

# Atmospheric Physics

## Lecture 7

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## THE Atmosphere in Equilibrium

## جو در حال تعادل

اکنون اثر میدان گرانشی زمین را بر توزیع انرژی و یا سرعت مولکولها و وابستگی فشار یا چگالی به ارتفاع بررسی می شود.

$$p\alpha = R^*T$$

State Equation

$$\frac{\partial p}{\partial z} = -\rho g$$

$$-\frac{\partial p}{\partial \phi} = \rho$$

$$\phi = gz$$

$$\frac{\partial p}{\partial \phi} = -\frac{p}{R^*T}$$

$$p = p_0 e^{-\frac{\phi}{R^*T}} \quad \rho = \rho_0 e^{-\frac{\phi}{R^*T}}$$

Because the density is proportional to the number of molecules per unit volume:

$$n = n_0 e^{-\frac{\phi}{R^*T}} = n_0 e^{-\frac{m\phi}{kT}}$$

The form of exponent is similar to the exponent in the velocity distribution function:

$$f(v_x) = \left( \frac{m}{2\pi kT} \right)^{1/2} e^{-\frac{mv^2}{2kT}}$$

Consider now two levels of constant geopotential so close together that collision occurring between the planes may be neglected

$$\frac{dn_1}{dv_1} = n_1 f(v_1) \quad \frac{n_2, T_2, \quad f(v_2)}{\phi_2}$$

$$v_1 = v(\phi_1) \quad \frac{n_1, T_1, \quad f(v_1)}{\phi_1}$$

$$v_2 = v(\phi_2)$$

$$\frac{dn_1}{dv_1} = n_1 \left( \frac{m}{2\pi kT} \right)^{1/2} \exp\left( -\frac{mv_1^2}{2kT} \right)$$

$$v(\phi_1)dn_1 = n_1 v(\phi_1) \left( \frac{m}{2\pi k T_1} \right)^{1/2} \exp\left(-\frac{mv^2(\phi_1)}{2kT_1}\right) dv_1$$

$$v(\phi_1)dn_1 = v(\phi_2)dn_2 =$$

$$= n_2 v(\phi_2) \left( \frac{m}{2\pi k T_2} \right)^{1/2} \exp\left(-\frac{mv^2(\phi_2)}{2kT_2}\right) dv_2 \quad *$$

$$\frac{1}{2} v^2(\phi_2) = \frac{1}{2} v^2(\phi_1) - \Delta\phi \quad *'$$

$$\Delta\phi = \phi_2 - \phi_1$$

$$v(\phi_2)dv_2 = v(\phi_1)dv_1 \quad *''$$

$$v(\phi_1)dn_1 = n_2 v(\phi_2) \left( \frac{m}{2\pi k T_2} \right)^{1/2} \exp \left[ -m \left( \frac{v^2(\phi_1) - 2\Delta\phi}{2kT_2} \right) \right] dv_2$$

$$v(\phi_1)dn_1 = n_2 \exp\left(\frac{m\Delta\phi}{kT_2}\right) v(\phi_1) \left( \frac{m}{2\pi k T_2} \right)^{1/2} \exp\left(-\frac{mv^2(\phi_1)}{2kT_2}\right) dv_1$$

$$1 = \frac{n_1}{n_2} e^{-\frac{m\Delta\phi}{kT}}$$

$$n = n_0 e^{-\frac{m\Delta\phi}{kT}}$$

The partial densities of two constituents are then represented by:

$$\rho_1 = \rho_{01} e^{\frac{-\phi}{R^*_1 T}} \quad \rho_2 = \rho_{02} e^{\frac{-\phi}{R^*_2 T}}$$



# CONSERVATION OF MASS

بقای جرم

$$\oiint \rho \vec{v} \cdot \hat{n} dA = - \iiint \frac{\partial \rho}{\partial t} dx dy dz$$

$$\oiint \rho \vec{v} \cdot \hat{n} dA = - \iiint \nabla \cdot (\rho \vec{v}) dx dy dz$$

$$\nabla \cdot (\rho \vec{v}) = - \frac{\partial \rho}{\partial t} \quad \text{equation of continuity}$$

$$\rho \nabla \cdot \vec{v} + \vec{v} \cdot \nabla \rho = - \frac{d\rho}{dt} + \vec{v} \cdot \nabla \rho$$

