Atmospheric Dynamics

Lecture 7

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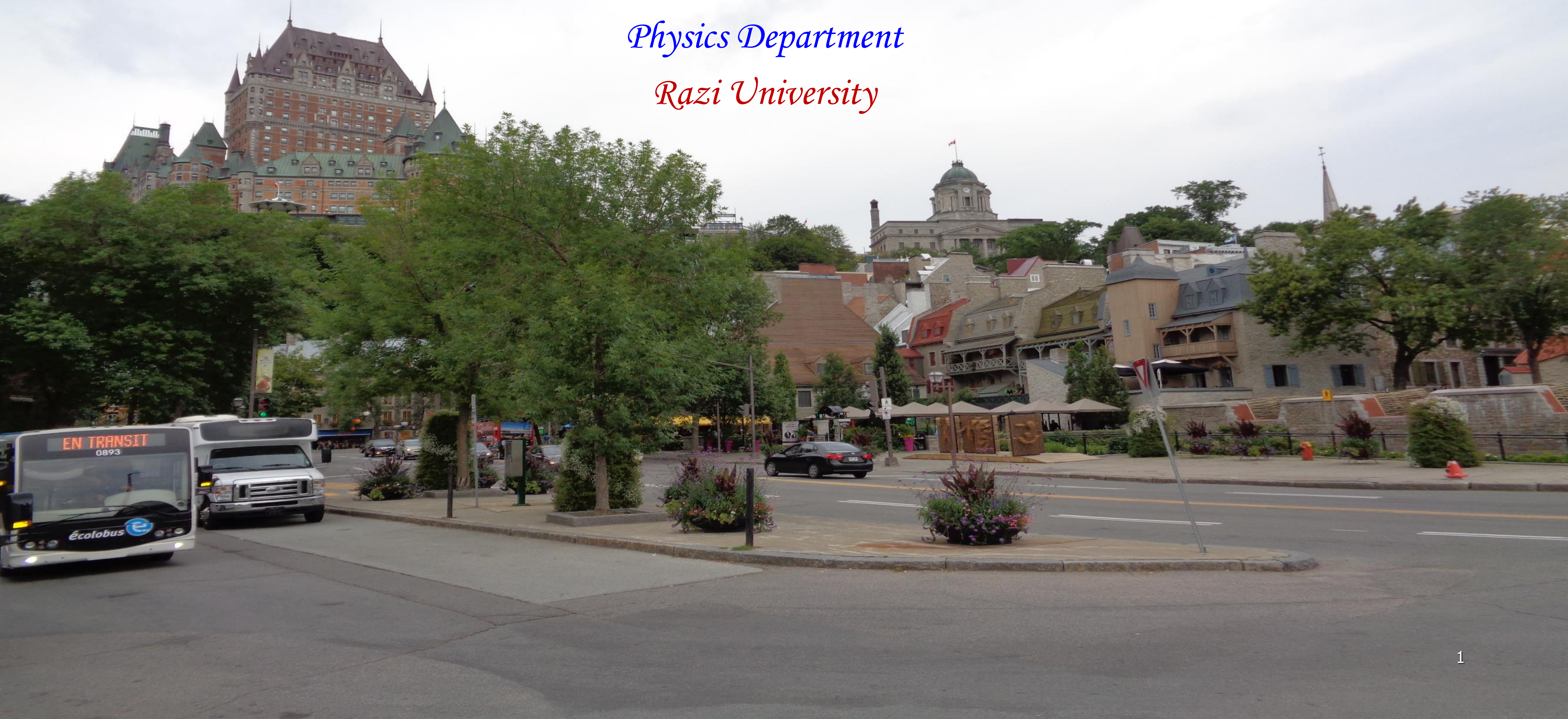


Conservation of Potential Vorticity

The conservation of potential vorticity couples changes in depth, relative vorticity, and changes in latitude. All three interact.

