Atmospheric Dynamics

Lecture 8

Sahraei

Physics Department

Razi University

https://sci.razi.ac.ir/~sahraei

Potential Vorticity

$$g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p}) = const$$

Ertel potential Vorticity

Isentropic Potential Vorticity

$$\frac{\zeta + f}{h} = const$$

Barotropic (Rossby) Potential Vorticity Equation

Conservation of Potential Vorticity

The conservation of potential vorticity couples changes in depth, relative vorticity, and changes in latitude. All three interact.

By using isentropic surfaces, atmospheric scientists are able to reduce the problem of tracking air parcel motion from a three dimensional (latitude, longitude, altitude) problem to a two dimensional (latitude, longitude) problem on an isentropic surface.

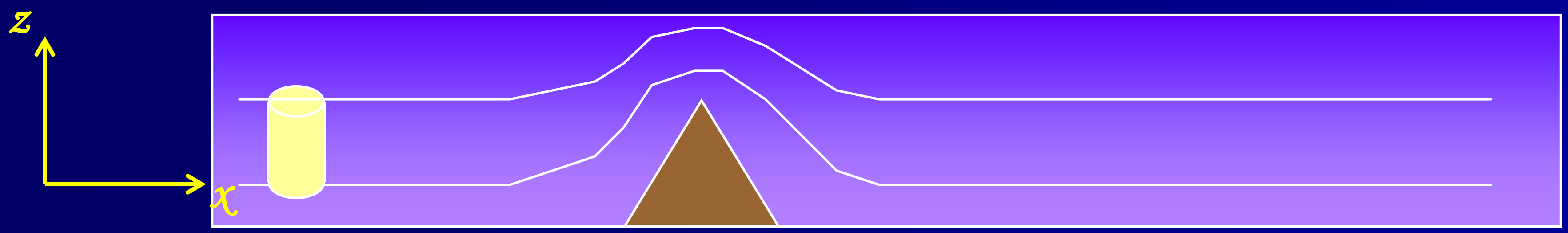
Conserved quantities

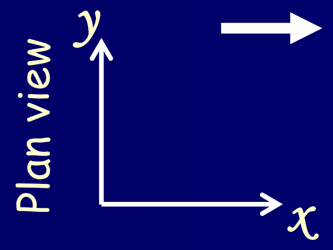
In this section, we explore the concepts of potential temperature and potential vorticity, two central concepts in understanding dynamical meteorological processes.

Both these quantities, potential temperature and potential vorticity, are referred to as conserved quantities. Like mixing ratio, they are quantities that are invariant or fixed for a particular air parcel even as the parcel moves about. Because of its fixed value, it allows the individual air parcel to be traced, and hence the conserved property acts as a tracer (see section 2.2).

