Atmospheric Physics

Lecture 5

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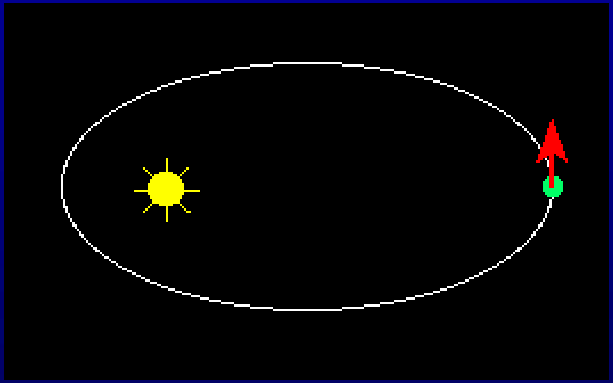
Gravitational Orbits



Newton's theories of dynamics and gravity provided a complete understanding of the interaction between gravitating bodies and the resulting orbits for planets and satellites. The various kinds of possible orbits are described further in this lecture

Newton found that ellipses were not the only orbital paths.

Energy

The **potential energy** of an object at point **r** in a gravitational field is the energy required to bring it to that point from infinity.

for objects of mass M and m separated by distance r:

$$U = -\int_{\mathbf{r}_\infty}^{\mathbf{r}} \mathbf{F}.d\mathbf{r} = \int_{\infty}^{r} G\frac{Mm}{r^2}dr = \left[-G\frac{Mm}{r}\right]_{\infty}^{r} = -G\frac{Mm}{r}$$

The **kinetic energy** of an object is: $\boxed{K = \frac{1}{2}m\mathrm{v}^2}$

What is orbital energy?

orbital energy = kinetic energy + gravitational potential energy

It is the combined kinetic and gravitational potential energy of an orbiting object.

$$\text{Recall}: \; E = U + K = \text{constant},$$

$$U = -\frac{GMm}{r} < 0, \qquad K = \frac{1}{2} m\text{v}^2 > 0$$

What does this mean dynamically?

e.g. Object m is said to be bound to object M with **binding energy** E if $E < 0$

$$\Rightarrow \quad E > 0 \Leftrightarrow \mathrm{v} > 0 \ \text{ at } r = \infty : \ \textbf{unbound}$$

$$E = 0 \Leftrightarrow \mathrm{v} \equiv 0 \ \text{ at } r = \infty$$

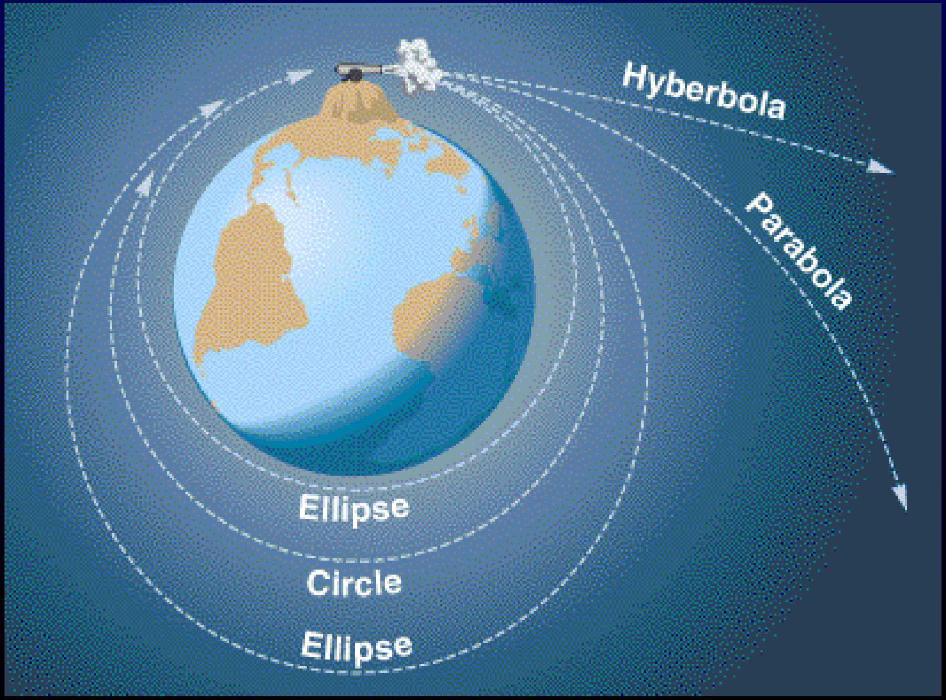
$$E < 0 \Leftrightarrow \mathrm{v} = 0 \ \text{ at } r < \infty : \ \textbf{bound}$$

The orbits of the planets generally are ellipses.

Some comets have parabolic orbits; this means that they pass the Sun once and then leave the Solar System, never to return. Other comets have elliptical orbits and thus orbit the Sun with specific periods.

possible orbital paths are:
ellipse (bound)
parabola (unbound)
hyperbola (unbound)

All are **“Conic Sections”**



The **escape velocity** at distance r from M is the speed required to reach ∞ (or, equivalently, for $E = 0$)

$$U + K = 0 \Rightarrow \frac{1}{2} m \mathrm{v}_{\mathrm{esc}}^2 = \frac{GMm}{r} \Rightarrow \mathrm{v}_{\mathrm{esc}} = \sqrt{\frac{2GM}{r}}$$

If an object gains enough energy so that its new orbit is unbound, we say that it has reached **escape velocity**.