Changes in the depth *h* of the flow causes changes in the relative vorticity. The concept is analogous with the way figure skaters decreases their spin by extending their arms and legs.

$$\frac{\zeta + f}{h} = const \qquad PV_E = g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p})$$

The conservation of potential vorticity is the air's equivalent of the conservation of angular momentum.

$$PV_E = g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p})$$

Ertel potential Vorticity

Isentropic Potential Vorticity

Relative vorticity is zero for stationay atmosphere

$$\zeta_\theta + f = 0 + f = f$$

$$PV_s = -gf\frac{\partial\theta}{\partial p}$$

$$g = 10m/s^2 \qquad f = 10^{-4}s^{-1} \qquad \frac{\partial\theta}{\partial p} = \frac{10K}{100hpa}$$

$$PV_s \cong 10m/s^2 \times 10^{-4}s^{-1} \times \frac{10K}{1000Pa} = 10^{-6}m^2s^{-1}Kkg^{-1}$$

Isentropic potential vorticity is of the order of:

$$P_s = 10^{-6} m^2 s^{-1} K kg^{-1} = 1 PVU$$

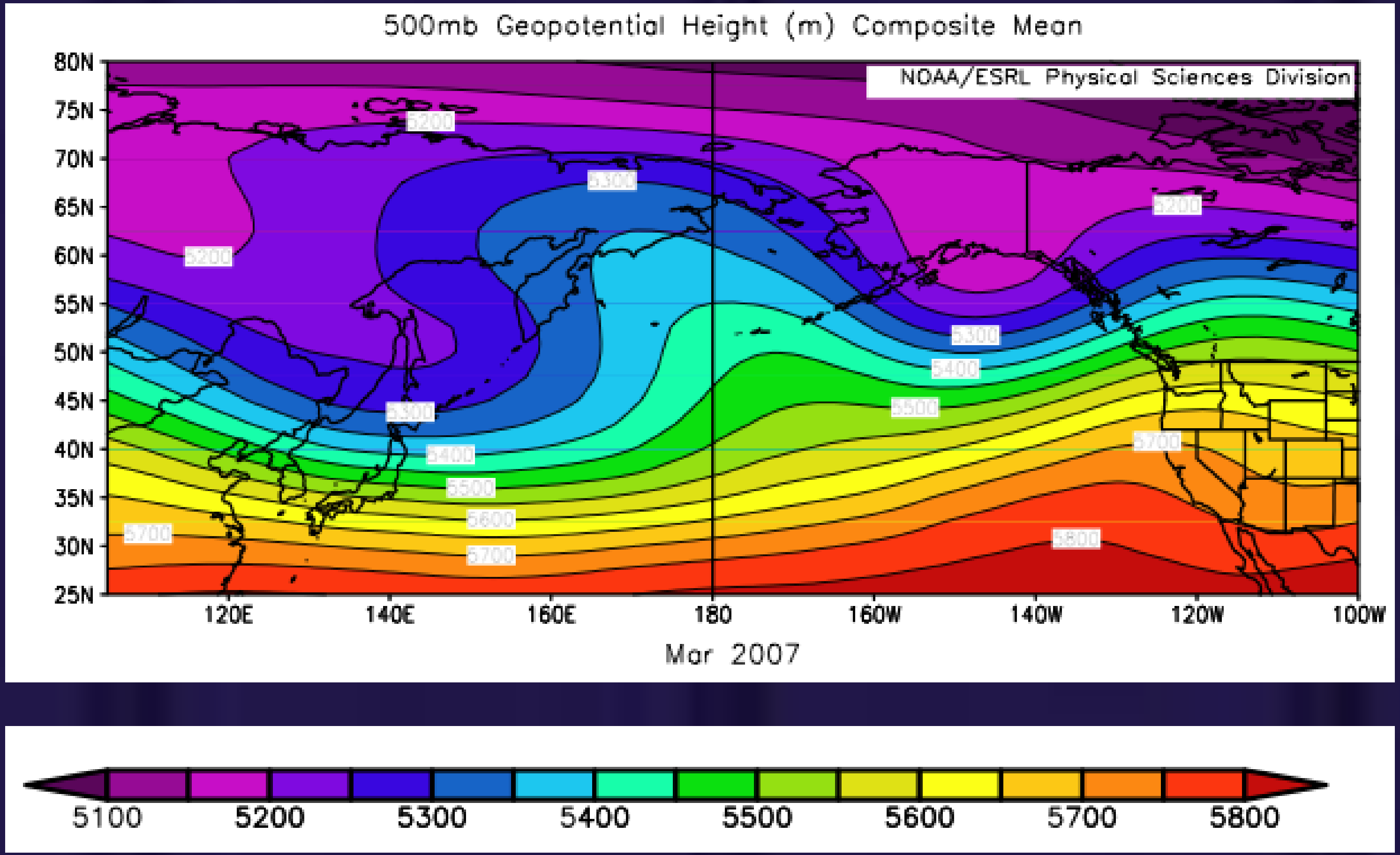
Potential Vorticity Unit

Values of IPV < 1.5 PVU are generally associated with tropospheric air

Values of IPV > 1.5 PVU are generally associated with stratospheric air

Comparison to Isobaric Analyses:

Regions of low geopotential heights correspond to regions with large PV values



Under the assumption of a three dimensional balance between the fields of mass, pressure and wind,

positive PV anomalies are connected with cyclonic vorticity

and negative PV anomalies with anticyclonic vorticity.

In the case of a positive PV anomaly, the isentropes are characterised by higher values than in the surrounding areas (indicating colder air).

