Atmospheric Dynamics

Lecture 6

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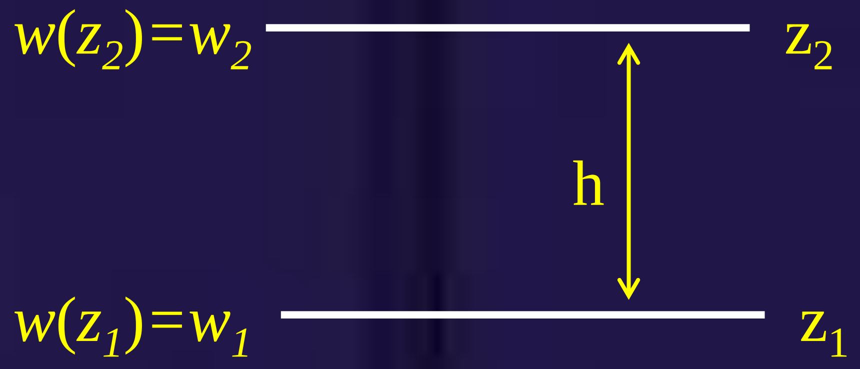
Potential Vorticity

$$\frac{1}{\rho}\frac{d\rho}{dt} + \nabla.\vec{V} = 0 \qquad \rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z}$$



$$\frac{d}{dt}(\zeta + f) \quad = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$$\frac{d}{dt}(\zeta + f) \quad = (\zeta + f)\frac{\partial w}{\partial z} \qquad *$$

$$\frac{\partial w}{\partial z} \approx \frac{\delta w}{\delta z} = \frac{w_2 - w_1}{z_2 - z_1} \quad = \frac{1}{h}(w_2 - w_1)$$

$$\frac{dz}{dt} = w$$

$$\frac{\partial w}{\partial z} \approx \frac{1}{h}\left(\frac{dz_2}{dt} - \frac{dz_1}{dt}\right) \quad = \frac{1}{h}\frac{d}{dt}(z_2 - z_1) \quad = \frac{1}{h}\frac{dh}{dt}$$

$$\frac{d(\zeta+f)}{dt} = (\zeta+f)\frac{1}{h}\frac{dh}{dt}$$

Absolute Vorticity

$$\frac{1}{(\zeta+f)}\frac{d(\zeta+f)}{dt} - \frac{1}{h}\frac{dh}{dt} = 0$$

$$\frac{d}{dt}[\ln(\zeta+f) - \ln h] = 0$$

$$\frac{d}{dt}\left[\ln\left(\frac{\zeta+f}{h}\right)\right] = 0$$

$$\frac{1}{\left(\frac{\zeta+f}{h}\right)}\frac{d}{dt}\left(\frac{\zeta+f}{h}\right) = 0$$

$$\frac{\zeta+f}{h} \neq 0$$

$$\frac{d}{dt}\left(\frac{\zeta+f}{h}\right) = 0$$

Barotropic (Rossby) Potential Vorticity Equation

$$\boxed{\frac{\zeta+f}{h} = const}$$

For a homogeneous, incompressible fluid flow

$$PV_R = \frac{\zeta + f}{h} \qquad \frac{d}{dt}(\zeta + f) = 0 \qquad \zeta + f = const$$

Which states the absolute vorticity is conseved following the horizontal motion.

Example (1)

$$(\zeta + f)_0 = (\zeta + f)_1$$

$$\zeta_0 + f_0 = \zeta_1 + f_1$$

$$5 \times 10^{-5} + 2\Omega \sin 3\,0 = \zeta_1 + 2\Omega \sin 9\,0$$

$$\zeta_1 = -2.3 \times 10^{-5} s^{-1}$$

Example (2)

$$\left(\frac{\zeta + f}{h}\right)_0 = \left(\frac{\zeta + f}{h}\right)_1$$

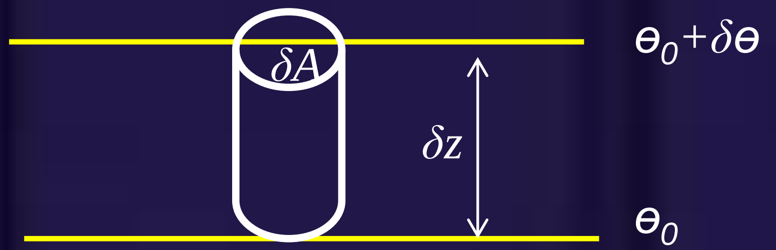
$$\frac{0 + 2\Omega \sin 6\,0}{10km} = \frac{\zeta_1 + 2\Omega \sin 4\,5}{(10 - 2.5)km}$$

$$\zeta_1 = -8.4 \times 10^{-6} s^{-1}$$

$$\zeta_{a1} = \zeta_1 + f_1 = 9.5 \times 10^{-5} s^{-1}$$

Ertel potential Vorticity

$$\delta m = \rho \delta V = \rho \delta A \delta z$$



$$\frac{\delta p}{\delta z} = -\rho g$$

$$\delta m = \rho \delta A \left(-\frac{\delta p}{\rho g}\right)$$

$$\frac{\delta m}{\delta \theta} = const$$

$$\delta A = -g \frac{\delta m}{\delta \theta} \frac{\delta \theta}{\delta p}$$

$$\delta A = \left(-g \frac{\delta \theta}{\delta p}\right) \times const$$

Kelvin's Circulation Theorem

Adiabatic flow can be described by Kelvin's circulation theorem:

$$\frac{d}{dt}(\delta C + 2\Omega\delta A\sin\varphi) = 0$$

where δC is evaluated for a closed loop encompassing the area δA on an isentropic surface.