m will move in a **circle** of radius r around M if

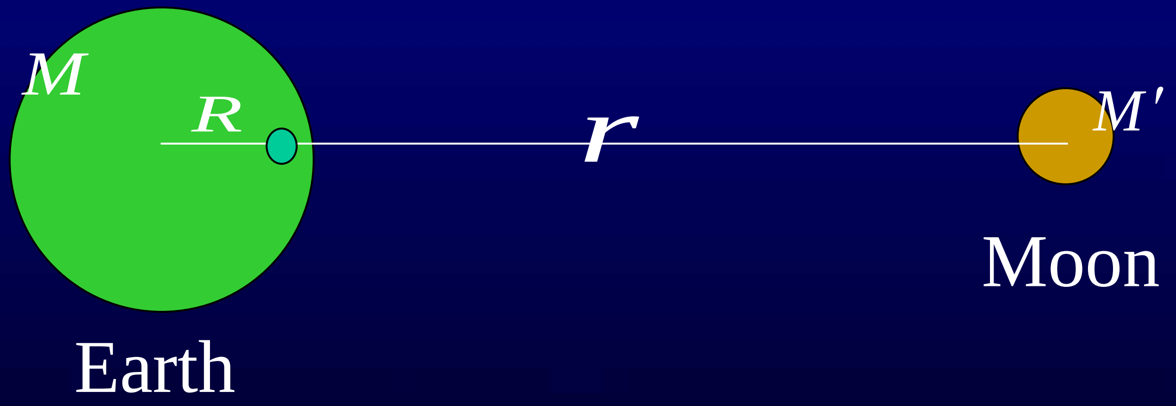
$$m a_{\mathrm{circ}} = \frac{m \mathrm{v}_{\mathrm{circ}}^2}{r} = -F = \frac{GMm}{r^2} \Rightarrow \mathrm{v}_{\mathrm{circ}} = \sqrt{\frac{GM}{r}}$$

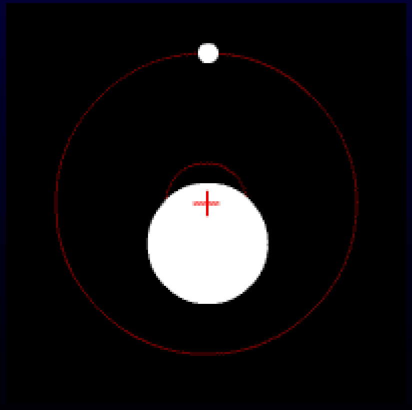
An orbit can only change if it gains/loses energy from another object, such as a gravitational encounter:

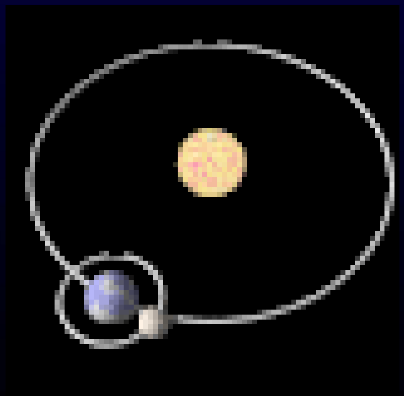
Tides in the Atmosphere

TIDE-PRODUCING FORCES

- **Earth + Moon = single system rotating about a common CM.** ***(T=27.3 days)***