The height of the tropopause has a local minimum. The corresponding pressure and wind field shows an area with low pressure and a cyclonic circulation.

In the case of a negative PV anomaly, the isentropes are characterised by lower values than in the surrounding areas (indicating warmer air).

The height of the tropopause has a local maximum. The corresponding pressure and wind field shows an area with high pressure and an anticyclonic circulation.

Consevation of absolute vorticity

Conservation of absolute vorticity following the motion provides a strong constraint on the flow, as can be shown by a simple example that again illustrates an asymmetry between westerly and easterly flow.

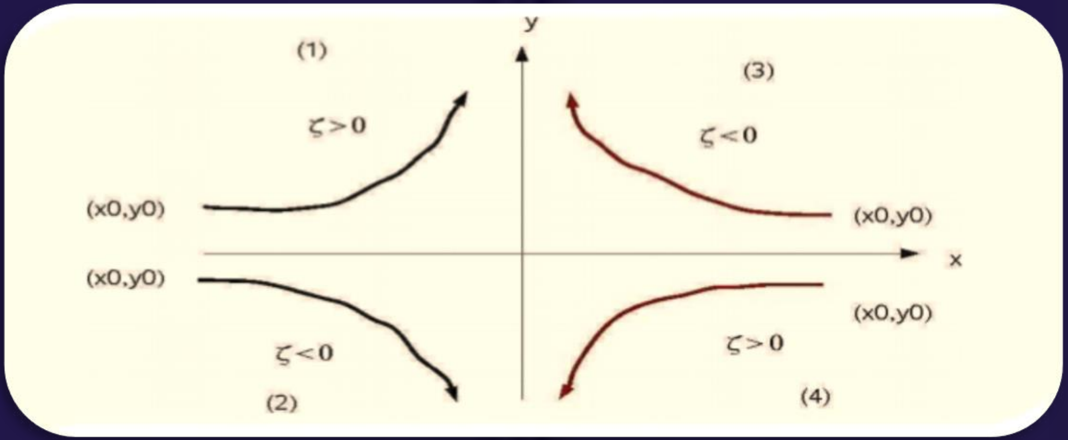
Suppose that at a certain point (x_0, y_0) the flow is in the zonal direction and the relative vorticity vanishes $(\zeta = 0)$ so that

$$\text{absolute vorticity} \quad \eta = \zeta + f \quad \rightarrow \quad \eta(x_0, y_0) = f_0$$

Then, if absolute vorticity is conserved, the motion at any point along a parcel trajectory that passes through (x_0, y_0) must satisfy

$$\zeta + f = f_0$$

$$\eta(x_0, y_0) = f_0$$



(1) $\zeta > 0$; f increases; $\zeta + f > f_0$; η not conserved

(2) $\zeta < 0$; f decreases; $\zeta + f < f_0$; η not conserved

(3) $\zeta < 0$; f increases; $\zeta + f = f_0$; η conserved

(4) $\zeta > 0$; f decreases; $\zeta + f = f_0$; η conserved

- for westerly flows (wind blows from west to east) the motion has to remain purely zonal if the absolute vorticity is conserved.

