Atmospheric Dynamics

Lecture 5

Physics Department

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Scale Analysis of the Vorticity Equation

$$\frac{\partial\zeta}{\partial t}+u\frac{\partial\zeta}{\partial x}+v\frac{\partial\zeta}{\partial y}+w\frac{\partial\zeta}{\partial z}+v\frac{\partial f}{\partial y}=-\left(\zeta+f\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)+\frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial x}\frac{\partial p}{\partial y}-\frac{\partial\rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

10^{-10} 10^{-11} 10^{-10} 10^{-9} 10^{-11} 10^{-11}

$$\frac{df}{dt}=\frac{\partial f}{\partial t}+u\frac{\partial f}{\partial x}+v\frac{\partial f}{\partial y}+w\frac{\partial f}{\partial z}$$

$$\frac{\partial(\zeta+f)}{\partial t}+u\frac{\partial(\zeta+f)}{\partial x}+v\frac{\partial(\zeta+f)}{\partial y}=-(\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

local tendency of absolute vorticity — horizontal advection of absolute vorticity — effects of divergence

Implications: For large-scale weather systems, the change of absolute vorticity following the fluid motion is approximately equal to the production of vorticity due to horizontal convergence or the destruction of vorticity due to horizontal divergence.

(Note that this does not hold at smaller scales, for which the vertical advection, tilting and baroclinic terms may also be important)

The inequality is used in the last three terms because in each case it is possible that the two parts of the expression might partially cancel so that the actual magnitude would be less than indicated.

In fact, this must be the case for the divergence term because if $\partial u/\partial x$ and $\partial v/\partial y$ were not nearly equal and opposite, the divergence term would be an order of magnitude greater than any other term and the equation could not be satisfied.

Therefore, scale analysis of the vorticity equation indicates that synoptic-scale motions must be quasi-nondivergent.

The divergence term will be small enough to be balanced by the vorticity advection terms only if

$$f\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \lesssim \frac{f_0 U}{L} \sim 10^{-9}\ \mathrm{s}^{-2}$$

$$\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) \lesssim \frac{WU}{HL} \sim 10^{-11}\ \mathrm{s}^{-2}$$

$$\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y}-\frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right) \lesssim \frac{\delta\rho\,\delta p}{\rho^2 L^2} \sim 10^{-11}\ \mathrm{s}^{-2}$$

$$\left|\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right| \leq 10^{-6}\, s^{-1}$$

so that the horizontal divergence must be small compared to the vorticity in synoptic-scale systems.

From the aforementioned scalings and the definition of the Rossby number, we see that

$$\left|\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)/\ f_0\right| \le R_0^{\ 2} \qquad \left|\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)/\ \zeta\right| \le R_0$$

Thus, the ratio of the horizontal divergence to the relative vorticity is the same magnitude as the ratio of relative vorticity to planetary vorticity.

$$\frac{\zeta}{f_0} \le \frac{U}{f_0 L} \equiv R_o \sim 10^{-1}$$

Retaining only the terms of order $10^{-10}\ s^{-2}$ in the vorticity equation yields the approximate form valid for synoptic-scale motions,

$$\frac{d_h(\zeta + f)}{dt} = -f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^* \qquad \frac{d_h}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$$

But for Intense Cyclonic Storms

$$\frac{d_h(\zeta + f)}{dt} = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^{**}$$

As mentioned earlier, * is not accurate in intense cyclonic storms

In intense cyclonic storms, the relative vorticity should be retained in the divergence term.

$$\frac{d_h(\zeta + f)}{dt} = -f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) *$$

Equation * states that the change of absolute vorticity following the horizontal motion on the synoptic scale is given approximately by the concentration or dilution of planetary vorticity caused by the convergence or divergence of the horizontal flow, respectively.

In ** however, it is the concentration or dilution of absolute vorticity that leads to changes in absolute vorticity following the motion.

$$\frac{d_h(\zeta + f)}{dt} = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) **$$

Physical Explanation of Significant Terms

$$\frac{\partial \zeta}{\partial t} = -u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} \quad - v\frac{\partial f}{\partial y} \quad -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Horizontal Advection of Relative Vorticity

The local relative vorticity will increase (decrease) if positive (negative) relative vorticity is advected toward the location

→ Positive Vorticity Advection (PVA)

→ Negative Vorticity Advection (NVA)

PVA often leads to a decrease in surface pressure
(intensification of surface lows)

$$\frac{\partial \zeta}{\partial t} = -u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} \quad - v\frac{\partial f}{\partial y} \quad - (\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