The vertical component of vorticity is given by $\zeta_\theta = \frac{\delta C}{\delta A},$

thus if the isentropic surface is approximately horizontal, for an infinitesimal parcel of air:

$$\boxed{\frac{d}{dt}\big(\delta A(\zeta_\theta + f)\big) = 0 \rightarrow \delta A(\zeta_\theta + f)} = const$$

relative vorticity on an isentropic surface

Coriolis parameter

$$\begin{cases} \delta A = (-g\dfrac{\delta\theta}{\delta p}) \times const \\ (\zeta_\theta + f)\delta A = const \end{cases}$$

$$(\zeta_\theta + f)(-g\frac{\delta\theta}{\delta p}) \times const = const$$

$$(\zeta_\theta + f)(-g\frac{\delta\theta}{\delta p}) = const$$

$$g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p}) = const$$

$$\zeta_\theta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)_\theta$$

$$\boxed{PV_E = g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p})}$$

Ertel potential Vorticity

Isentropic Potential Vorticity

Potential vorticity is conserved following adiabatic, frictionless flow

Due to the conservation of PV, there is a close relationship between absolute vorticity and static stability. The diagram below shows a cylinder with the top and bottom defined by two isentropic surfaces.

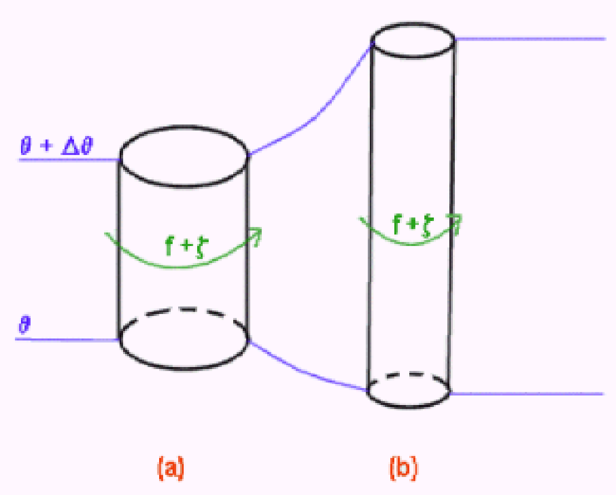
Difference in potential temperature between the top and bottom is the same for the two cylinders. If PV is conserved, and the cylinder is stretched as shown in (b), then static stability is decreasing and absolute vorticity must increase. Alternatively, if one goes from (b) to (a), then static stability is increasing and absolute vorticity must decrease.

$$\frac{\partial\theta}{\partial p} \to 0 \quad (adiabatic), \quad \varsigma_\theta \to very \quad large$$

$$\boxed{PV_E = g(\zeta_\theta + f)(-\frac{\partial\theta}{\partial p})}$$

$$\frac{\partial\theta}{\partial p} \to \quad (increasing), \quad \varsigma_\theta \to (decreasing)$$

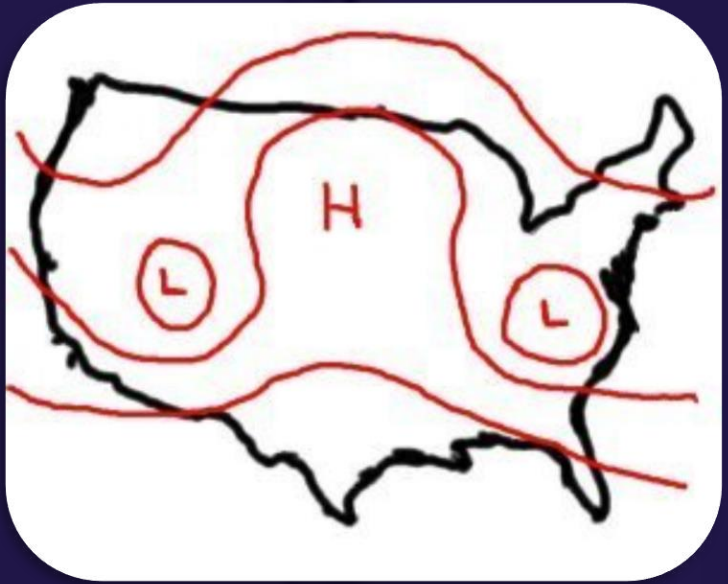
$$\frac{\partial\theta}{\partial p} \to 0 \quad (decreasing), \quad \varsigma_\theta \to (increasing)$$



There are several reasons why many meteorologists think that the consideration of IPV charts are useful.

First of all, PV is a conserved quantity in adiabatic, frictionless flow. The conservation of potential vorticity is a powerful constraint on the large scale motions of the atmosphere.

PV centres may be identified on a series of analyses and can be used to describe the evolution of flow patterns during significant synoptic events such as rapid cyclogenesis, blocking and retrogression of longwaves.



Secondly, it is possible to deduce the T, p and wind fields from the PV distribution if a number of assumptions are made.

For example, one assumption involves the specification of a balance condition which relates the mass field to the motion field.

The simplest balance condition is the *quasi-geostrophic* approximation.
One must also specify an initial reference state and appropriate boundary conditions.

Once this is done, however, the spatial distribution of PV then becomes a source term in the equations, the flow field being derived entirely from this term.

Later, an analogy will be made with static electric charge distributions and their associated electric fields.

Finally, certain atmospheric processes may be described in terms of the interaction of PV anomalies with the background structure of the atmosphere.

For example, when a strong upper-level PV anomaly moves over a low-level baroclinic zone, cyclogenesis usually results.

There is no need to invoke secondary circulations (vertical motions) as drivers of the development.

In addition, a superposition principle may be used to describe the interaction of PV anomalies at different levels in the atmosphere, interactions which lead to changes in the circulations at these levels.