\partial y} \right) - \left(\frac{\partial w}{\partial z} \frac{\partial \theta}{\partial z} \right) \right\}$$

Gradient in diabatic heating

$$F = \frac{1}{|\nabla\theta|}\Bigg(\frac{\partial\theta}{\partial x}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right] - \left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}\right) - \left(\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial y}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right] - \left(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}\right) - \left(\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right) - \left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial z}\left\{\left(\frac{p_0^{\kappa}}{C_p}\right)\left[\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)\right] - \left(\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}\right) - \left(\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}\right) - \left(\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right)\right\}\Bigg)$$

the diabatic heating rate

$$F = \frac{1}{|\nabla\theta|}\frac{\partial\theta}{\partial x}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right]\right\}$$

Gradient in diabatic heating in x direction

$$F = \frac{1}{|\nabla\theta|}\frac{\partial\theta}{\partial y}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right]\right\}$$

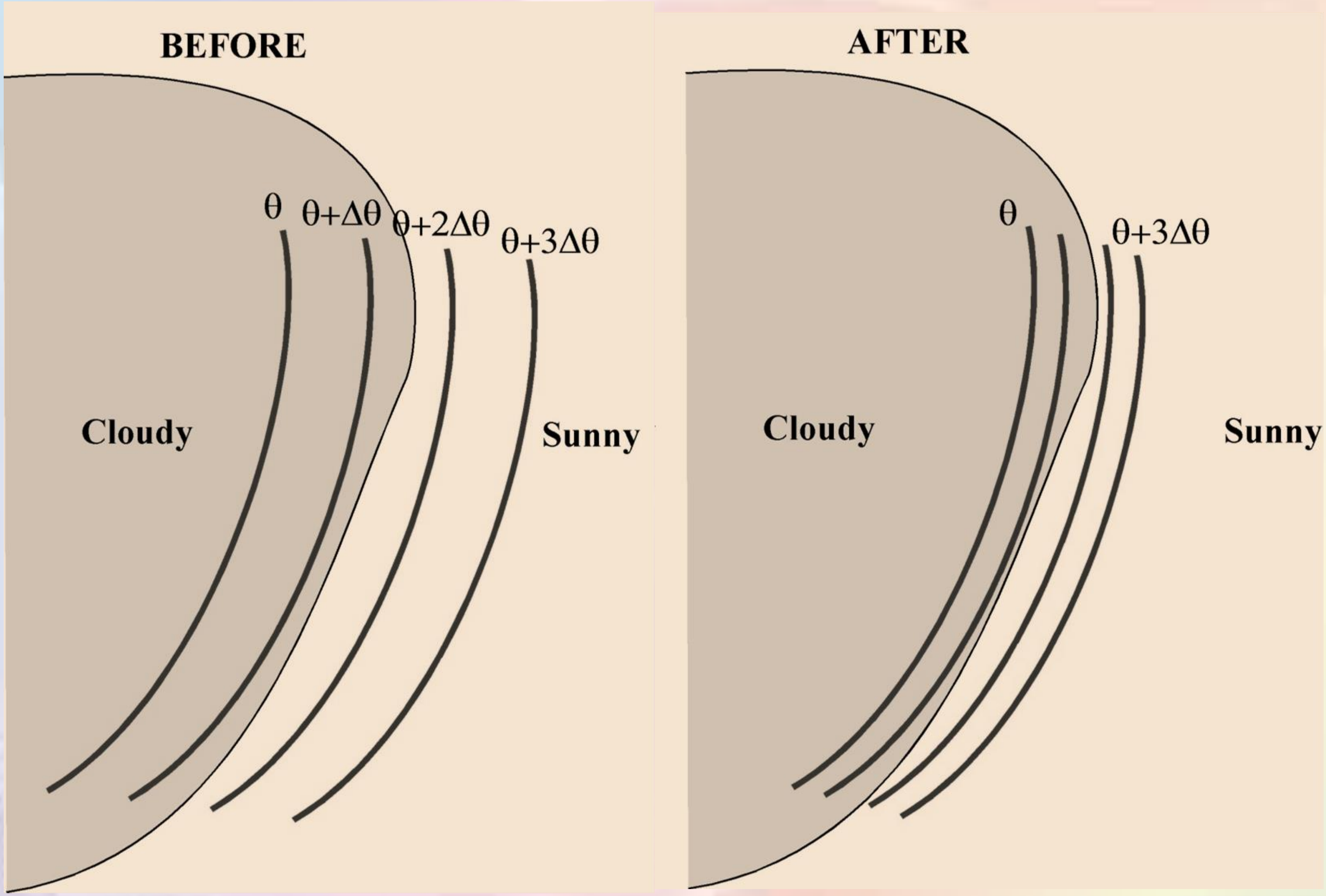
Gradient in diabatic heating in y direction

Temperature gradient

Horizontal gradient in diabatic heating or cooling rate

If $\frac{\partial\theta}{\partial x}$ and $\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)$ have the same sign, it means the diabatic heating will increase the temperature gradient.

Differential solar heating during daytime



Diabatic heating term

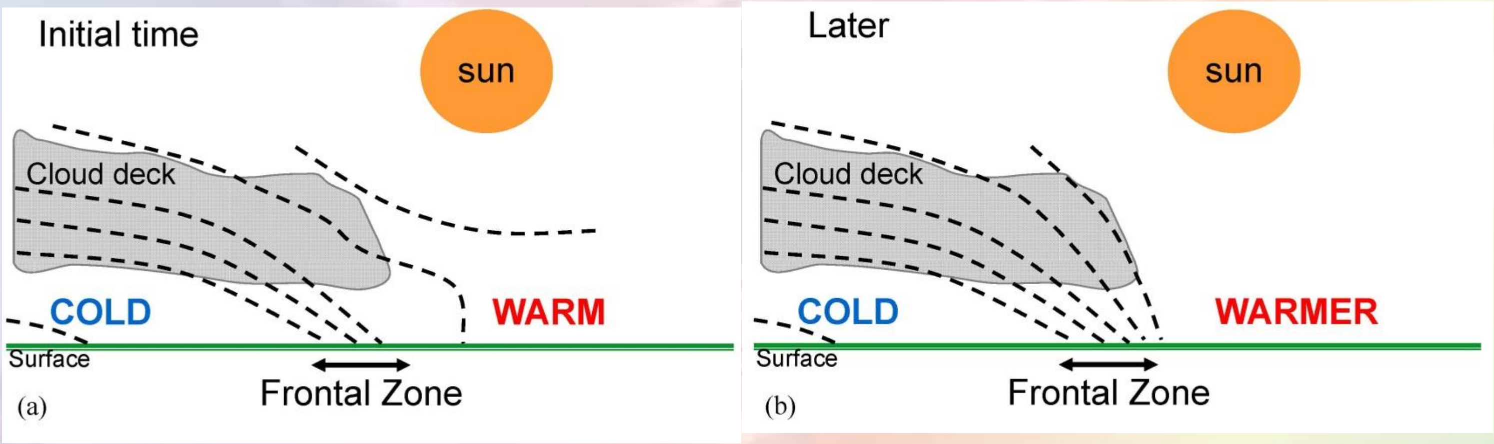
The *differential* diabatic heating term takes into account all diabatic processes together:

Differential solar radiation, differential surface heating due to soil characteristics, differential heat surface flux

One example: differential solar radiation

Assume the diabatic heating rate in the warm air exceeds the diabatic heating rate in the cold air

In that example, $\frac{\partial}{\partial y}\left(\frac{d\theta}{dt}\right)$ would be positive, and *F* positive



Example: positive contribution to *F* along a front: differential diabatic heating

$$F = \frac{1}{|\nabla\theta|}\left(\frac{\partial\theta}{\partial x}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right)\right\}\right.$$

$$+\frac{\partial\theta}{\partial y}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$\left.+\frac{\partial\theta}{\partial z}\left\{\left(\frac{p_0^{\kappa}}{C_p}\right)\left[\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right)\right\}\right)$$

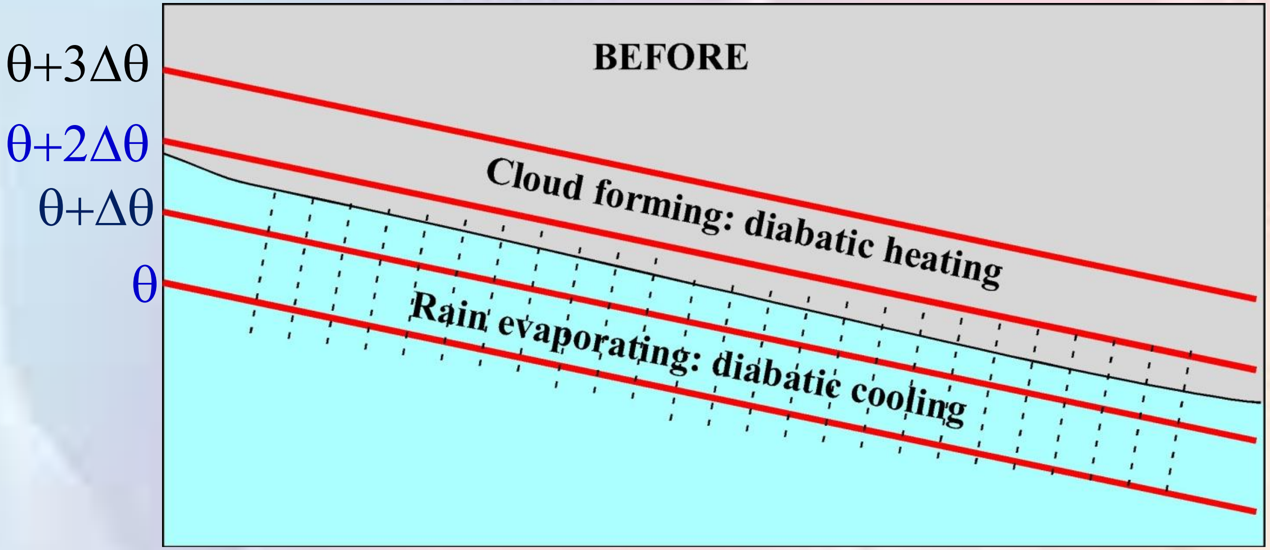
Vertical cross section of potential temperature

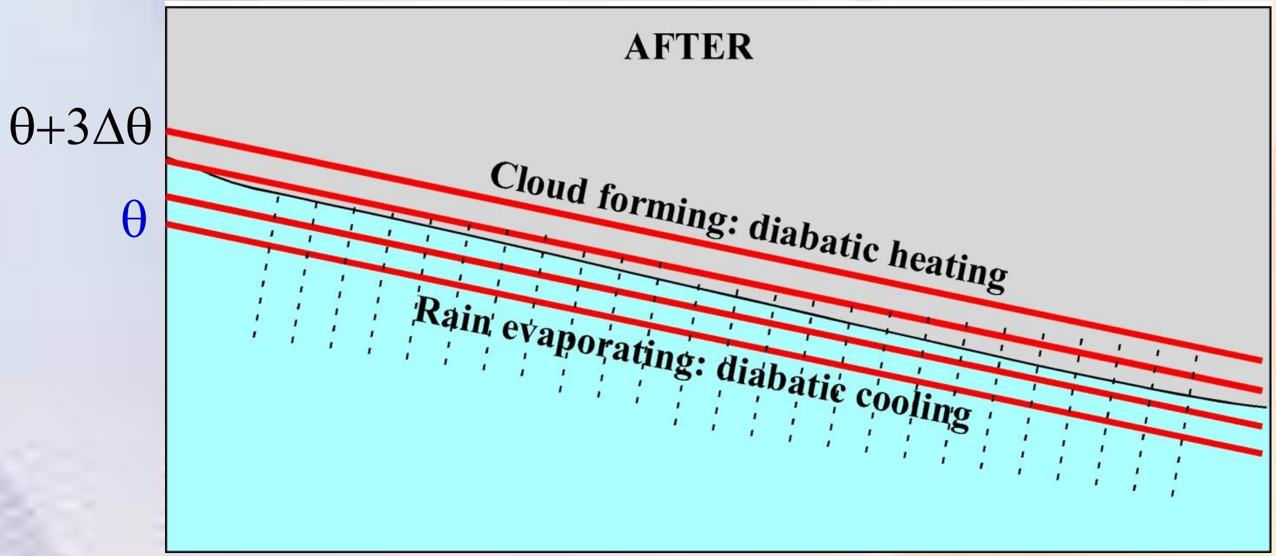
$$F = \frac{\partial\theta/\partial z}{|\nabla\theta|}\left\{\left(\frac{p_0^{\kappa}}{C_p}\right)\left[\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)\right]\right\}$$

adjustment for specific heat of air

weighting factor

vertical gradient in diabatic heating or cooling rate adjusted for pressure altitude


θ+3Δθ
θ+2Δθ
θ+Δθ
θ
BEFORE
Cloud forming: diabatic heating
Rain evaporating: diabatic cooling


θ+3Δθ
θ
AFTER
Cloud forming: diabatic heating
Rain evaporating: diabatic cooling

Horizontal Deformation

$$F = \frac{1}{|\nabla \theta|}\Bigg(\frac{\partial \theta}{\partial x}\left\{ \frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right] - \left(\frac{\partial u}{\partial x}\frac{\partial \theta}{\partial x}\right) - \left(\frac{\partial v}{\partial x}\frac{\partial \theta}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial \theta}{\partial z}\right)\right\}$$

$$+ \frac{\partial \theta}{\partial y}\left\{ \frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right] - \left(\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial x}\right) - \left(\frac{\partial v}{\partial y}\frac{\partial \theta}{\partial y}\right) - \left(\frac{\partial w}{\partial y}\frac{\partial \theta}{\partial z}\right)\right\}$$

the contribution to frontogenesis due to horizontal deformation flow.

