Atmospheric Aerosols

Lecture 12

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Concept of Drag

Drag is the retarding force exerted on a moving body in a fluid medium

It does not attempt to turn the object, simply to slow it down

It is a function of the speed of the body, the size (and shape) of the body, and the fluid through which it is moving

A common form of the drag force due to wind (air) acting on an object can be found by:

C_D A dimensionless Coef.

A_P Area projected in flow direction (a circle here)

$$F_{\text{drag}} = \frac{1}{2} C_D A_p \rho u_\infty^2$$

free stream velocity: vel. relative to object in same direction as Drag

Corrections to Stokes' Law: The Drag Coefficient

Stokes' law has been derived for $Re \ll 1$, neglecting the inertial terms in the equation of motion.

If Re= 1, the drag predicted by Stokes' law is 13% low, due to the errors introduced by the assumption that inertial terms are negligible.

To account for these terms, the drag force is usually expressed in terms of an empirical drag coefficient C_D as

$$F_{\text{drag}} = \frac{1}{2} C_D A_p \rho u_\infty^2$$

Diameter, μm	Re
0.1	7×10^{-9}
1	2.8×10^{-6}
10	2.5×10^{-3}
20	0.02
60	0.4
100	2
300	20

where A_p is the projected area of the body normal to the flow

Thus for a spherical particle of diameter D_p

$$F_{\text{drag}} = \frac{1}{8} \pi C_D \rho D_p^2 u_\infty^2$$

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where the following correlations are available for the drag coefficient as a function of the Reynolds number:

$$C_D = \frac{24}{\text{Re}} \qquad \text{Re} \lesssim 1 \quad \text{(Stokes' law)}$$

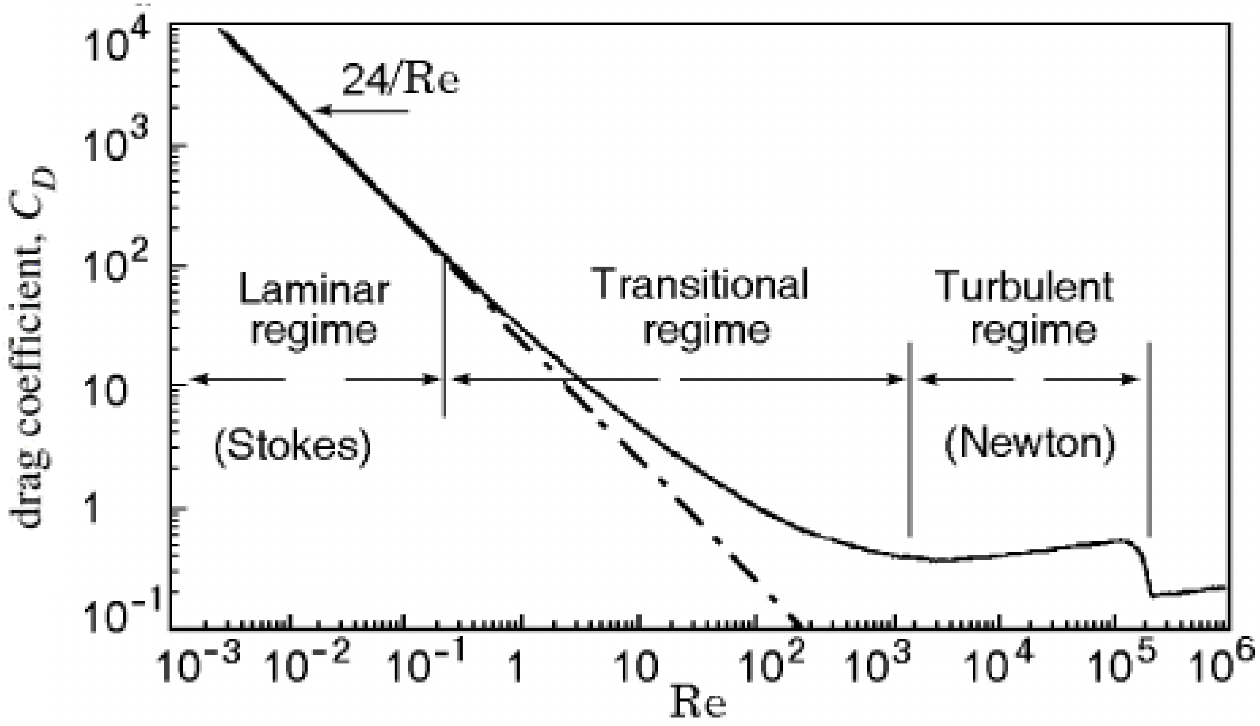
$$C_D = 18.5\ \text{Re}^{-0.6} \qquad \text{Re} \gtrsim 1$$