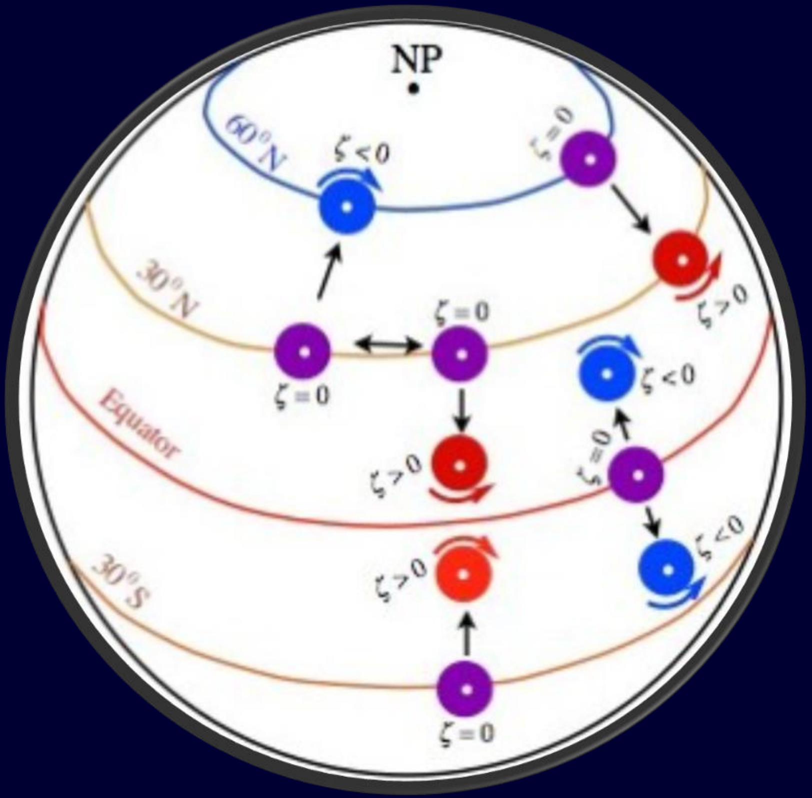
Meridional Advection of Planetary Vorticity

The local relative vorticity will decrease (increase) if the local flow is southerly (northerly) due to the advection of planetary vorticity (minimum at Equator; maximum at poles)

Divergence Term

The local relative vorticity will increase (decrease) if local convergence (divergence) exists

Absolute vorticity (planetary vorticity + relative vorticity) is conserved as fluid columns of constant height change latitude during their movement in the oceans and atmosphere