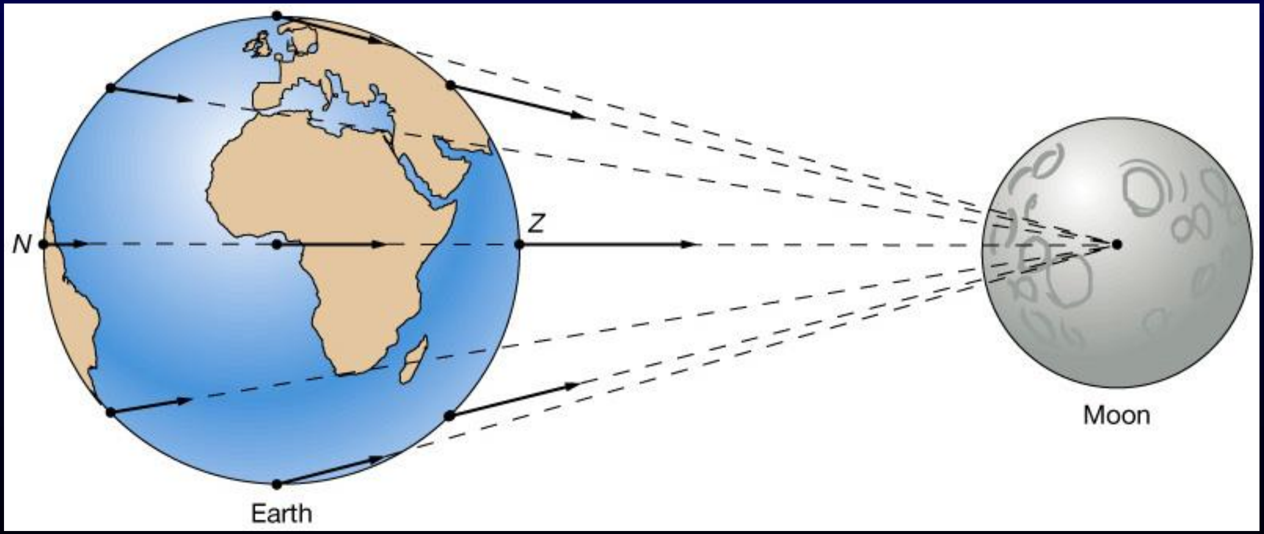






Gravitational forces on Earth due to the Moon

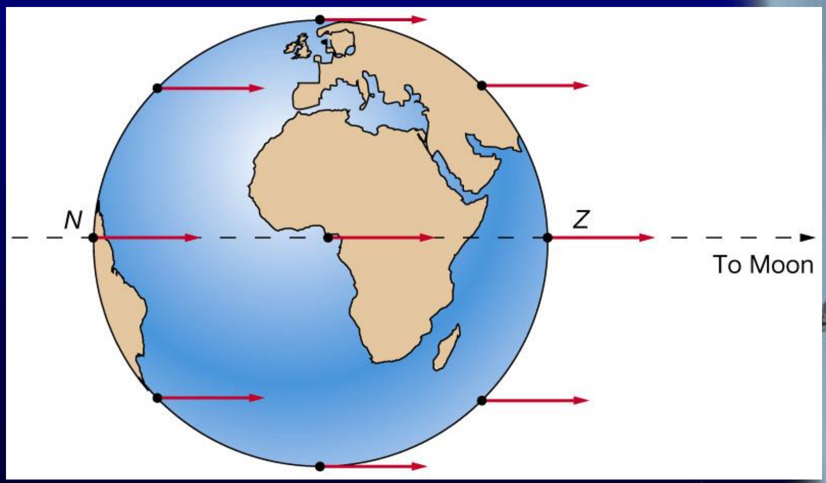
- Force decreases with increasing distance
- Force is directed toward the Moon’s center of mass



Centripetal forces on Earth due to the Moon

All points on and within the Earth will experience the *same centrifugal force.(Fc)*

- Force is the same everywhere on Earth
- Force is directed perpendicular to Earth's center everywhere on Earth



F_c acting on the Earth-Moon system = Fg Earth-Moon

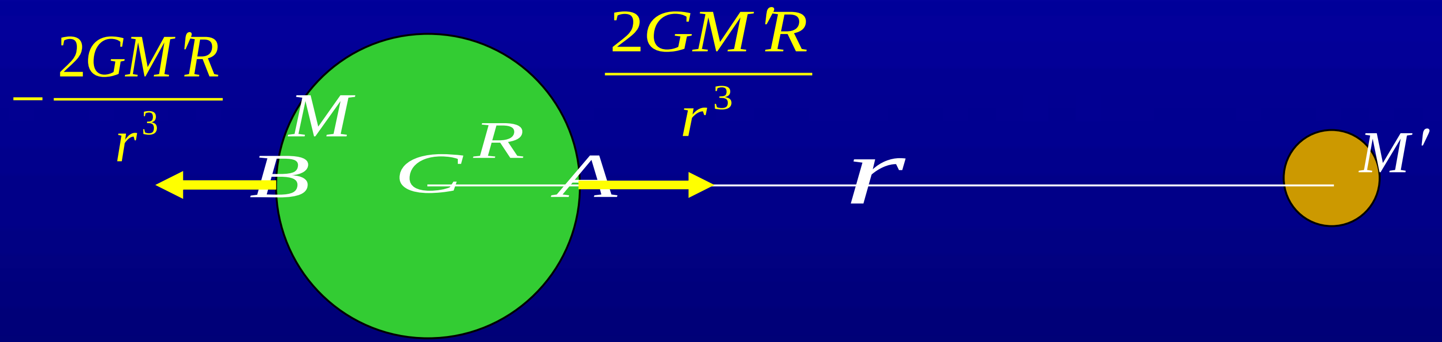
system in *equilibrium*

What causes tides?

- Tides are created by the imbalance between two forces:

1. **Gravitational force of the Moon and Sun on Earth**
   - If mass increases (↑), then gravitational force increases (↑)
   - If distance increases (↑), then gravitational force greatly decreases (↓↓)

2. Centripetal (center-seeking) force required to keep bodies in nearly circular orbits



$$F_c = \frac{GM'}{r^2}$$

$$F_A = \frac{GM'}{(r-R)^2} = \frac{GM'}{r^2(1-\frac{2R}{r})}$$

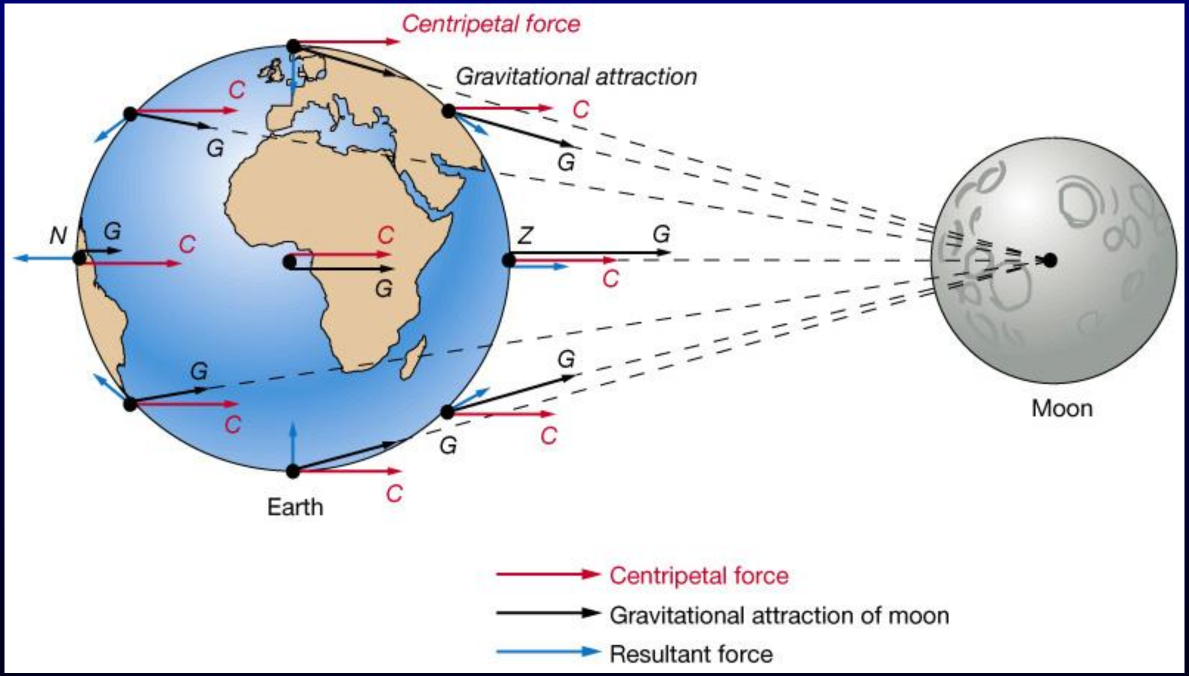
$$TPF_A = G\frac{M'}{(r-R)^2} - G\frac{M'}{r^2}$$