Note for C_D = 24/Re, the drag force calculated by (above Eq.) is equivalent to Stokes' law. $F_{drag} = 3\Pi\mu D_p u_\infty$

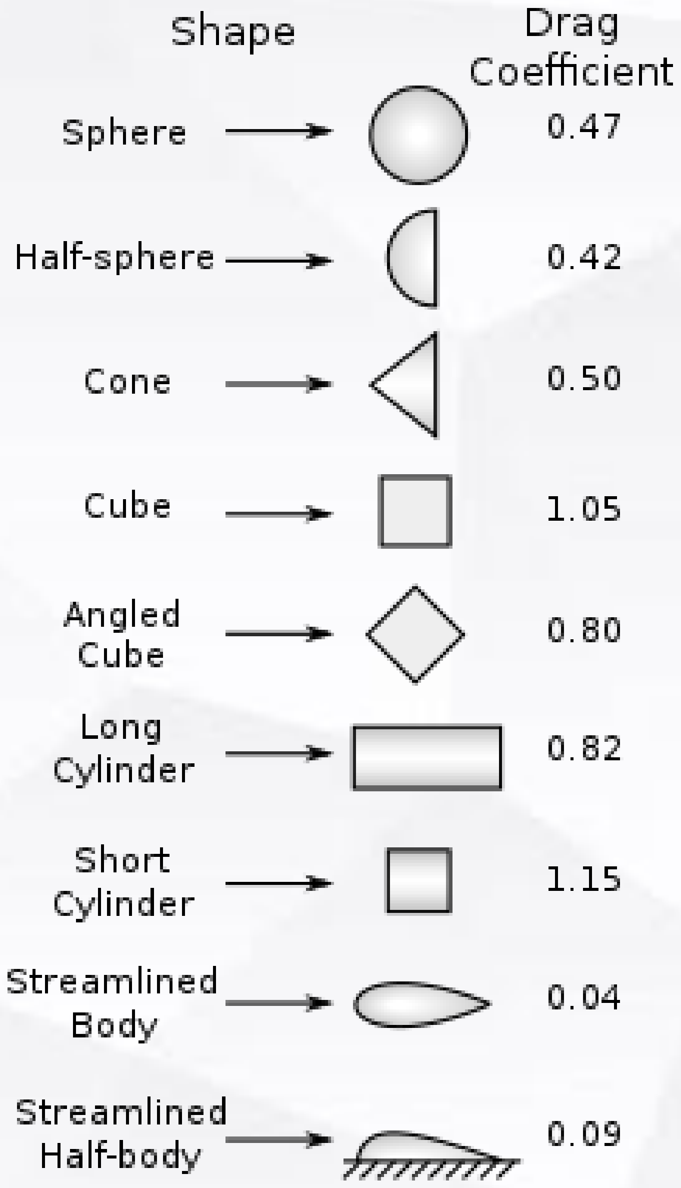
$$Re = u_\infty D_p \rho / \mu$$

Drag coefficient of a sphere as a function of Reynolds number

$$F_{drag} = C_D \rho g d^2 v^2$$



Drag coefficients in fluids with Reynolds number approximately 10^4



Measured Drag Coefficients

Reynolds Number for Particles in Air Falling at Their Terminal Velocities at 298 K

Diameter, μm	Re
0.1	7×10^{-9}
1	2.8×10^{-6}
10	2.5×10^{-3}
20	0.02
60	0.4
100	2
300	20

$$F_{\text{drag}} = \frac{3\pi \mu u_\infty D_p}{C_c}$$

$$C_c = 1 + \frac{2\lambda}{D_p}\left[1.257 + 0.4 \exp\left(-\frac{1.1 D_p}{2\lambda}\right)\right]$$

Creeping vs Inviscid Flows

Creeping Flows

Viscosity goes to ∞ (very low Reynolds number)

Left hand side of the momentum equation is not important (can be taken to vanish).

Friction is more important than inertia.

Inviscid Flows

Viscosity goes to zero (very large Reynolds number)

Left hand side of the momentum equation is important. Right hand side of the momentum equation includes pressure only.

Inertia is more important than friction.