$$F = -\left\{ \frac{\partial \theta / \partial x}{|\nabla \theta|}\left(\frac{\partial u}{\partial x}\frac{\partial \theta}{\partial x}\right) + \frac{\partial \theta / \partial y}{|\nabla \theta|}\left(\frac{\partial v}{\partial y}\frac{\partial \theta}{\partial y}\right)\right\}$$

Confluence terms
or
Stretching
deformation

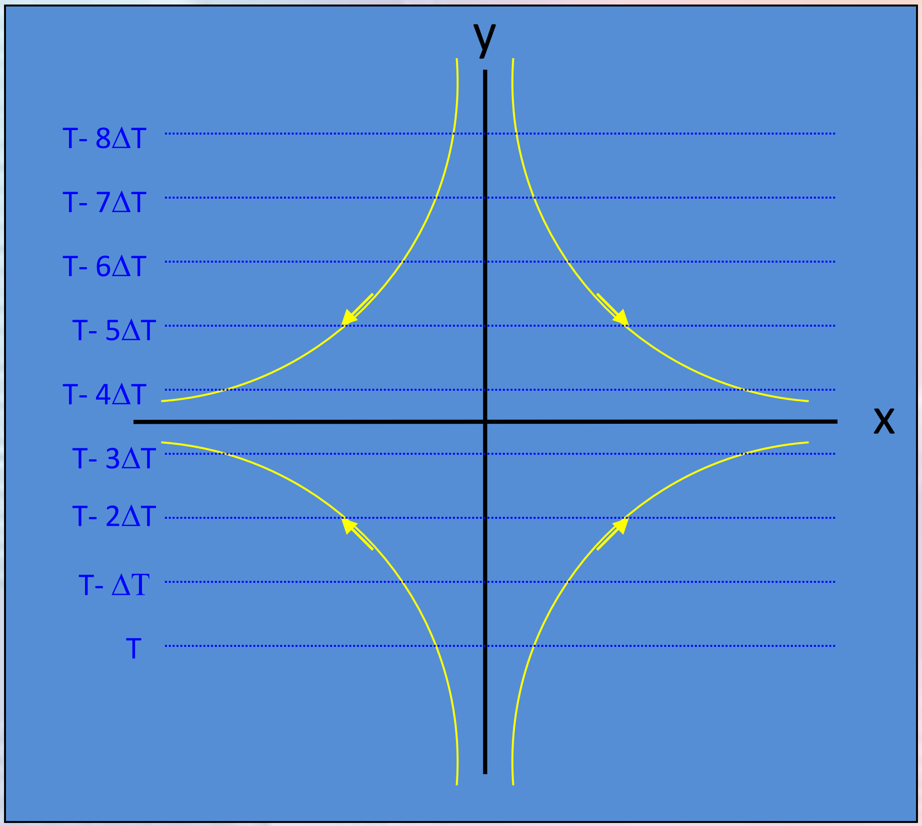
Shearing
deformation

Deformation
acting on
temperature gradient

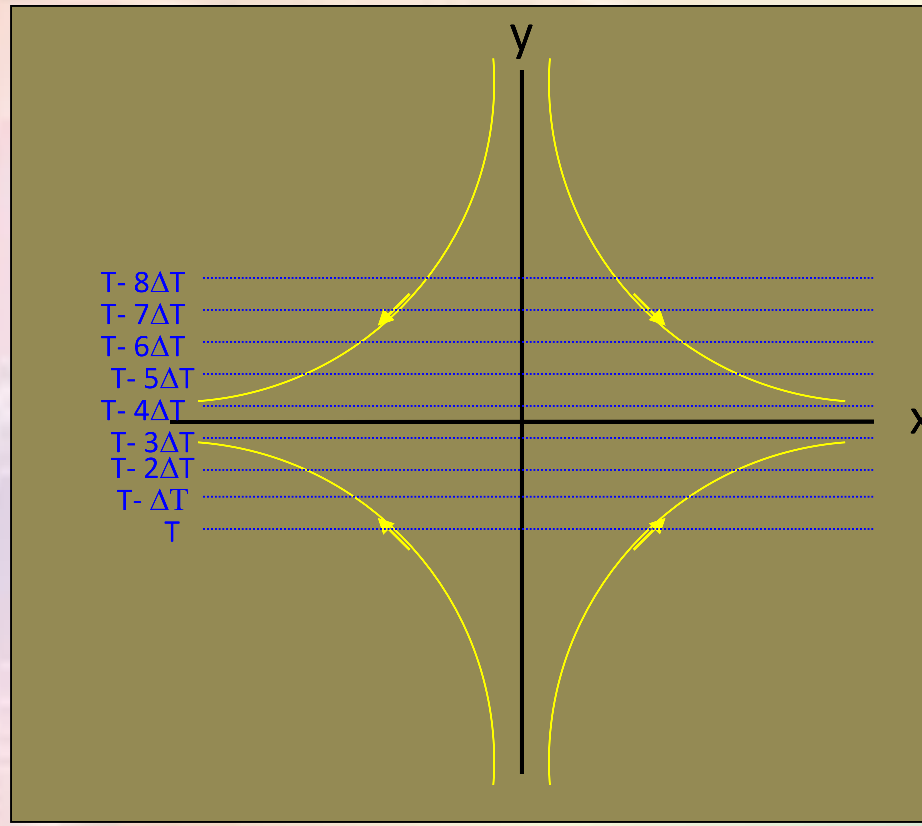
Stretching Deformation

$$F = -\left\{ \frac{\partial\theta/\partial x}{|\nabla\theta|}\left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}\right) + \frac{\partial\theta/\partial y}{|\nabla\theta|}\left(\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right) \right\}$$