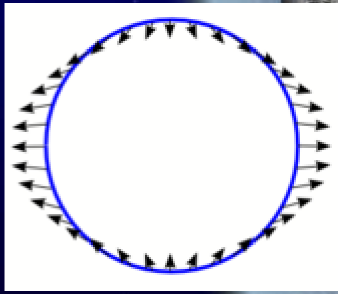
$$\Rightarrow TPF_A \approx 2G\frac{M'R}{r^3} = 1.13\times10^{-6}\ \ \mathrm{N/kg}$$

Resultant forces

Resultant forces are:

The difference between gravitational (G) and centripetal (C) forces

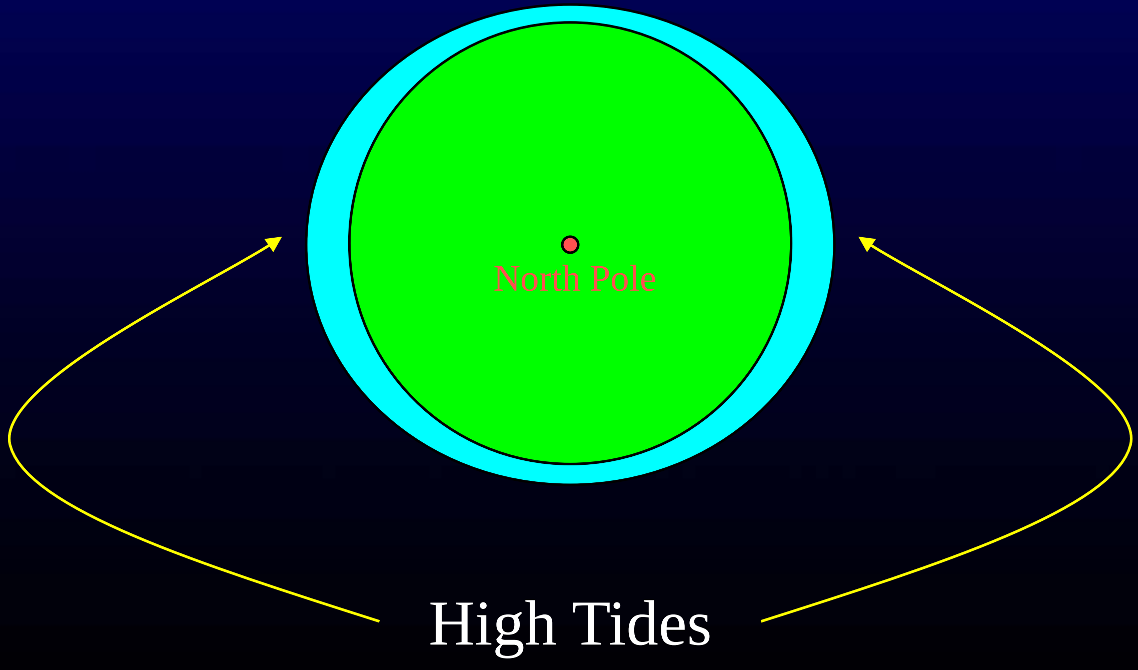


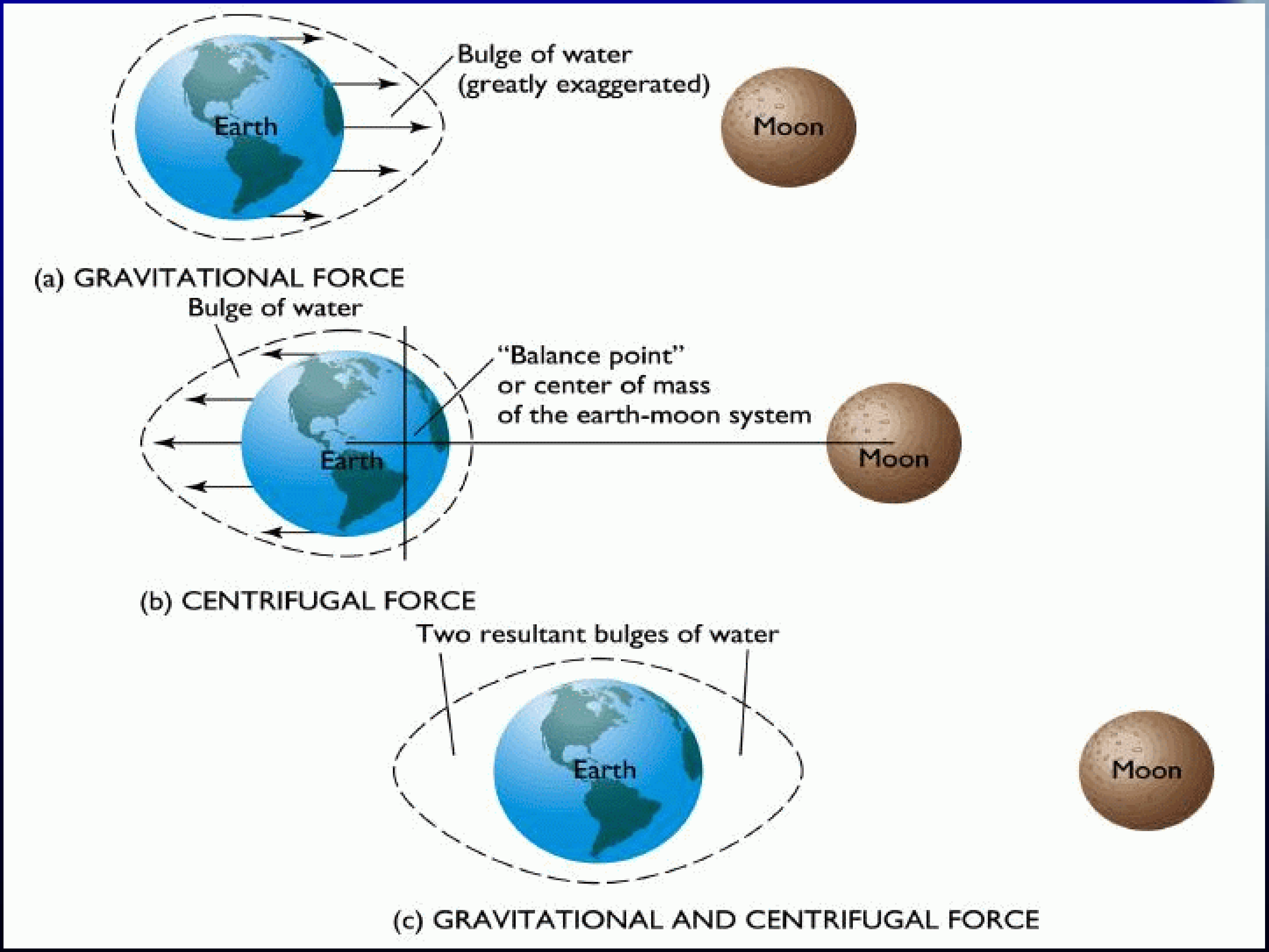


Directed away from Moon on the side of Earth opposite Moon

Directed toward Moon on the side of Earth facing Moon