- for easterly flows (wind blows from east to west) conservation of absolute vorticity is possible both for northward and southward curvature.

Whereas trajectories that curve southward must have

$$\zeta = f_0 - f > 0$$

However, as indicated in the figure if the flow is westerly, northward curvature downstream implies

$$\zeta > 0$$

whereas southward curvature implies $\zeta < 0$

Thus, westerly zonal flow must remain purely zonal if absolute vorticity is to be conserved following the motion.

The easterly flow case, also shown in Fig. Is just the opposite.

Northward and southward curvatures are associated with negative and positive relative vorticities, respectively.

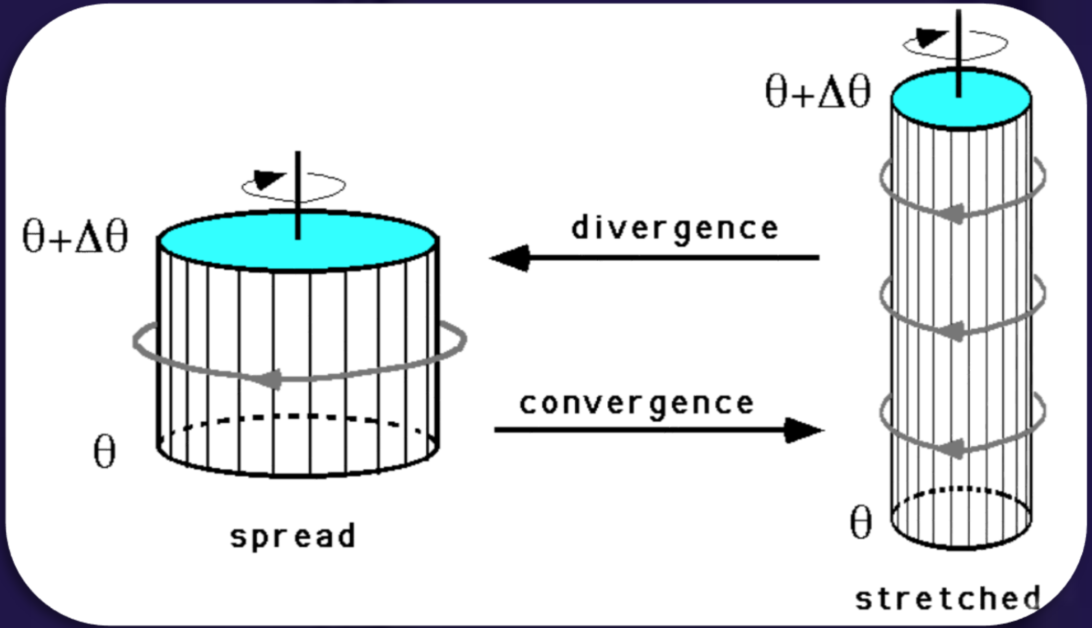
Hence, an easterly current can curve either to the north or to the south and still conserve absolute vorticity.

Because f increases toward the north, trajectories that curve northward in the downstream direction must have

$$\zeta = f_0 - f < 0$$

The potential vorticity can only be changed by diabatic heating or friction

$$\frac{\zeta + f}{h} = const$$



The vortex of air on the left in is broad and slow. When the air converges, the column stretches, i.e. h increases. To maintain potential vorticity, the air spins faster (ξ increases), resulting in the streched vortex on the right.

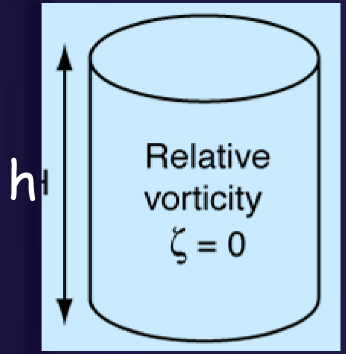
Divergence, on the other hand, causes vortex spreading and slows down the rate of spin.