Time = t



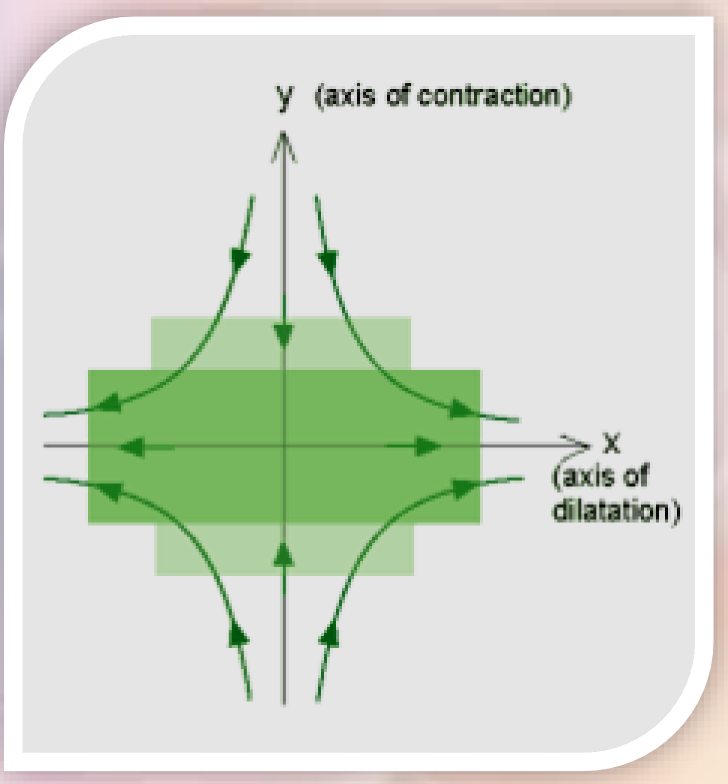
Time = t + Δt

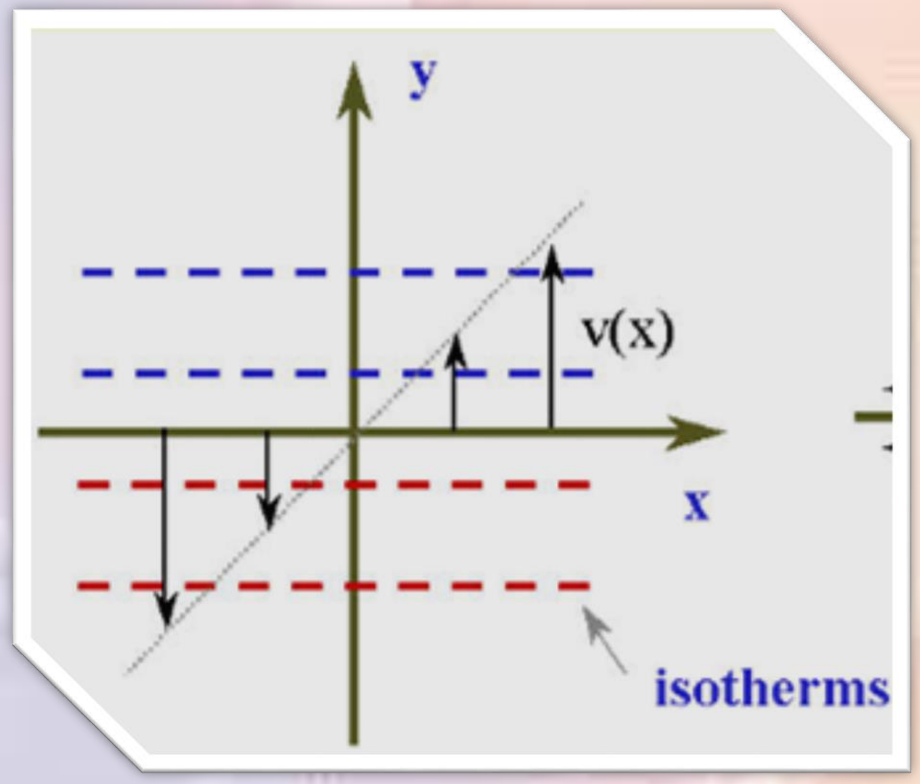


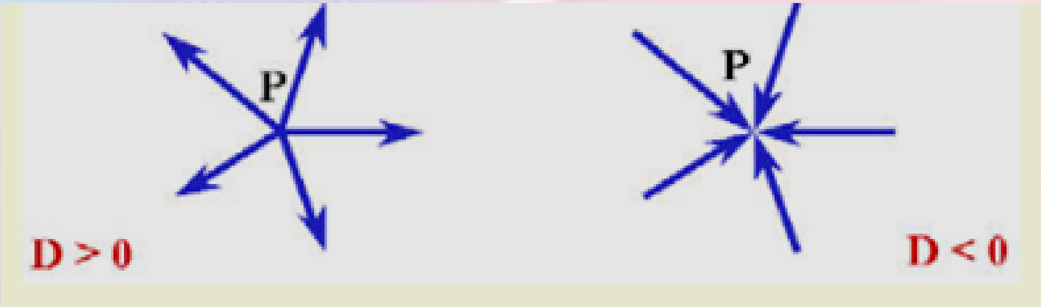
Stretching deformation

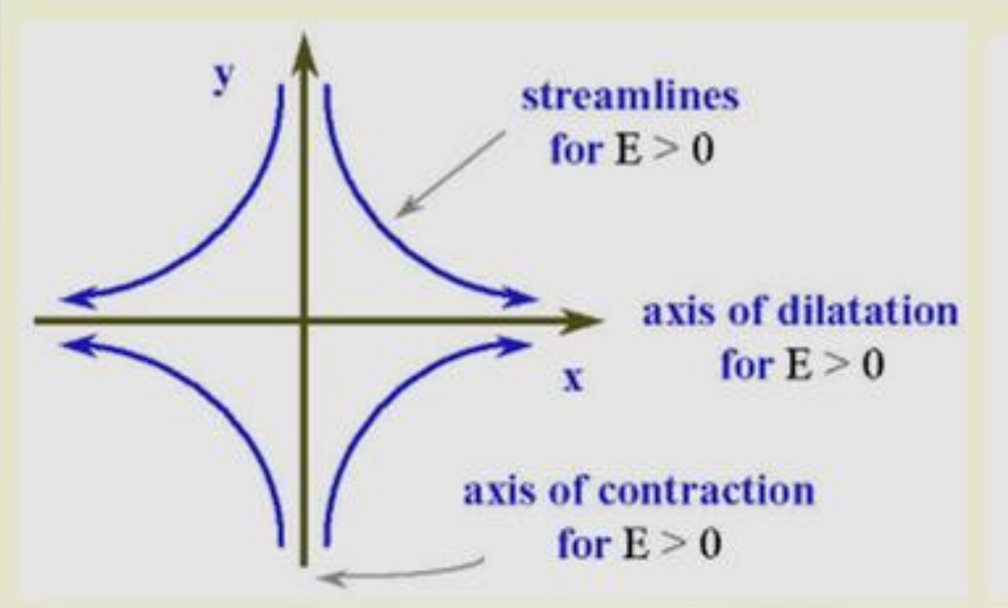
Green lines and arrows show the wind field. Light green square is an element, which stretches into the darker green rectangle.