- **Differences in gravity on different parts of the Earth produce tidal bulges.**

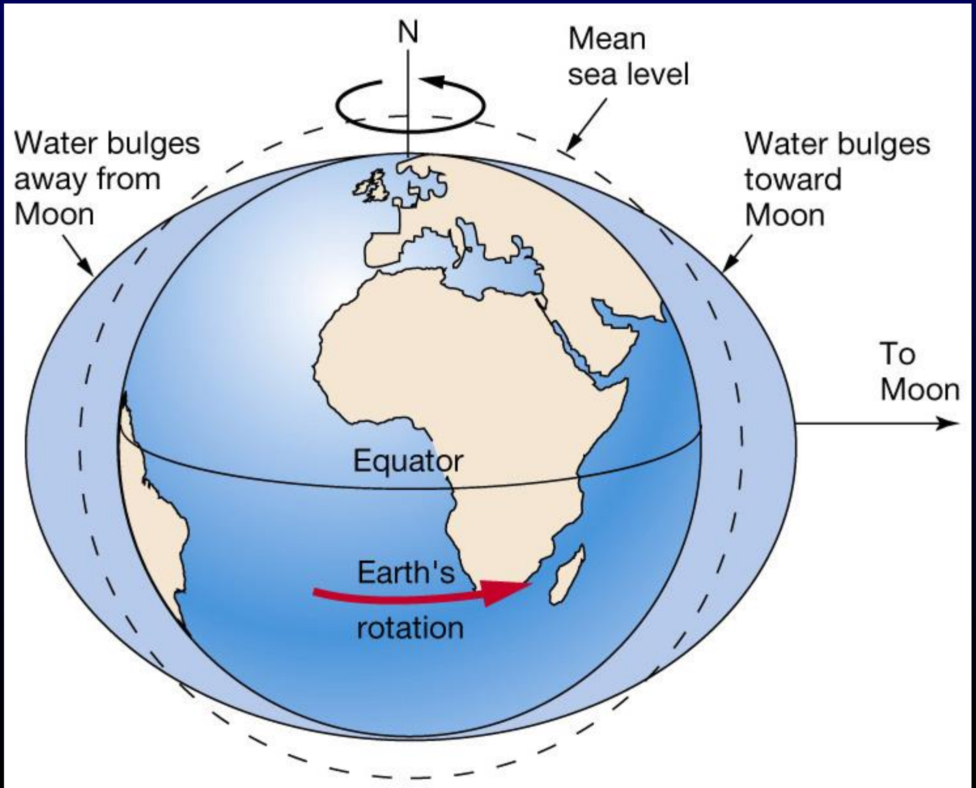




Bulge of water
(greatly exaggerated)
Earth
Moon
(a) GRAVITATIONAL FORCE
Bulge of water
"Balance point"
or center of mass
of the earth-moon system
Earth
Moon
(b) CENTRIFUGAL FORCE
Two resultant bulges of water
Earth
Moon
(c) GRAVITATIONAL AND CENTRIFUGAL FORCE