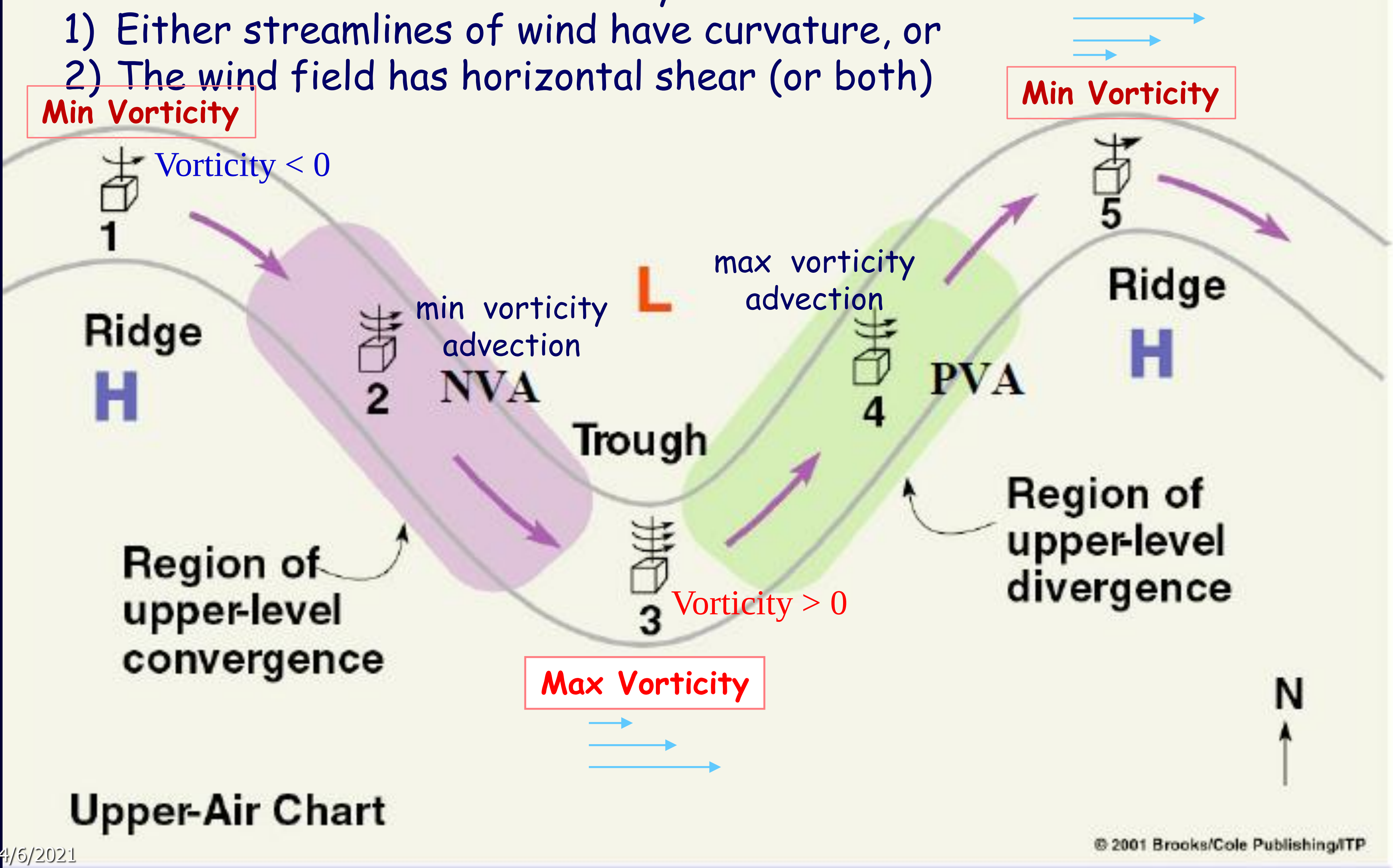


Assume that a column placed on 30°N has no rotational motion (ζ = 0) initially. If it moves either way along this latitude then its absolute vorticity will not change, and its relative vorticity will continue to remain zero. However, if a fluid column on the equator with (ζ = 0), moves northward or southward, then negative (anticyclonic) vorticity will be generated in order to conserve the absolute vorticity.

Vorticity Advection

We've seen that relative vorticity is non-zero for two reasons:

1) Either streamlines of wind have curvature, or
2) The wind field has horizontal shear (or both)



Positive vorticity advection (PVA)

found where air blows from regions of higher vorticity toward lower vorticity

significant because main mechanism to reduce vorticity is divergence

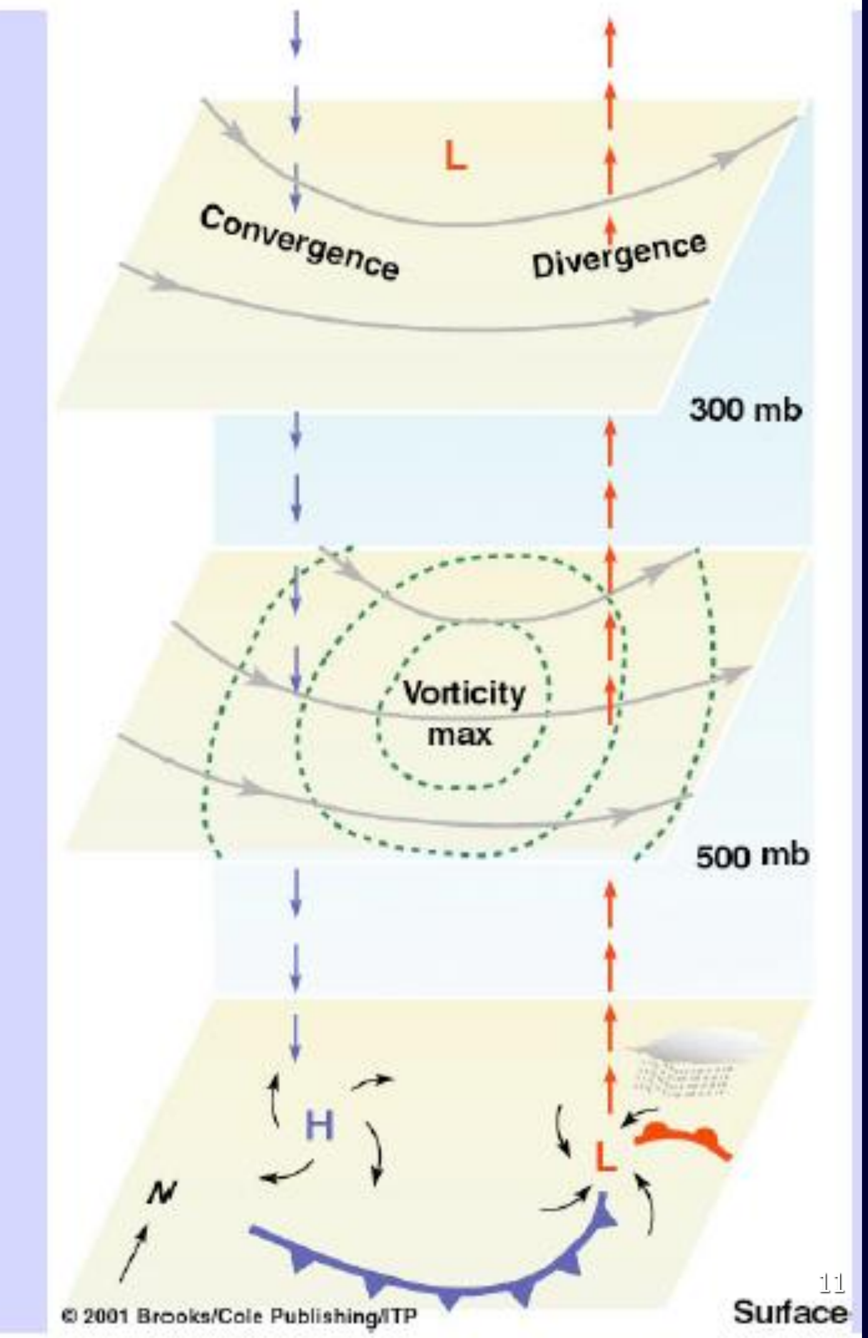
that is, in regions of PVA there tends to be divergence, which implies upward motions beneath these areas, surface convergence and surface pressure falls