Mountain lee waves

Assume air with no vorticity moving west to east towards a mountain range

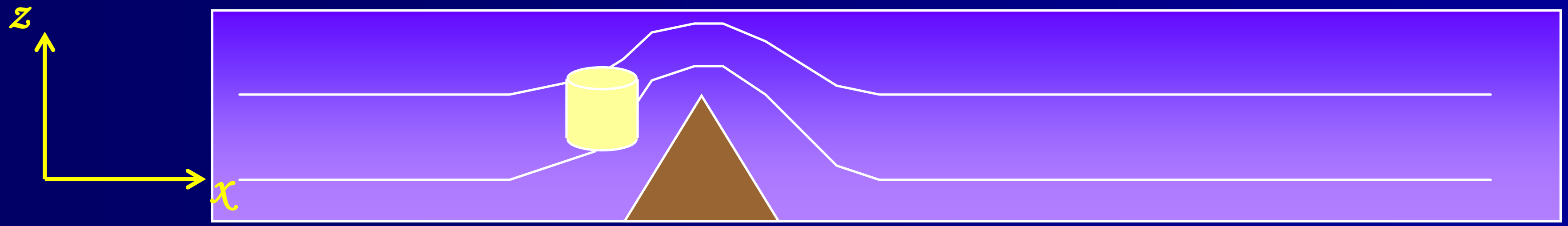


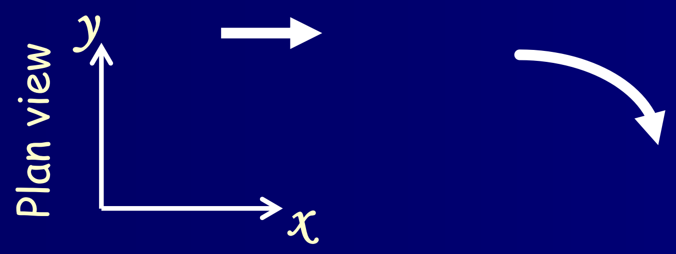


$$\frac{\zeta}{h} = const$$

Vortex shrinks, h reduced

ζ becomes -ve, anticyclonic spin

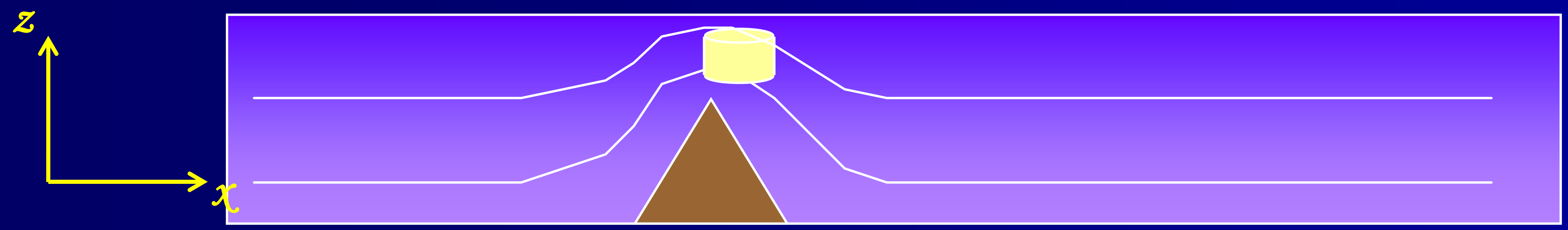




$$\frac{\zeta}{h} = const$$

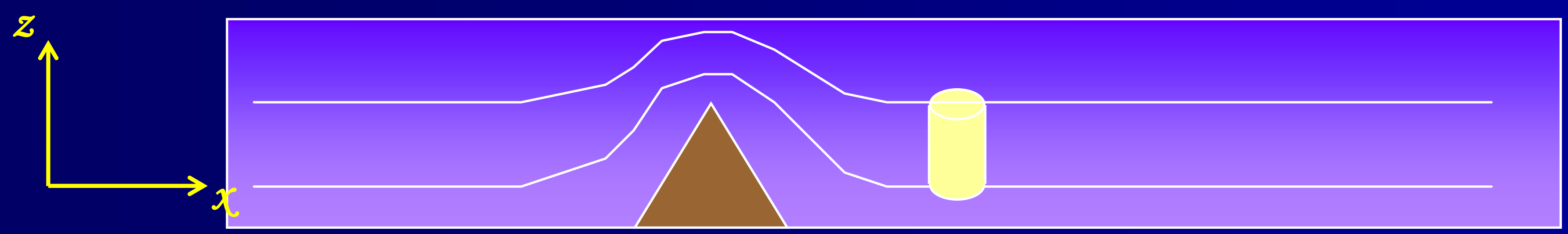
Vortex starts to stretch, h increases

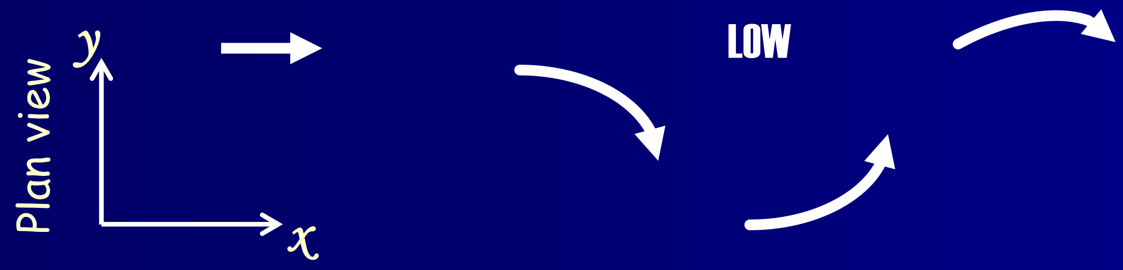
ζ becomes +ve, cyclonic spin



$$\frac{\zeta}{h} = const$$

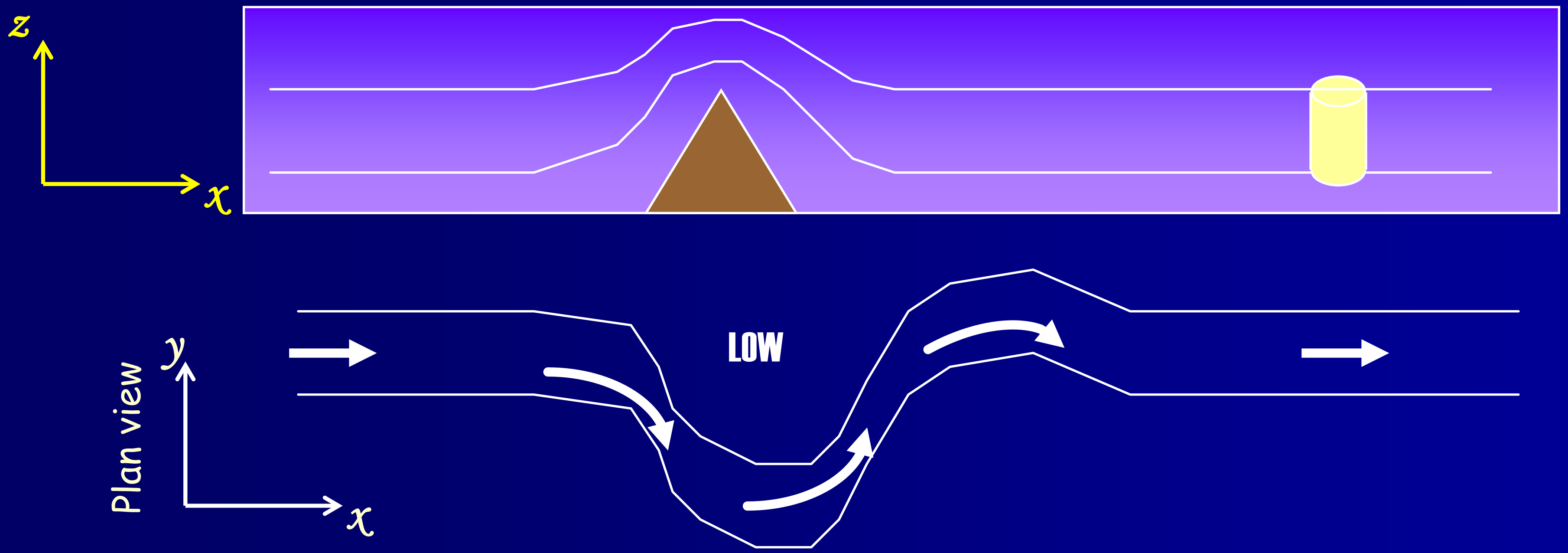
Back to original trajectory





Mountain lee waves

Trough of low pressure in the lee of the mountain range