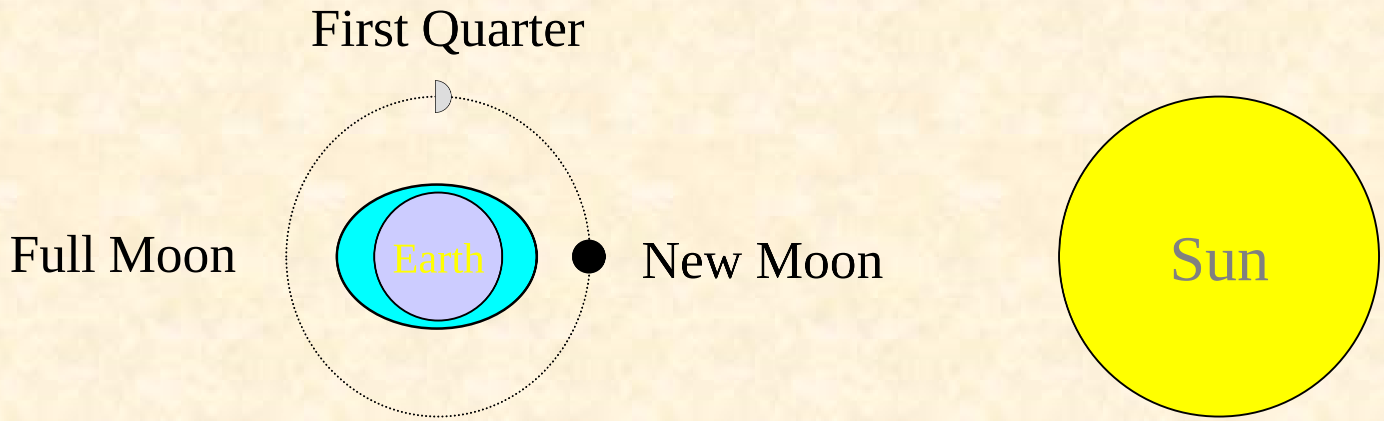
Tidal bulges

- Tide-generating forces produce 2 bulges:
  1. Away from Moon on side of Earth opposite Moon
  2. Toward Moon on side of Earth facing Moon
- Earth rotates into and out of tidal bulges, creating high and low tides



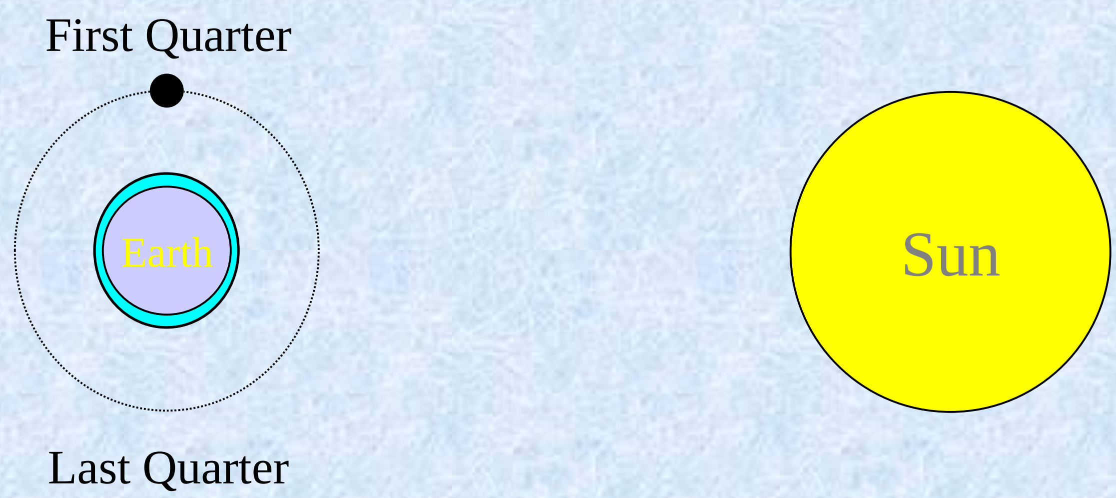
Earth-Moon-Sun positions and the monthly tidal cycle

For example, particularly large tides are experienced in the Earth's oceans when the Sun and the Moon are lined up with the Earth at new and full phases of the Moon. These are called *spring tides* (the name is not associated with the season of Spring). The amount of enhancement in Earth's tides is about the same whether the Sun and Moon are lined up on opposite sides of the Earth (full Lunar phase) or on the same side (new Lunar phase).



Earth-Moon-Sun positions and the monthly tidal cycle

Conversely, when the Moon is at first quarter or last quarter phase (meaning that it is located at right angles to the Earth-Sun line), the Sun and Moon interfere with each other in producing tidal bulges and tides are generally weaker; these are called *neap tides*.




خسته نباشید

What have we learned?

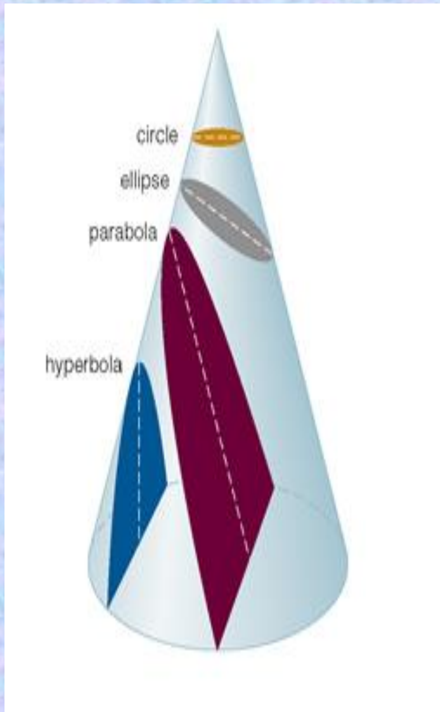
- Will a spacecraft passing by a planet be “sucked in”?