Negative vorticity advection (NVA)

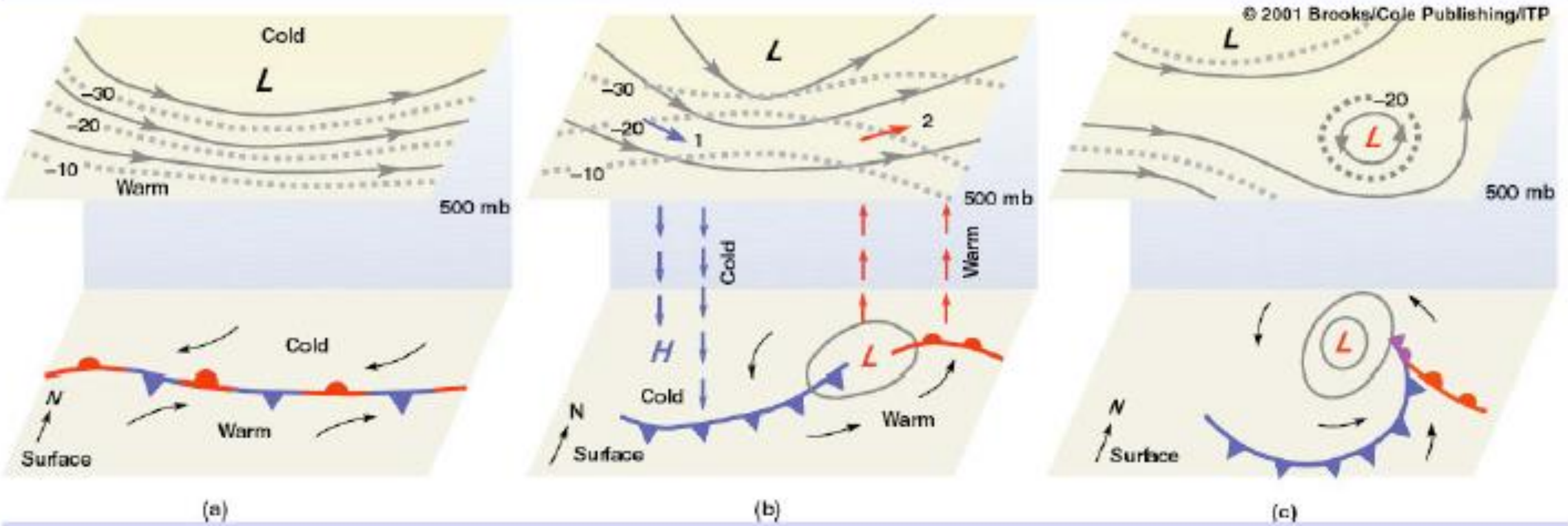
found where air blows from regions of lower vorticity towards higher vorticity