# CONSERVATION OF ENERGY

## بقای انرژی

$$de_t = du + d\phi + \frac{1}{2} dv^2$$

- Kinetic
- Internal
- Potential (of various kinds)
- Chemical
- Nuclear

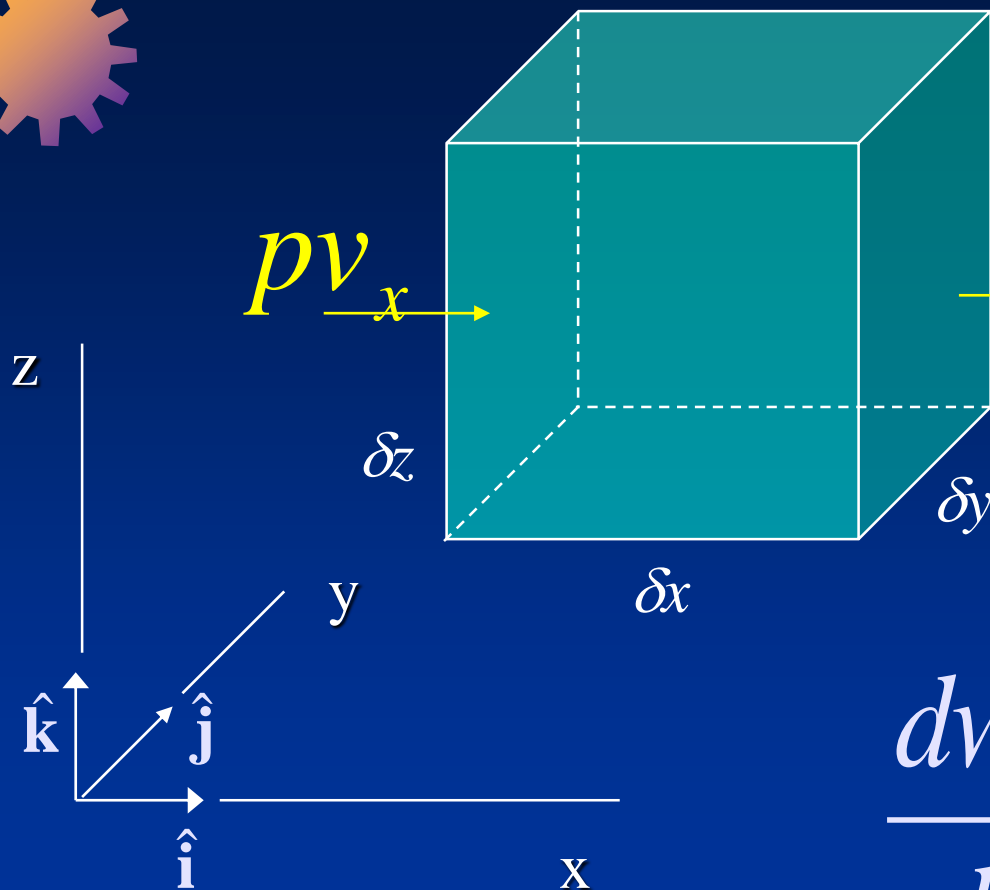
اصل بقای انرژی مستلزم آن است که تغییرات انرژی کل برابر مجموع گرمای مبادله شده سیستم و کار انجام یافته بر روی آن سیستم است.

$$de_t = dh + dw$$





محاسبه کار



$$\left(p + \frac{\partial p}{\partial x} dx\right) \left(v_x + \frac{\partial v_x}{\partial x} dx\right)$$

$$\frac{dw_1}{dt} = p v_x dy dz$$

$$\frac{dw_2}{dt} = \left(p + \frac{\partial p}{\partial x} dx\right) \left(v_x + \frac{\partial v_x}{\partial x} dx\right) dy dz$$



$$dw_x = dw_1 - dw_2$$

$$\frac{dw_x}{dt} = -\left(p \frac{\partial v_x}{\partial x} + v_x \frac{\partial p}{\partial x}\right) dx dy dz$$

$$\frac{dw_y}{dt} = -\left(p \frac{\partial v_y}{\partial y} + v_y \frac{\partial p}{\partial y}\right) dx dy dz$$

$$\frac{dw_z}{dt} = -\left(p \frac{\partial v_z}{\partial z} + v_z \frac{\partial p}{\partial z}\right) dx dy dz$$

$$\frac{dw}{dt} = -(p\nabla\cdot\vec{v} + \vec{v}\cdot\nabla p)dxdydz$$

$$\frac{dw}{dt} = -\frac{1}{\rho}(p\nabla\cdot\vec{v} + \vec{v}\cdot\nabla p)$$



First Law of Thermodynamics

$$\frac{dw}{dt} = -\frac{1}{\rho}\left(-\frac{1}{\rho}\frac{d\rho}{dt}p + v\cdot\nabla p\right)$$

$$\frac{dw}{dt} = -p\frac{d\alpha}{dt} - \alpha v\cdot\nabla p$$

$$dh = de_t - dw =$$

$$= du + \cancel{d\phi} + \frac{1}{2} \cancel{d(v^2)} + p d\alpha + \alpha v \cancel{\nabla p dt}$$

$$\frac{dv_z}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g - f_z$$

$$v_z \frac{dv_z}{dt} = -\alpha v_z \frac{\partial p}{\partial z} - g v_z$$

$$v_z = \frac{dz}{dt}, \quad d\phi = g dz, \quad g = \frac{d\phi}{dz}$$





$$\frac{1}{2} \frac{dv_z^2}{dt} + \frac{d\phi}{dt} = -\alpha v_z \frac{\partial p}{\partial z}$$

$$\frac{1}{2} dv^2 + d\phi = -\alpha \vec{v} \cdot \nabla p dt$$

$$dh = du + p d\alpha$$

This result is called the *first law of thermodynamics*. It may be used as the definition of heat to the system per unit mass.