  - No. Energy must be conserved, so an object’s orbital energy cannot change unless it gains or loses energy to something else.

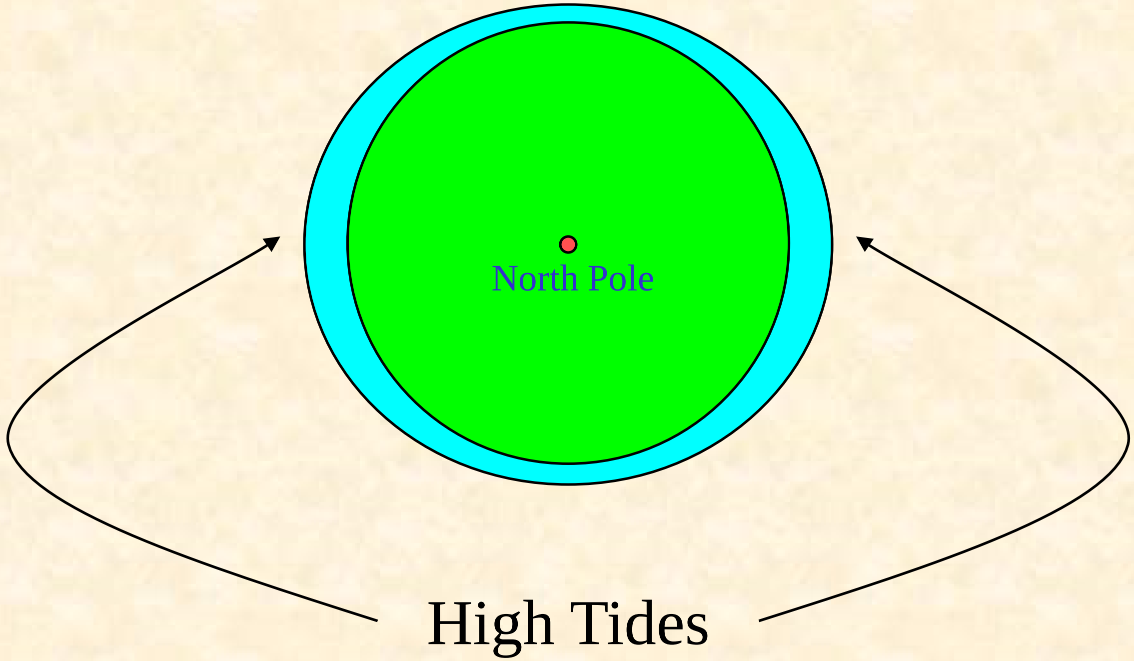
- How can an object achieve escape velocity?

  It must be given an orbit that is unbound so it will not return. It must be given enough energy to change a bound orbit to an unbound one.

orbital energy = kinetic energy + gravitational potential energy



- **Differences in gravity on different parts of the Earth produce tidal bulges.**



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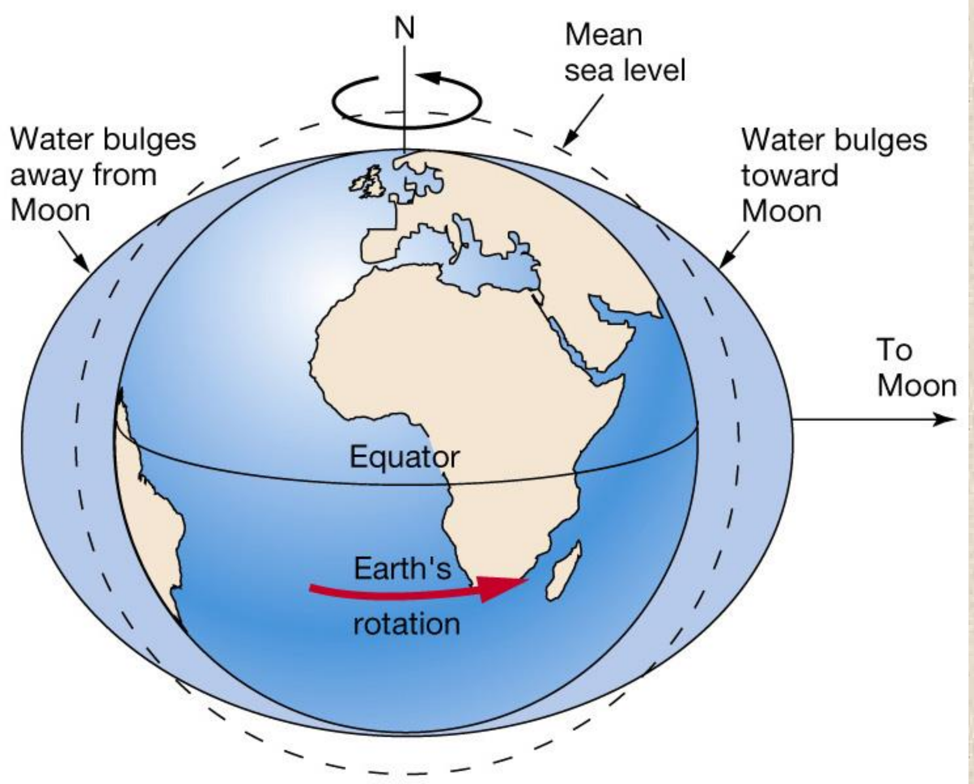