The area of the element stays the same. Here x-axis is the axis of dilatation and y-axis is the axis of contraction.




y
v(x)
x
isotherms


P
P
D > 0
D < 0

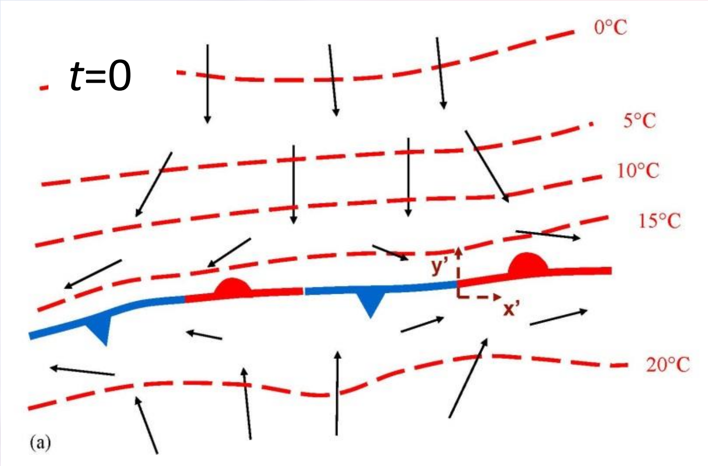

y
x
streamlines
for E > 0
axis of dilatation
for E > 0
axis of contraction
for E > 0

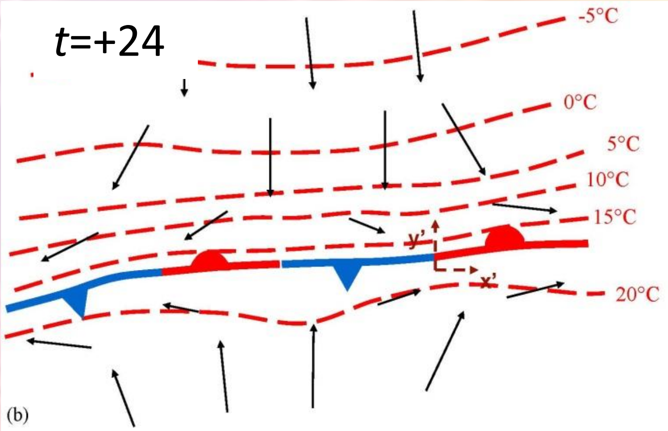
Confluence term

Confluence frontogenesis describes the change in front strength due to *stretching*. If the isotherms are stretching (spreading out), there is frontolysis. If they are compacting, frontogenesis is occurring.

Along the front, $\frac{\partial \theta}{\partial y}$ is negative. Here $\frac{\partial v}{\partial y}$ is also negative, giving a positive contribution to F (again note the rotation of the coordinate system!!)