Values of Cc as a function of the particle diameter Dp in air at 25°C are given in Table:

Slip Correction Factor *Cc* for Spherical Particles in Air at 298 K and 1 atm

D_p, µm	C_c
0.001	216
0.002	108
0.005	43.6
0.01	22.2
0.02	11.4
0.05	4.95
0.1	2.85
0.2	1.865
0.5	1.326
1.0	1.164
2.0	1.082
5.0	1.032
10.0	1.016
20.0	1.008
50.0	1.003
100.0	1.0016

The slip correction factor is generally neglected for particles exceeding 10 µm in diameter, as the correction is less than 2%.

On the other hand, the drag force for a 0.1 µm in diameter particle is reduced by almost a factor of 3 as a result of this slip correction.

GRAVITATIONAL SETTLING OF AN AEROSOL PARTICLE

To derive the equation of motion for a particle of mass m_p, let us begin with a force balance on the particle, which we write in vector form as

$$m_p \frac{d\mathbf{v}}{dt} = \sum_i \mathbf{F}_i$$

For a particle falling in a fluid there are two forces acting on it, the gravitational force $m_p g$ and the drag force F_{drag}. Therefore, for Re < 0.1, the equation of motion becomes

$$m_p \frac{d\mathbf{v}}{dt} = m_p\mathbf{g} + \frac{3\pi\mu D_p}{C_c}(\mathbf{u} - \mathbf{v})$$

This equation can be rewritten as

$$\tau \frac{d\mathbf{v}}{dt} = \tau\mathbf{g} + \mathbf{u} - \mathbf{v} \qquad \tau = \frac{m_p C_c}{3\pi\mu D_p}$$

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Let us consider the case of a particle in a quiescent fluid (u = 0) starting with zero velocity and let us take the *z* axis as positive downward.

Then the equation of motion becomes $$\tau \frac{dv_z}{dt} = \tau g - v_z \quad v_z(0) = 0$$

and its solution is $$v_z(t) = \tau g[1 - \exp(-t/\tau)]$$

For $t \gg \tau$, the particle attains a characterstic velocity, called its terminal settling velocity $v_t = \tau g$ or

$$\tau = \frac{m_p C_c}{3\pi\mu D_p} \qquad v_t = \frac{m_p C_c g}{3\pi\mu D_p} \quad *$$

For a spherical particle of density ρ_p in a fluid of density ρ,

$$m_p = (\pi/6) D_p^3 (\rho_p - \rho)$$

where the factor $(\rho_p - \rho)$ is needed to account for both gravity and buoyancy.

since generally $\rho_p \gg \rho$ $$m_p = (\pi/6) D_p^3 \rho_p$$

and * can be rewritten in the more convenient form: $$v_t = \frac{1}{18} \frac{D_p^2 \rho_p g C_c}{\mu}$$

The timescale τ indicates the time required by the particle to reach this terminal settling velocity and is given in Table.

Characteristic Time Required for Reaching Terminal Settling Velocity

The relaxation time x also describes the time required by a particle entering a fluid stream, to approach the velocity of the stream.

Thus the characteristic time of most particles of interest to achieve steady motion in air is extremely short.

D_p, µm	τ, s
0.05	4×10^{-8}
0.1	9.2×10^{-8}
0.5	1×10^{-6}
1.0	3.6×10^{-6}
5.0	7.9×10^{-5}
10.0	3.14×10^{-4}
50.0	7.7×10^{-3}

Diameter, μm	Re
0.1	7×10^{-9}
1	2.8×10^{-6}
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Our analysis so far is applicable to Re < 0.1 or particles smaller than about 20 μm (Table).

For larger particles, one needs to use the drag coefficient as an empirical means of representing the drag force for higher Reynolds numbers.

The equation along the direction of motion of the particle in scalar form, assuming no gas velocity, is then

$$m_p \frac{dv_z}{dt} = m_p g - \frac{1}{8}\pi \frac{C_D}{C_c} \rho D_p^2 v_z^2$$

At steady-state $v_z = v_t$, the particle reaches its terminal velocity given by $v_t = \left(\frac{4gD_pC_c\rho_p}{3C_D\rho}\right)^{1/2}$

However, as C_D is a function of Re and therefore v_t, we have only an implicit expression for v_t in the equation.