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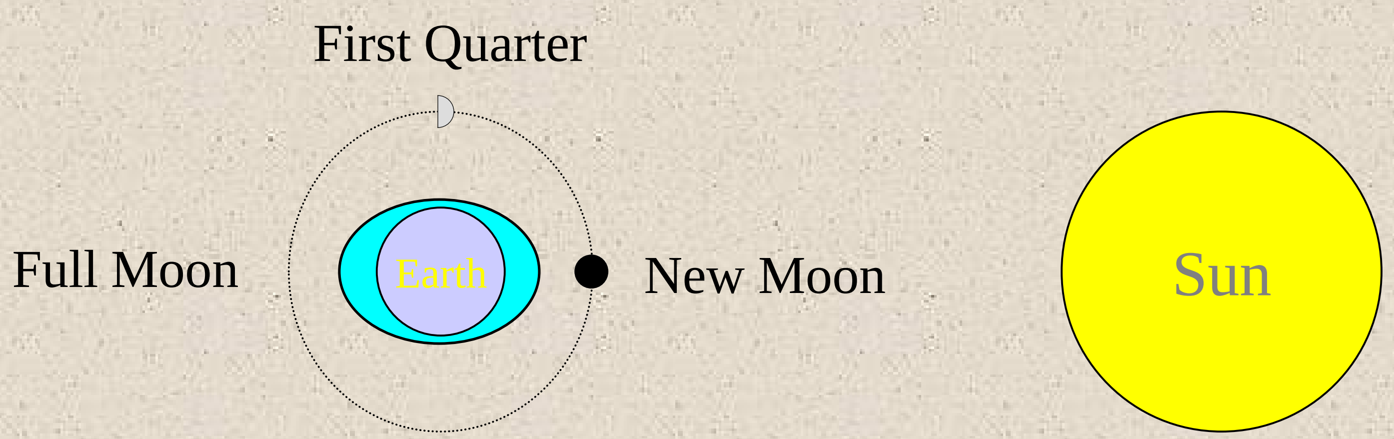

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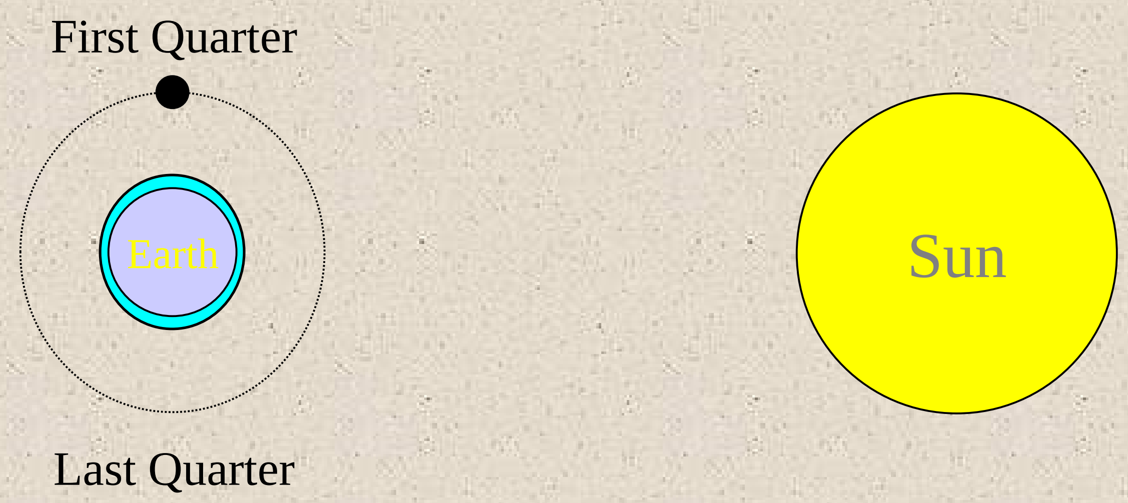

Question: How about tides due to the sun?

The son's gravitational force on Earth is 180 times as large as that of the moon's pull on Earth. So, what about ocean tides due to the sun??
Why are these not 180 times as strong as those due to the moon?

Because tides happen due to *differences* in grav pulls on one side of earth c.f. other side.

Because the sun is so far away, the $1/d^2$ factor flattens out, so the difference in its F at opposite points on the earth is very small: 0.017 %

Whereas for the moon, the difference in its grav F at opposite points on the earth is much larger: 6.7 %

Still, 180 is a big factor in the actual size of the force – and means that despite the tiny % difference, there *are* tides due to the sun, which are about half as high as those due to the moon

(180 x 0.017 % = 3 %, which is about half of 6.7 %)

What have we learned?

- Why are there two high tides on Earth each day?
  - The Moon’s gravity stretches Earth along the Earth-Moon line, so that it bulges both toward and away from the Moon.
- Why are tides on Earth caused primarily by the Moon rather than by the Sun?
  - Earth’s gravitational attraction to the sun is stronger than its gravitational attraction to the Moon, but tides are caused by the *difference* between the strength of the gravitational attraction across Earth’s diameter. This difference is greater for the gravitational force due to the Moon, because the Moon is so much closer than the Sun.

Inclination

The moon does not rotate around the earth's equator, but follows an orbit that is inclined in relation to the earth's axis.
Because of this, northern and southern latitudes commonly face only one high tide and one low tide in a day, called diurnal tides.
The inclination of the moon changes in relation to the earth on a 19 year cycle.

The earth's inclination in relation to the sun also effects the tides. The sun's inclination follows a year-long cycle, and is in highest inclination in the summer and winter months. During these months the "bulges" in the ocean are offset the most from the equator, and it is most likely to encounter only one tide cycle per day, or diurnal tides.