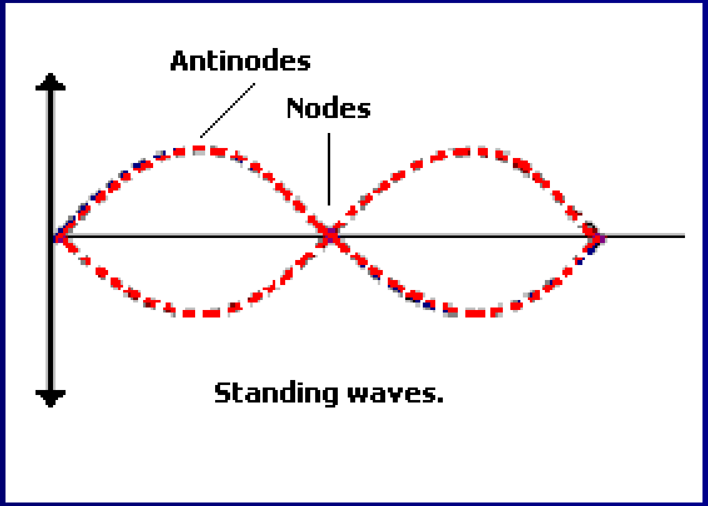
Lee wave: a standing wave generated on the sheltered side of a mountain by an air current passing over or around it, and often made visible by the formation of clouds. (The lee side is the side that is downstream from the wind.)

Lee waves cause updrafts and downdrafts

The wave appears stationary because its nodes and anti-nodes do not move.

What is actually happening though is that there is a wave called an incident wave created by the vibrations, which reaches the end of the medium and bounces back as a reflected wave.

The two waves occur at the same time with continued vibration and combine to form the pattern you can see.