This means along the front, confluence acts in a frontogenetical sense

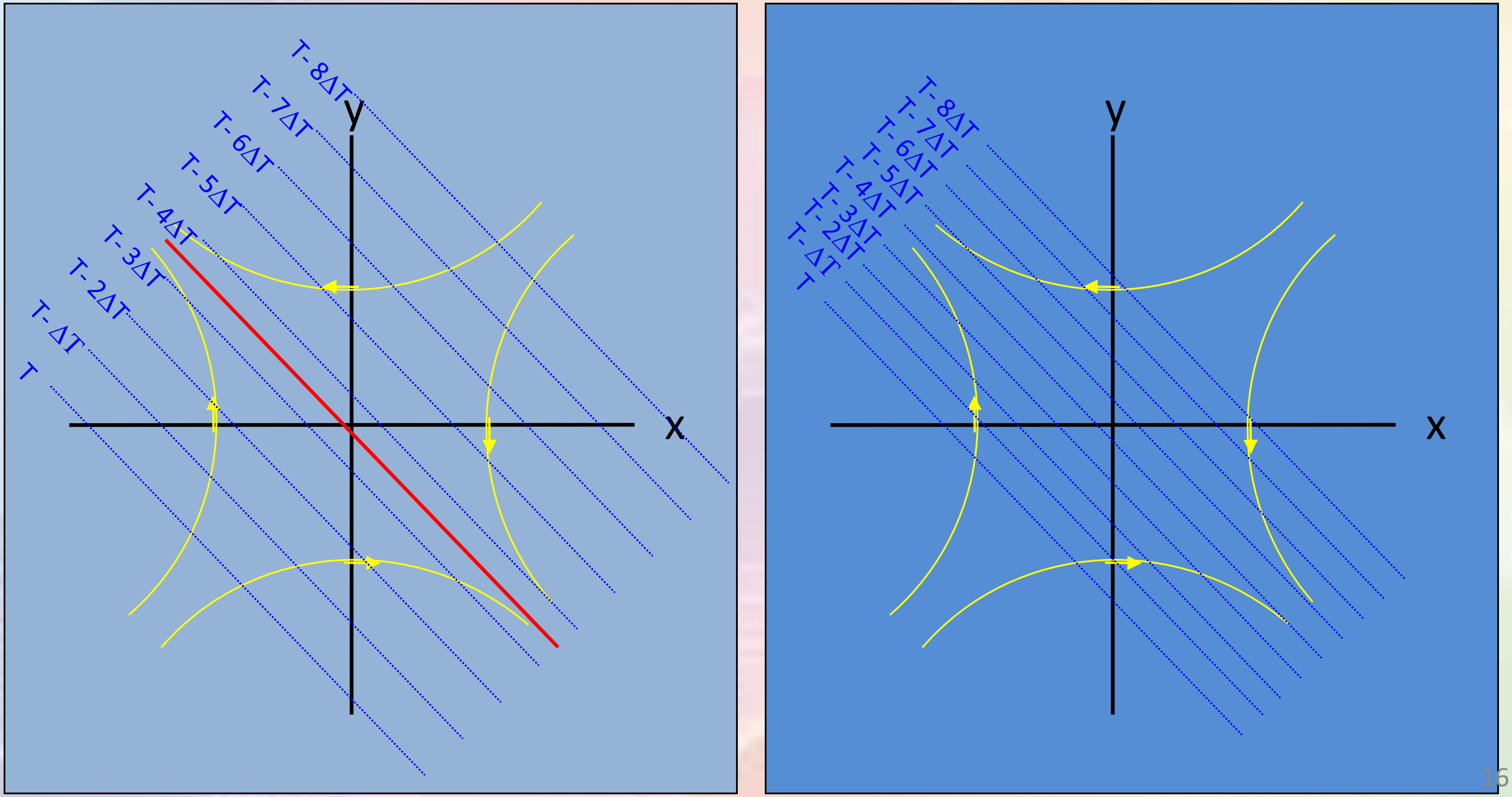




Example: positive contribution to *F* along the front: confluence frontogenesis

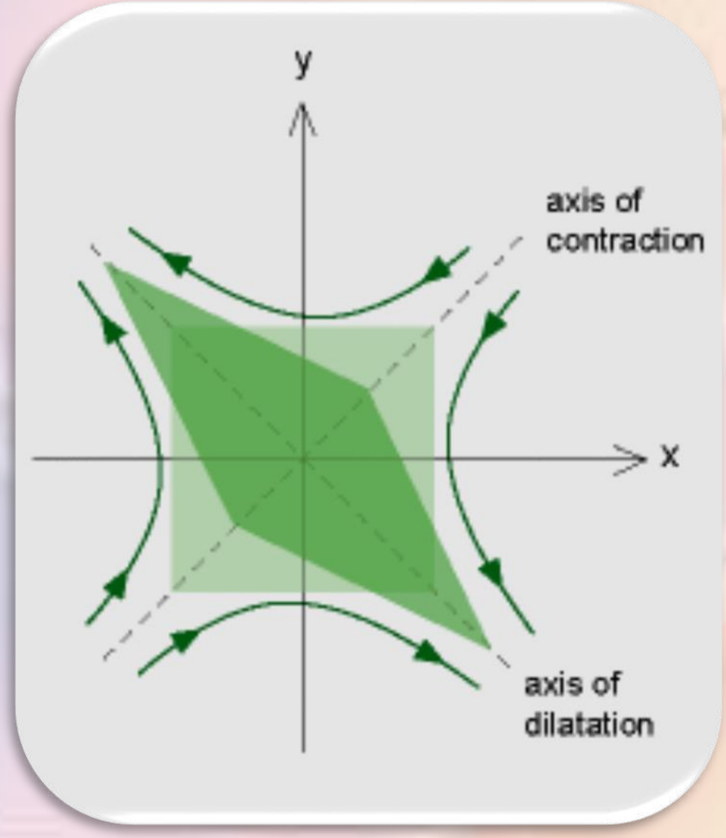
Shearing Deformation

$$F = -\left\{ \frac{\partial\theta / \partial x}{|\nabla\theta|} \left(\frac{\partial v}{\partial x} \frac{\partial\theta}{\partial y} \right) + \frac{\partial\theta / \partial y}{|\nabla\theta|} \left(\frac{\partial u}{\partial y} \frac{\partial\theta}{\partial x} \right) \right\}$$



Shearing deformation

The situation is the same as on the left, but now the axes of dilatation and contraction are at 45° angle to the x- and y-axes.

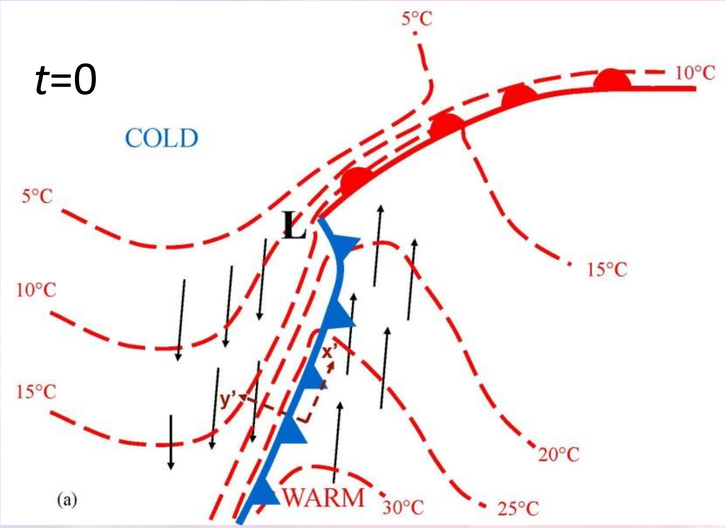


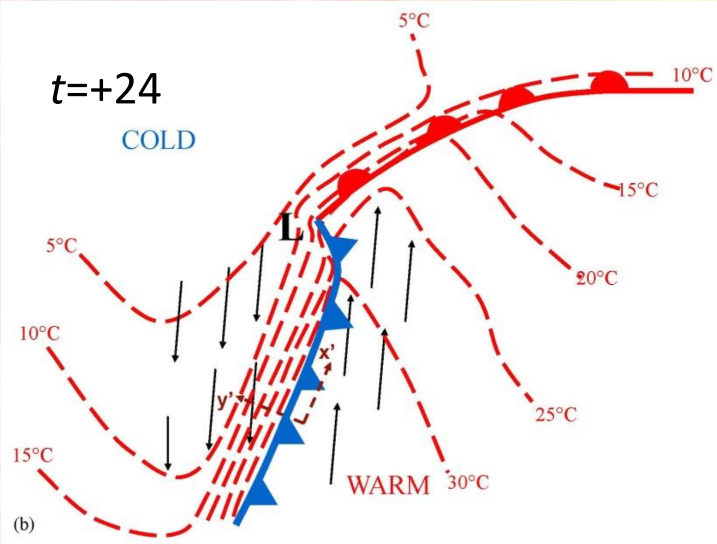
Shearing term

Shear frontogenesis describes the change in front strength due to differential temperature advection by the front-parallel wind component

Along the cold front, both $\frac{\partial \theta}{\partial x}$ and $\frac{\partial u}{\partial y}$ are negative, giving a positive contribution to *F* (note the rotation of the coordinate system!!)

This means cold-air advection in the cold air, and warm-air advection in the warm air.





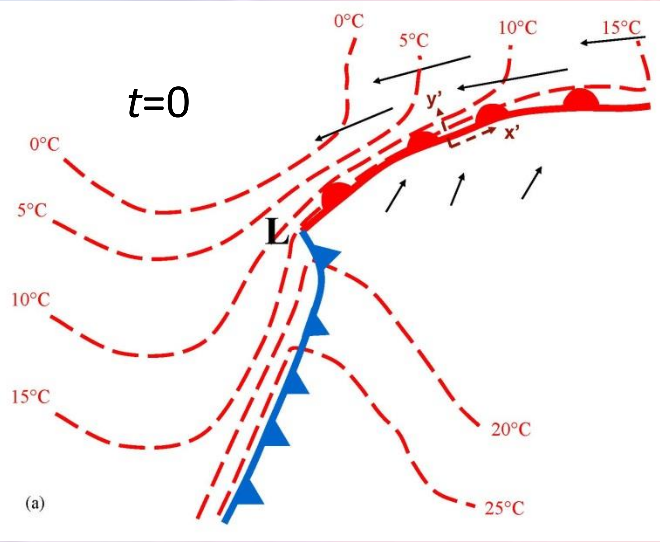
Example: positive contribution to *F* along the cold front: shearing frontogenesis

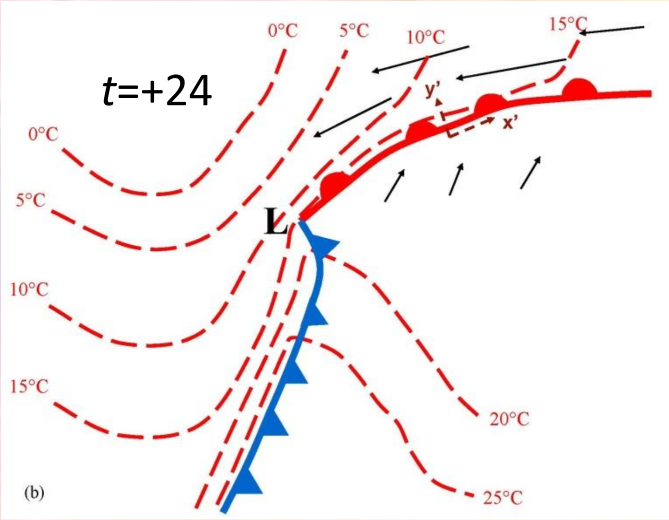
Shearing term

Shear frontogenesis describes the change in front strength due to differential temperature advection by the *front-parallel* wind component

Along the warm front, $\frac{\partial \theta}{\partial x}$ is positive, but $\frac{\partial u}{\partial y}$ is negative, giving a negative contribution to *F* (again note the rotation of the coordinate system!!)