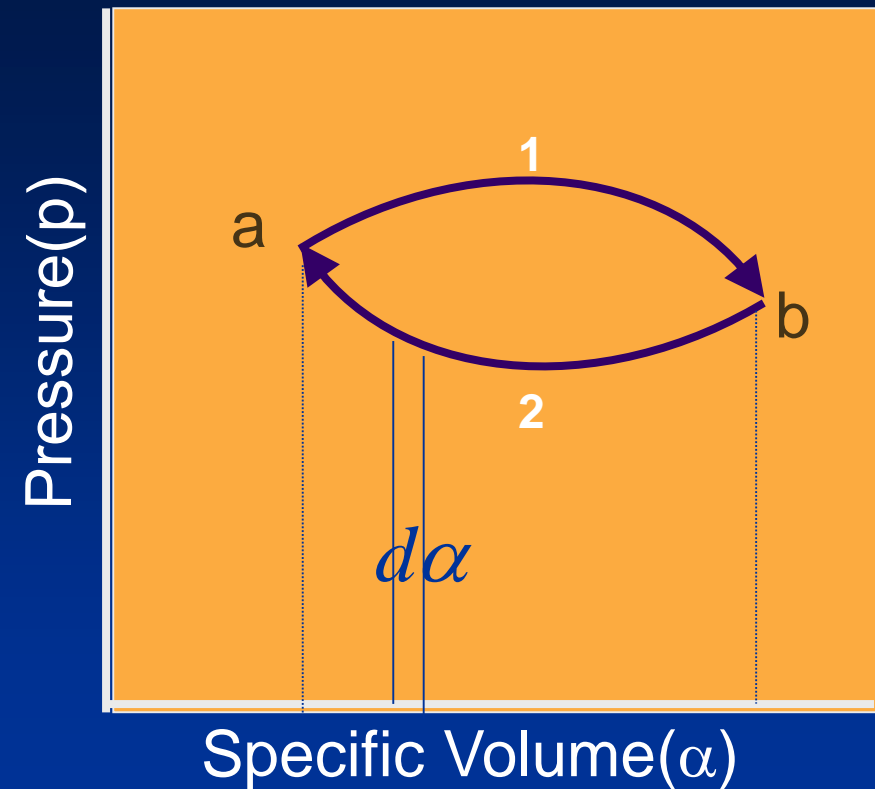
The internal energy is by definition the kinetic energy of molecular motions of the molecules, and therefore is proportional to the temperature.



If the volume is held constant  $d\alpha=0$ , and all the heat added to the system is used for the increase of internal energy.

The last term in above Equation, the expansion work, can be represented in a graph of  $p$  versus  $\alpha$  as illustrated in Fig.





The area enclosed by the two paths represents the work per unit mass performed by the system in a complete cycle.

If the state is changed from **a** to **b** along curve 1, then the area under the curve represents the expansion work done by the system

$$\int_a^b p d\alpha$$

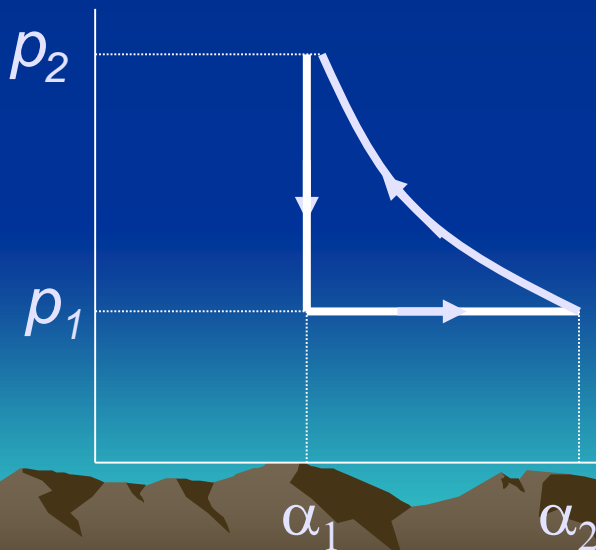
If the state now is changed from **b** to **a** along curve 2, then an amount of work is done on the system which corresponds to the area under this new curve.

The complete integration described is called the line integral around a closed path and is written



$$\oint p d\alpha$$

**Problem:** What is the expansion work done by a system in the cyclic process shown in the adjoining figure?



$$W = \int_{\alpha_1}^{\alpha_2} p_1 d\alpha - \frac{RT}{M} \int_{p_1}^{p_2} \frac{dp}{p} + 0$$

$$= p_1(\alpha_2 - \alpha_1) + \frac{RT}{M} \ln \frac{p_1}{p_2}$$

For the stated conditions

$$p_1 = \frac{RT}{M\alpha_2} \quad \text{and} \quad p_2 = \frac{RT}{M\alpha_2}$$

$$W = \frac{RT}{M} \left[ 1 - \frac{\alpha_1}{\alpha_2} + \ln \frac{\alpha_1}{\alpha_2} \right]$$