$$g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p}) = const$$

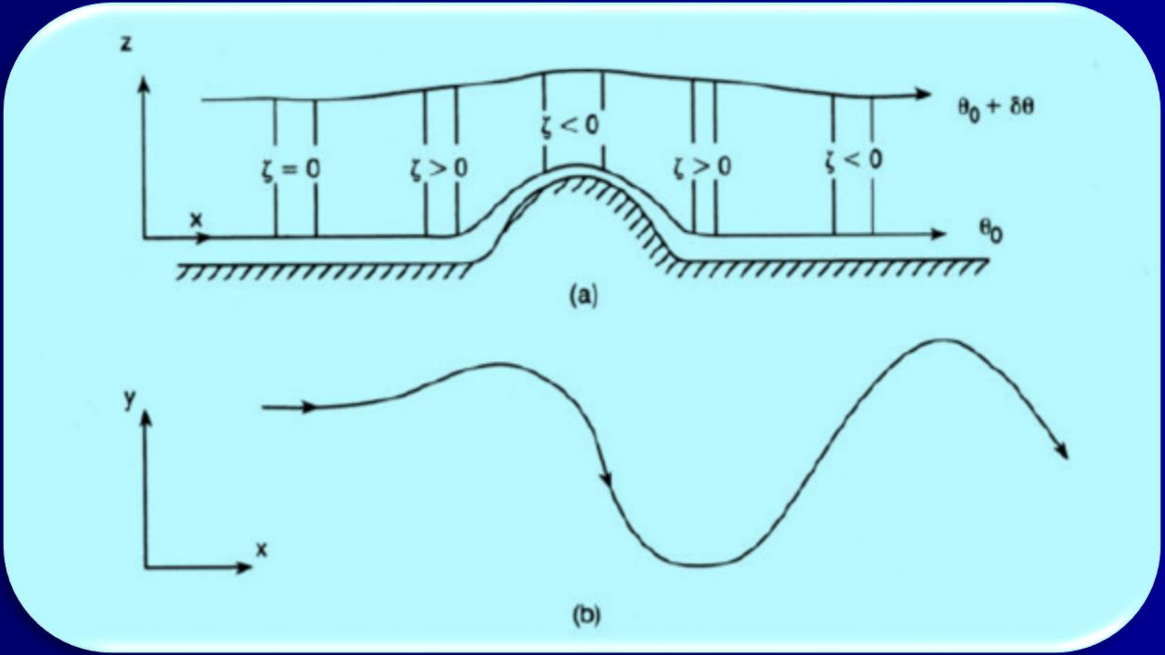
Conservation of potential vorticity

$$\frac{\zeta + f}{h} = const$$

Westerly Flow Over a Barrier

Consider a westerly flow of air encountering a north-south mountain barrier. Upstream of the barrier, assume the flow is zonal and uniform, thus $\zeta = 0$.

When the depth of the vortex changes following motion, its absolute vorticity must change to maintain conservation of potential vorticity



Conservation of potential vorticity

$$\frac{\zeta + f}{h} = const$$

For westerly flow impinging on an infinitely long mountain range…

(a) upstream, zonal flow is uniform

$$(\delta u / \delta y = 0, \quad v = 0), \quad \zeta = 0$$

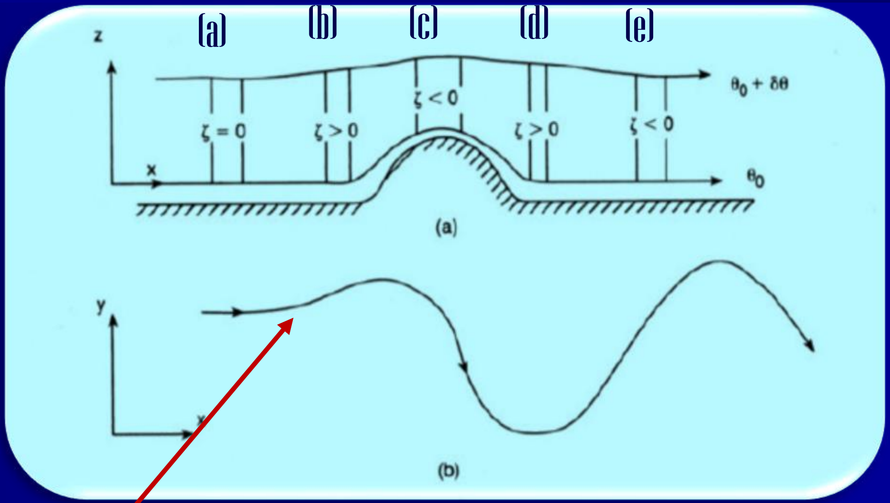
(b) deflection of upper θ surface upstream of barrier

→ *increases h*

→ absolute vorticity must increase

→ air column turns cyclonically

$$-\frac{\partial \theta}{\partial p} \quad decreases$$



(Holton 2004, p. 98)