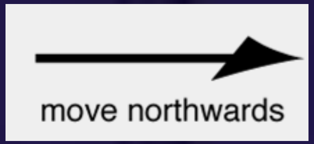
Conservation of potential vorticity (relative plus planetary)

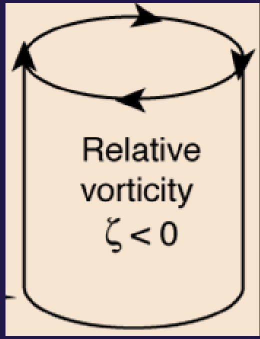
Conservation of potential vorticity in the absence of stretching (N.H.)

(Balance of planetary vorticity and relative vorticity)

$$\frac{\zeta + f}{h} = const$$







$$PV = \frac{f(\varphi_1)}{h}$$

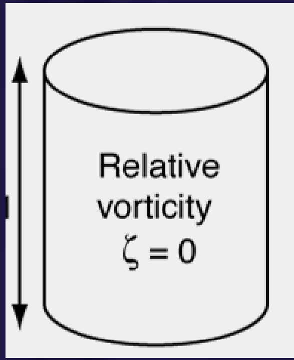
$$PV = \frac{f(\varphi_2) + \zeta}{h}$$

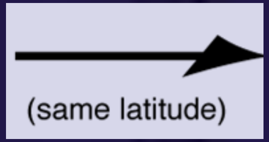
$$\frac{f(\varphi_1)}{h} = \frac{f(\varphi_2) + \zeta}{h}$$

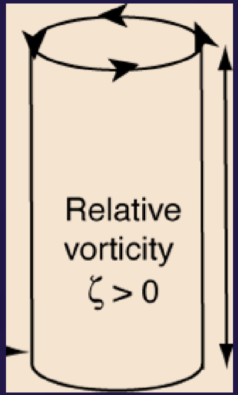
Conservation of potential vorticity (relative and stretching)

Conservation of potential vorticity in the absence of planetary vorticity change (N.H.) (Balance of planetary vorticity and stretching)

h1







h2

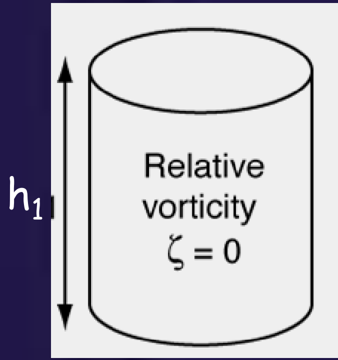
$$PV = \frac{f(\varphi_1)}{h_1}$$

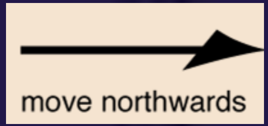
$$PV = \frac{f(\varphi_1) + \zeta}{h_2}$$

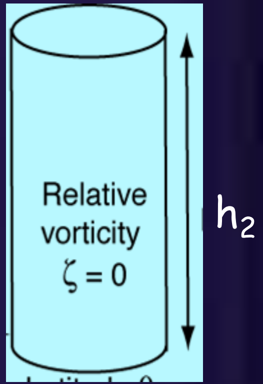
$$\frac{f(\varphi_1)}{h_1} = \frac{f(\varphi_1) + \zeta}{h_2}$$

Conservation of potential vorticity (planetary and stretching)

Conservation of potential vorticity in the absence of relative vorticity change (N.H.) (Balance of planetary vorticity and stretching)







$$PV = \frac{f(\varphi_1)}{h_1}$$

$$PV = \frac{f(\varphi_2)}{h_2}$$

$$\frac{f(\varphi_1)}{h_1} = \frac{f(\varphi_2)}{h_2}$$

$$PV_E = g(\zeta_\theta + f)(-\frac{\partial \theta}{\partial p})$$

As the air flows over the mountain, the potential temperature is conserved, so the 300K isentrope (ϴ in Fig) bends over the mountain. Air aloft, at the 320K isentrope (ϴ+Δϴ), is lifted much less as it passes the range.

Therefore Δϴ is reduced over the mountain chain, and to keep the potential vorticity constant, the absolute vorticity (f + ξ) must be reduced equally.