One needs then to solve $v_t = \left(\frac{4gD_pC_c\rho_p}{3C_D\rho}\right)^{1/2}$ numerically with C_D calculated by

$$C_D = \frac{24}{\mathrm{Re}} \qquad \mathrm{Re} \lesssim 1 \quad \text{(Stokes' law)}$$

$$C_D = 18.5\ \mathrm{Re}^{-0.6} \qquad \mathrm{Re} \gtrsim 1$$

or one can use the following technique:

If we form the product $C_D\,\mathrm{Re}^2 = \frac{C_D v_t^2 D_p^2 \rho^2}{\mu^2}$

and substitute into this the v_t given by above equation, we obtain $C_D\,\mathrm{Re}^2 = \frac{4D_p^3\rho\rho_p g C_c}{3\mu^2}$

C_DRe^2 can be calculated from

$$C_D = \frac{24}{\text{Re}} \qquad \text{Re} \lesssim 1 \quad \text{(Stokes' law)}$$

$$C_D = 18.5\ \text{Re}^{-0.6} \qquad \text{Re} \gtrsim 1$$

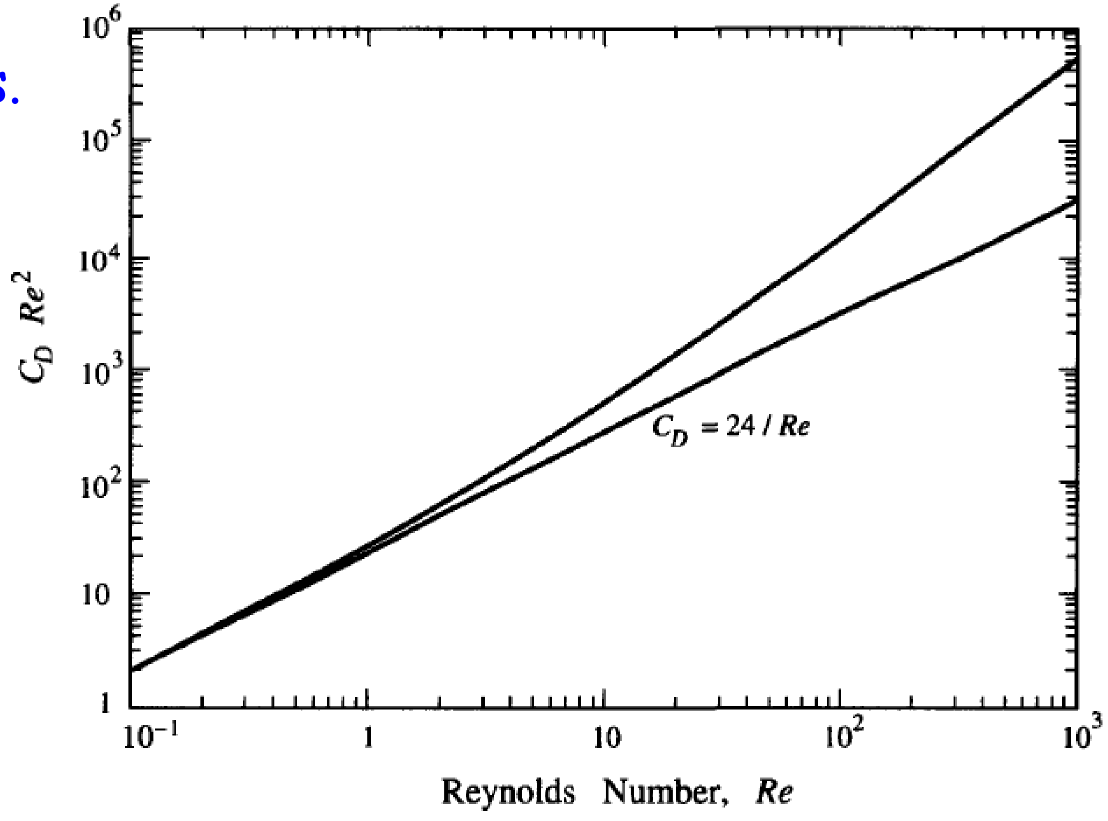
The terminal velocity can now be calculated as follows.

First, we calculate C_DRe^2 using $C_D\,\text{Re}^2 = \dfrac{4D_p^3\rho\rho_p g C_c}{3\mu^2}$

Then using Figure, we calculate Re.

Then $v_t = \dfrac{\mu \text{Re}}{\rho D_p}$

and there is no need to solve the system of nonlinear algebraic equations.



C_DRe^2 as a function of Re for a sphere