$$g(\zeta_\theta + f)(-\frac{\partial \theta}{\partial p}) = const$$

$$g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p}) = const$$

Conservation of potential vorticity

$$\frac{\zeta + f}{h} = const$$

For westerly flow impinging on an infinitely long mountain range…

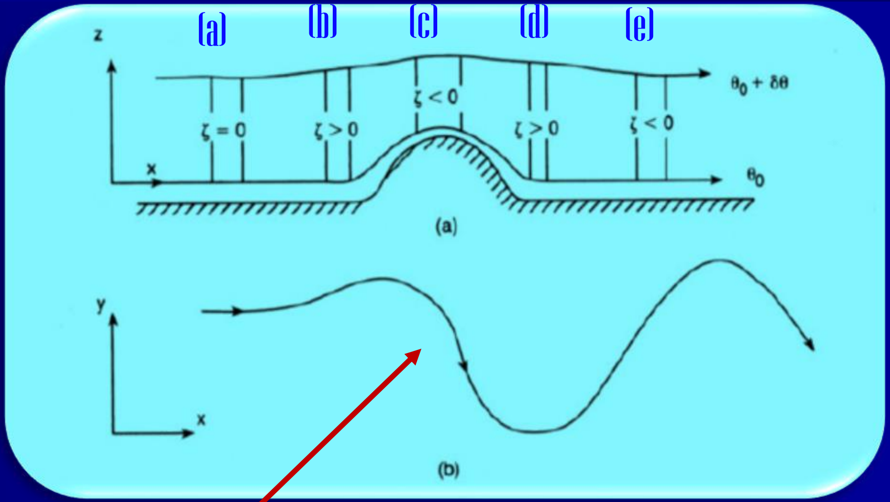
poleward drift in (b) also causes increase in f

(c) as column crosses mountain, h decreases

→absolute vorticity must decrease

→ ζ becomes negative

→air column drifts equatorward

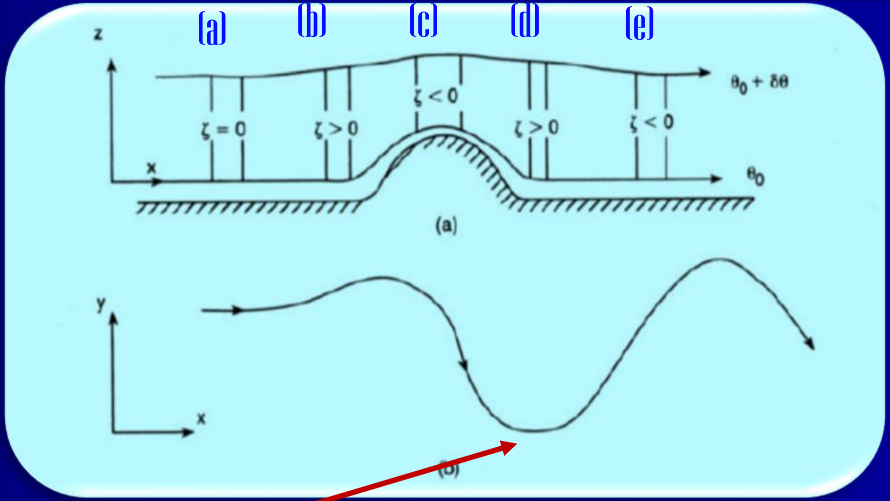


$$g(\zeta_\theta + f)(-\frac{\partial \theta}{\partial p}) = const$$

Conservation of potential vorticity

$$\frac{\zeta + f}{h} = const$$

equatorward drift in (c) also causes decrease in *f*



(d) as column crosses mountain, *h increases*

→ absolute vorticity must increase

→ ζ becomes positive

→ air column drifts poleward

When the air column has passed over the barrier and returned to its original depth, it will be south of its original latitude so that *f* will be smaller and the relative vorticity must be positive.

Thus, the trajectory must have cyclonic curvature and the column will be deflected poleward. When the parcel returns again to its original latitude, it will still have a poleward velocity component and will continue poleward gradually, acquiring anticyclonic curvature until its direction is again reversed.

The parcel will then move downstream, conserving potential vorticity by following a wave-like trajectory in the horizontal plane.

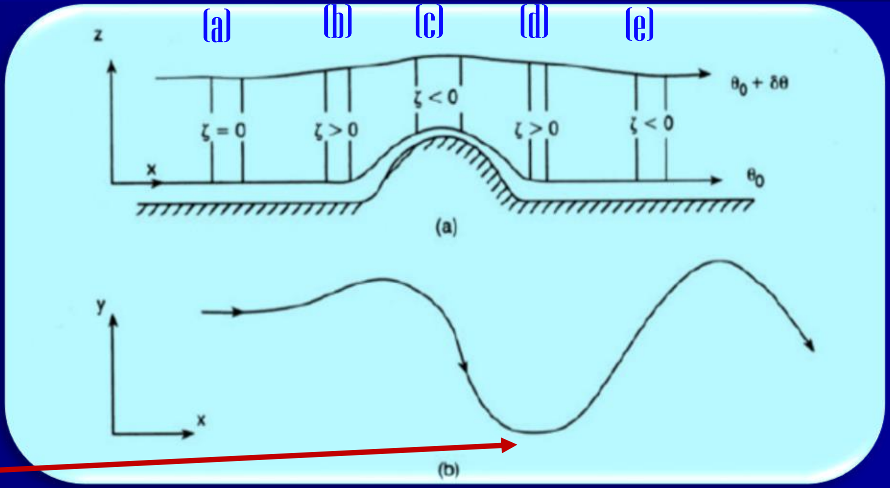
Therefore, steady westerly flow over a large-scale ridge will result in an anticyclonic flow over the mountain, a cyclonic flow pattern to the east of the barrier, followed by wave train downstream.