This means along the warm front, shearing acts in a frontolytical sense





Example: negative contribution to *F* along the warm front: shearing frontolysis

Vertical shear acting on a horizontal temperature gradient (also called vertical deformation term)

$$F = \frac{1}{|\nabla\theta|}\Bigg(\frac{\partial\theta}{\partial x}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial y}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\}$$

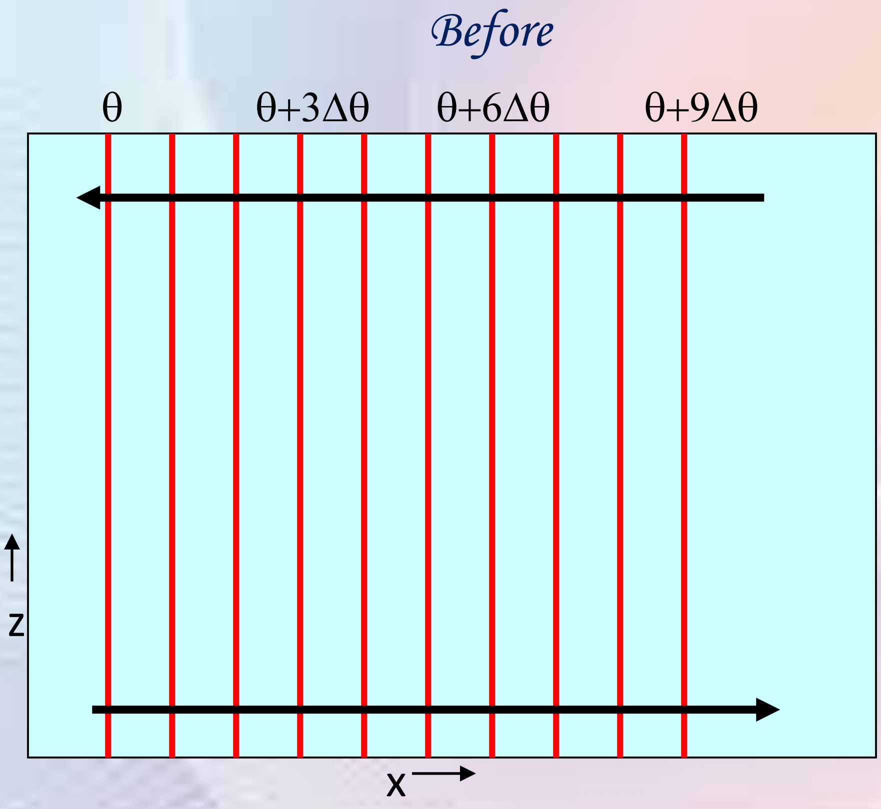
$$+\frac{\partial\theta}{\partial z}\left\{\left(\frac{p_0^{\kappa}}{C_p}\right)\left[\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right)\right\}$$

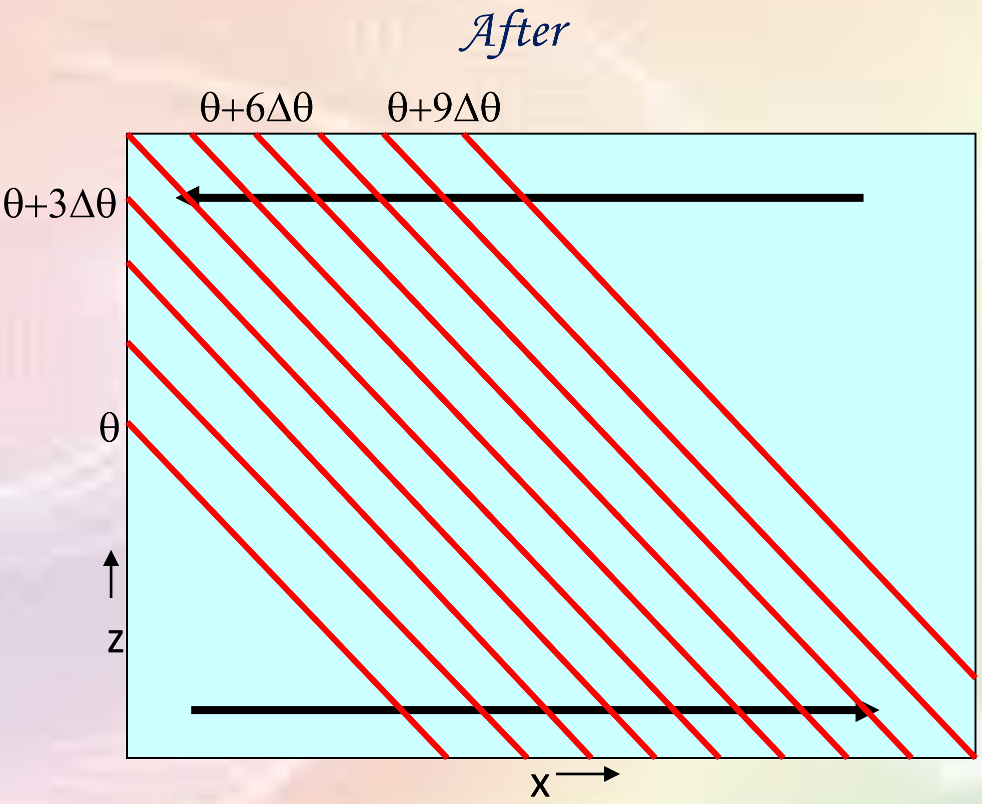
Vertical shear of E-W wind Component acting on a horizontal temp gradient in x direction

Vertical shear of N-S wind component acting on a horizontal temp gradient in y direction

Vertical shear acting on a horizontal temperature gradient

$$F = -\frac{\partial\theta/\partial z}{|\nabla\theta|}\left(\left(\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}\right)+\left(\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}\right)\right)$$





$$F=\frac{1}{|\nabla\theta|}\Bigg(\frac{\partial\theta}{\partial x}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial y}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial z}\left\{\left(\frac{p_0^{\kappa}}{C_p}\right)\left[\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right)\right\}$$

The contribution
to frontogenesis due to tilting.

$$F = \frac{1}{|\nabla\theta|}\Bigg(\frac{\partial\theta}{\partial x}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial y}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial z}\left\{\left(\frac{p_0^{\kappa}}{C_p}\right)\left[\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right)\right\}\Bigg)$$

Tilting terms

$$F = -\left\{\frac{\partial\theta/\partial x}{|\nabla\theta|}\left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right)+\frac{\partial\theta/\partial y}{|\nabla\theta|}\left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\}$$