$$g(\zeta_\theta + f)(-\frac{\partial \theta}{\partial p}) = const$$

Conservation of potential vorticity

$$\frac{\zeta + f}{h} = const$$

(e) alternating series of ridges and troughs downstream of mountain range

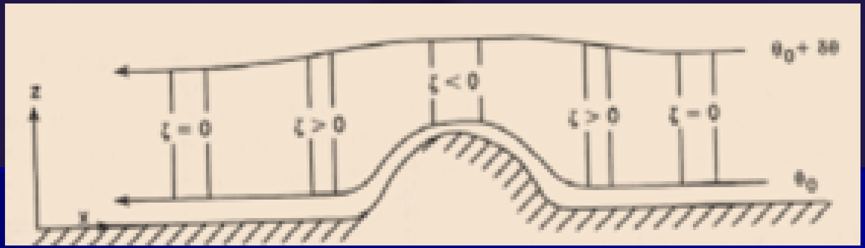


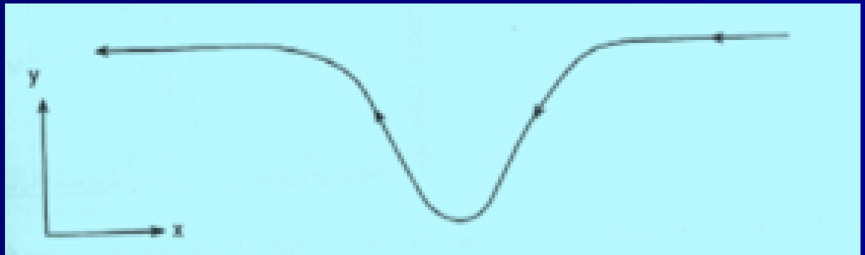
cyclonic flow pattern immediately to the east of the mountains (lee side trough)

The cyclonic flow downstream of a mountain barrier is known as a lee-side mountain trough, or lee trough.

Easterly Flow Over a Barrier

$$\frac{\zeta + f}{h} = const$$





Thus, the dependence of the Coriolis parameter on latitude creates a dramatic difference between westerly and easterly flow over large-scale topographic barriers.

In the case of a westerly wind, the barrier generates a wave-like disturbance in the streamlines that extends far downstream. However, in the case of an easterly wind, the disturbance in the streamlines damps out away from the barrier.

For a constant absolute vorticity value, as f decreases moving equatorward, cyclonic relative vorticity increases (large positive values)

For a constant absolute vorticity value, as f increases moving poleward, cyclonic relative vorticity decreases (small positive values)

Conclusion:
The dependence of the Coriolis parameter on latitude creates a dramatic difference between westerly and easterly flows over large-scale topographic barriers. For westerly flows, topographic barriers generate wavelike disturbances in the streamlines that extend far downstream. For easterly flows, the disturbance in the streamlines damps out away from the barrier.

Westerly Flow Over a Barrier in the southern hemisphere

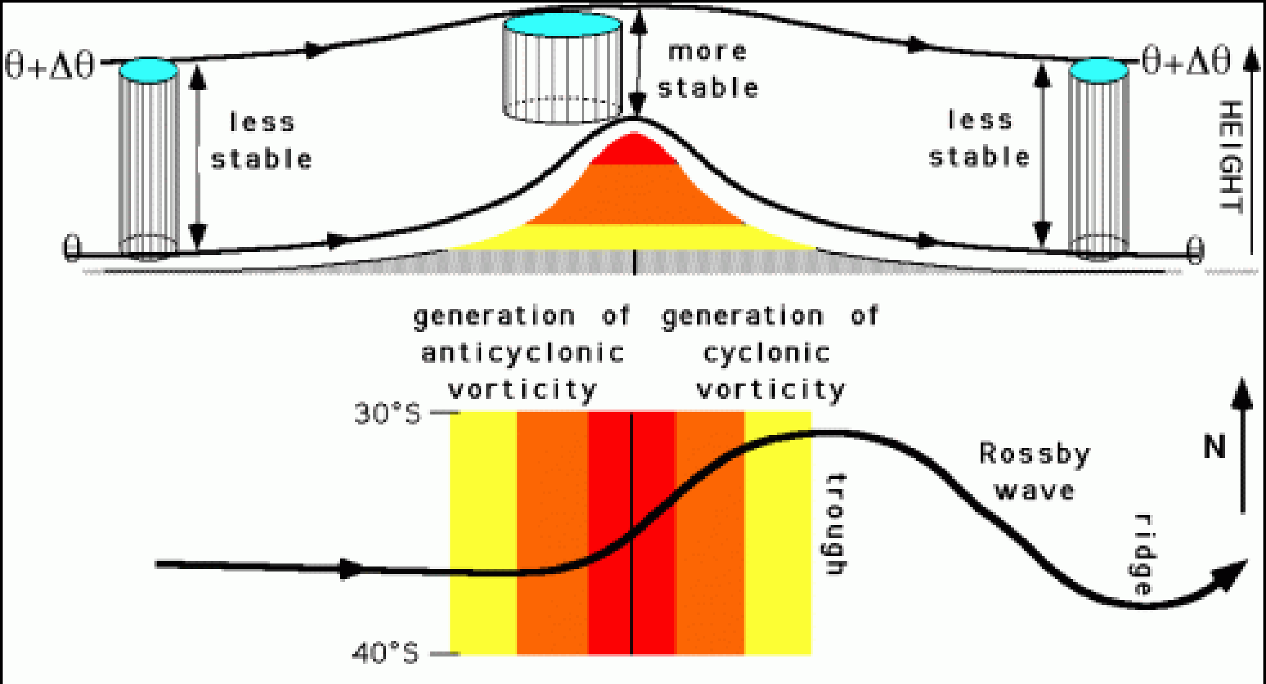
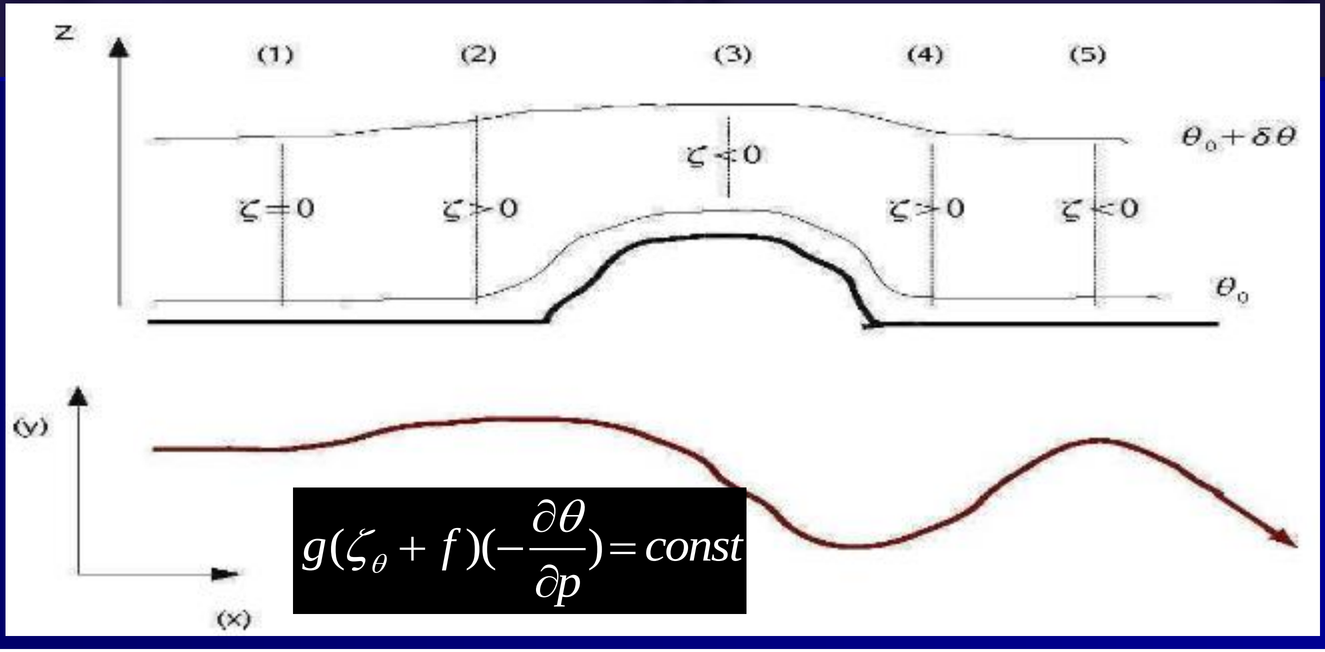


Fig 12.K.4 The formation of a trough in the lee of a mountain range in the southern hemisphere. The top figure is a vertical cross section, and the bottom one is a plan view. The mountain ridge is at the same location in both figures.