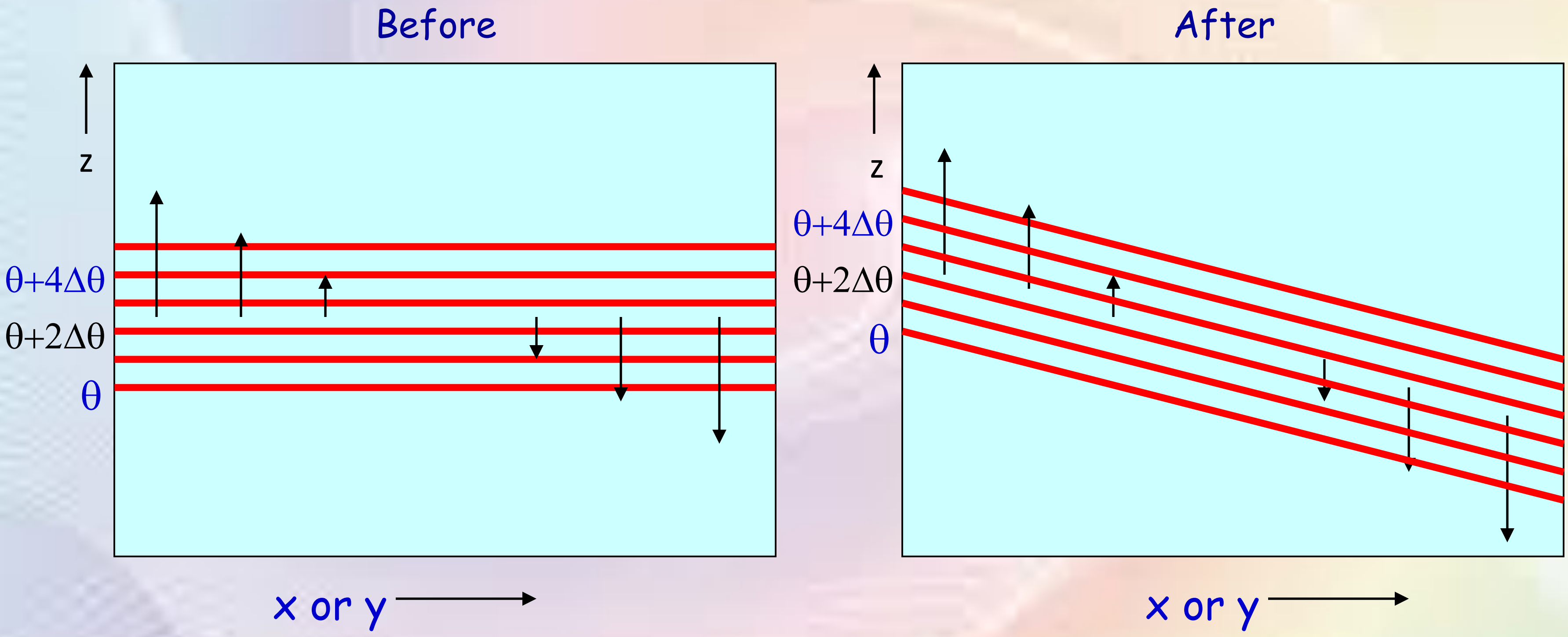
Tilting of vertical θ Gradient (E-W direction)

Tilting of vertical θ Gradient (N-S direction)

Weighting factor

Tilting terms

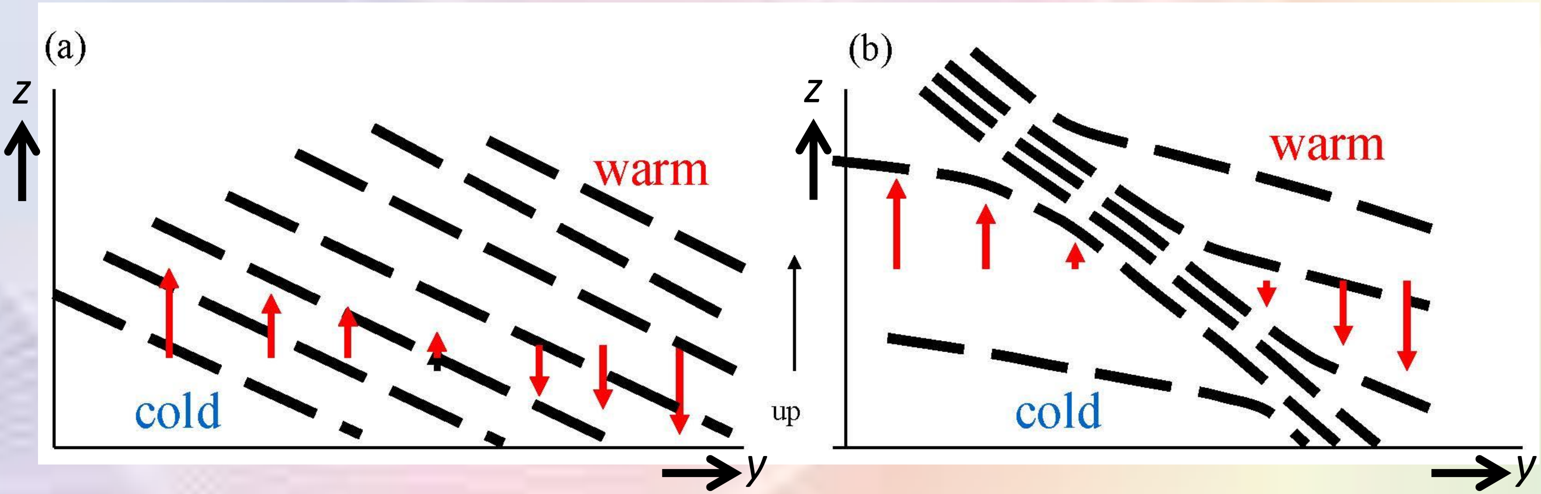
$$F = -\left\{ \frac{\partial\theta/\partial x}{|\nabla\theta|}\left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right) + \frac{\partial\theta/\partial y}{|\nabla\theta|}\left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right) \right\}$$



Tilting term

Near the Earth's surface, vertical motion is usually fairly small.
But higher aloft, it can be strong.
Thus tilting usually acts to strengthen fronts *above* the Earth's surface.

Consider the following example: here $\frac{\partial \theta}{\partial z}$ is positive (temperature decreases above the surface), and $\frac{\partial w}{\partial y}$ is also positive (rising motion in the cold air, sinking in the warm air).



Example: positive contribution to *F* along a front: tilting

$$F=\frac{1}{|\nabla\theta|}\Bigg(\frac{\partial\theta}{\partial x}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial y}\left\{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}\right)\right\}$$

$$+\frac{\partial\theta}{\partial z}\left\{\left(\frac{p_0^{\kappa}}{C_p}\right)\left[\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)\right]-\left(\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}\right)-\left(\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}\right)-\left(\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right)\right\}\Bigg)$$

Compression
of vertical
θ gradient
by differential
vertical motion

Vertical Divergence
the contribution
to frontogenesis due to divergence.

Differential vertical motion
(also called divergence term because
$\partial w/\partial z$ is related to divergence through continuity equation)

Differential vertical motion

$$F = -\left\{ \frac{\partial\theta / \partial z}{|\nabla\theta|} \left(\frac{\partial w}{\partial z} \frac{\partial\theta}{\partial z} \right) \right\}$$