Westerly flow

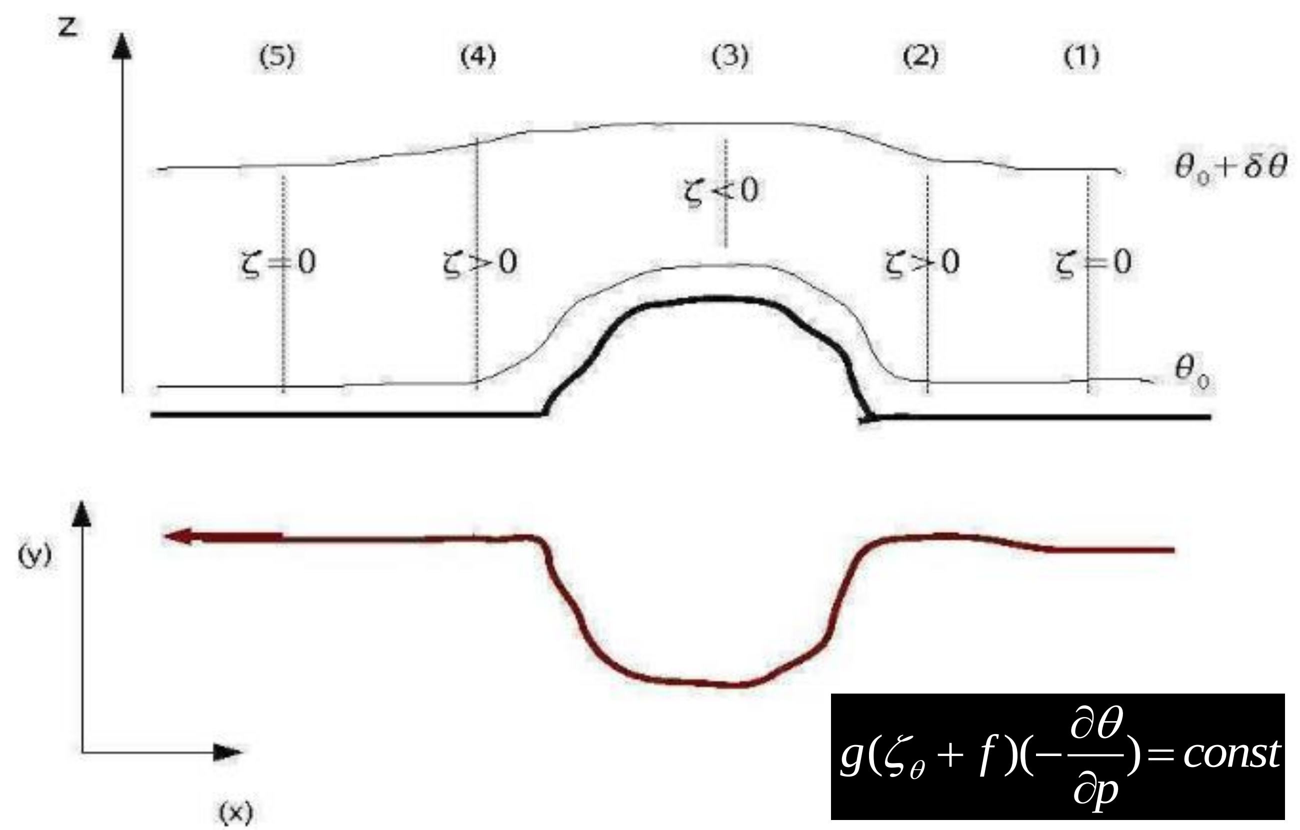
- isentropic surface θ_0 follows approx. the topography
- $\theta_0 + \delta\theta$ isentropic surface is deflected vertically, but the vertical displacement at upper levels is spread horizontally; it extends upstream and downstream of the barrier and has a smaller amplitude in the vertical



$$g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p}) = const$$

(1) zonal flow ($\zeta = 0$)
(2) $\zeta > 0$ (vertical stretching of air columns upstream of the topographic barrier); $-\partial\theta/\partial p$ decreases; ζ must become positive to conserve potential vorticity; air column turns cyclonically as it approaches the topographic barrier; poleward drift-f increases)
(3) $\zeta < 0$ (vertical extent of the air parcel decreases; relative vorticity must become negative; anticyclonic vorticity and southward displacement)
(4) $\zeta > 0$ (air column has passed over the mountain and returns to original depth; is located south of original latitude; f is smaller and relative vorticity must be positive; the trajectory must have cyclonic curvature)
(5) $\zeta < 0$ (the parcel returns to its original latitude; still has a poleward velocity component and will continue poleward, gradually acquiring anticyclonic curvature until its direction is again reversed)

EASTERLY FLOW



$$g(\zeta_\theta + f)(-\frac{\partial \theta}{\partial p}) = const$$

(1) $\zeta = 0$ (zonal flow)
(2) $\zeta > 0$ (vertical stretching; cyclonic flow; equatorward component of motion; f decreases)
(3) $\zeta < 0$ (vertical contraction; decrease of absolute vorticity due to both anticyclonic relative vorticity and a decrease in f owing to the equatorward motion)
(4) $\zeta > 0$ (the same depth as in (2))
(5) $\zeta = 0$ (purely zonal flow)