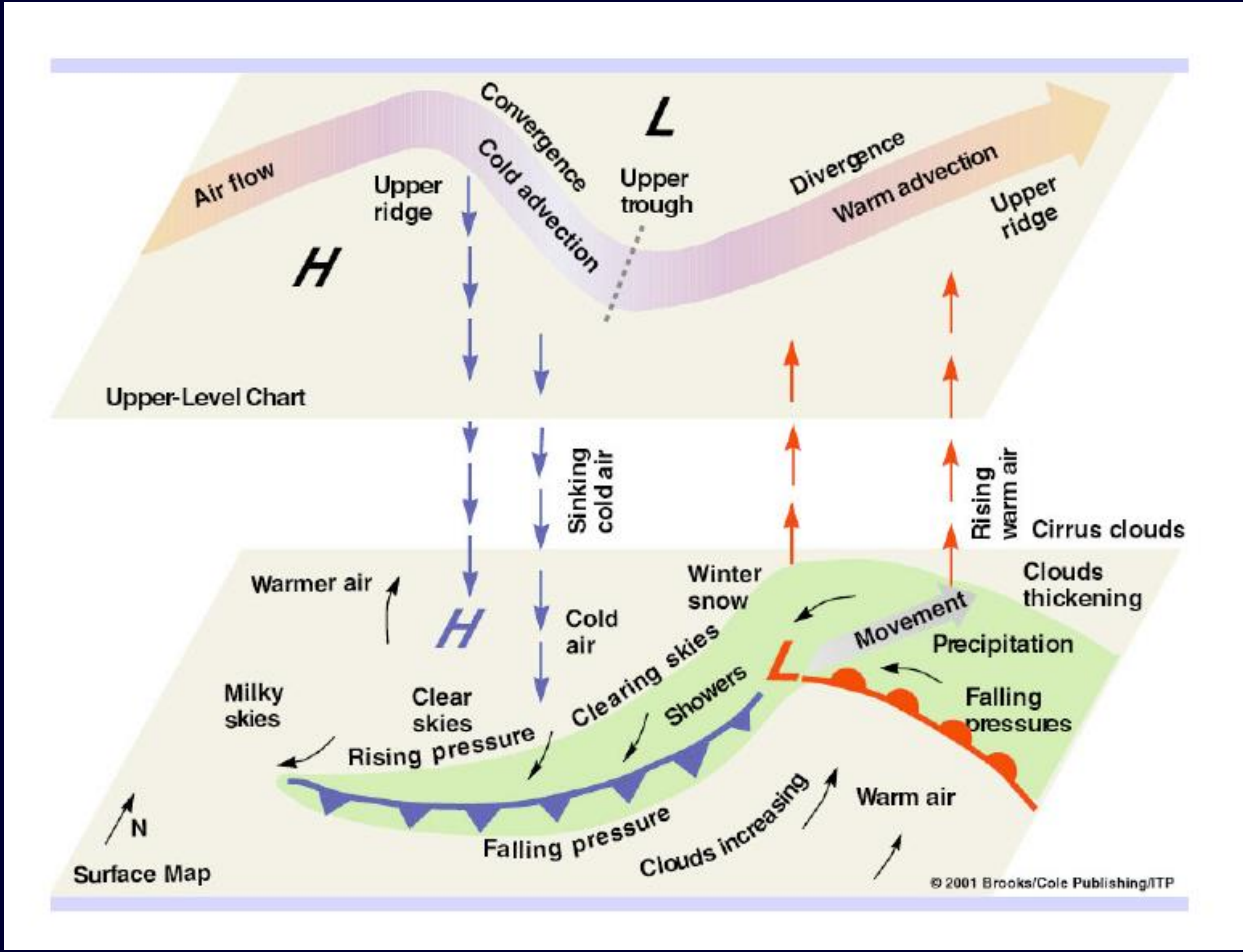
main mechanism to increase vorticity is convergence

when there is NVA in upper levels, there tends to be downward motion below, surface divergence and surface pressure rises



Development of a Baroclinic Wave




Air flow
Upper
ridge
H
Convergence
Cold advection
L
Upper
trough
Divergence
Warm advection
Upper
ridge
Upper-Level Chart
Sinking
cold air
Rising
warm air
Cirrus clouds
Warmer air
H
Cold
air
Winter
snow
Clouds
thickening
Movement
Precipitation
Clearing skies
Showers
L
Milky
skies
Clear
skies
Rising pressure
Falling
pressures
Falling pressure
Clouds increasing
Warm air
N
Surface Map
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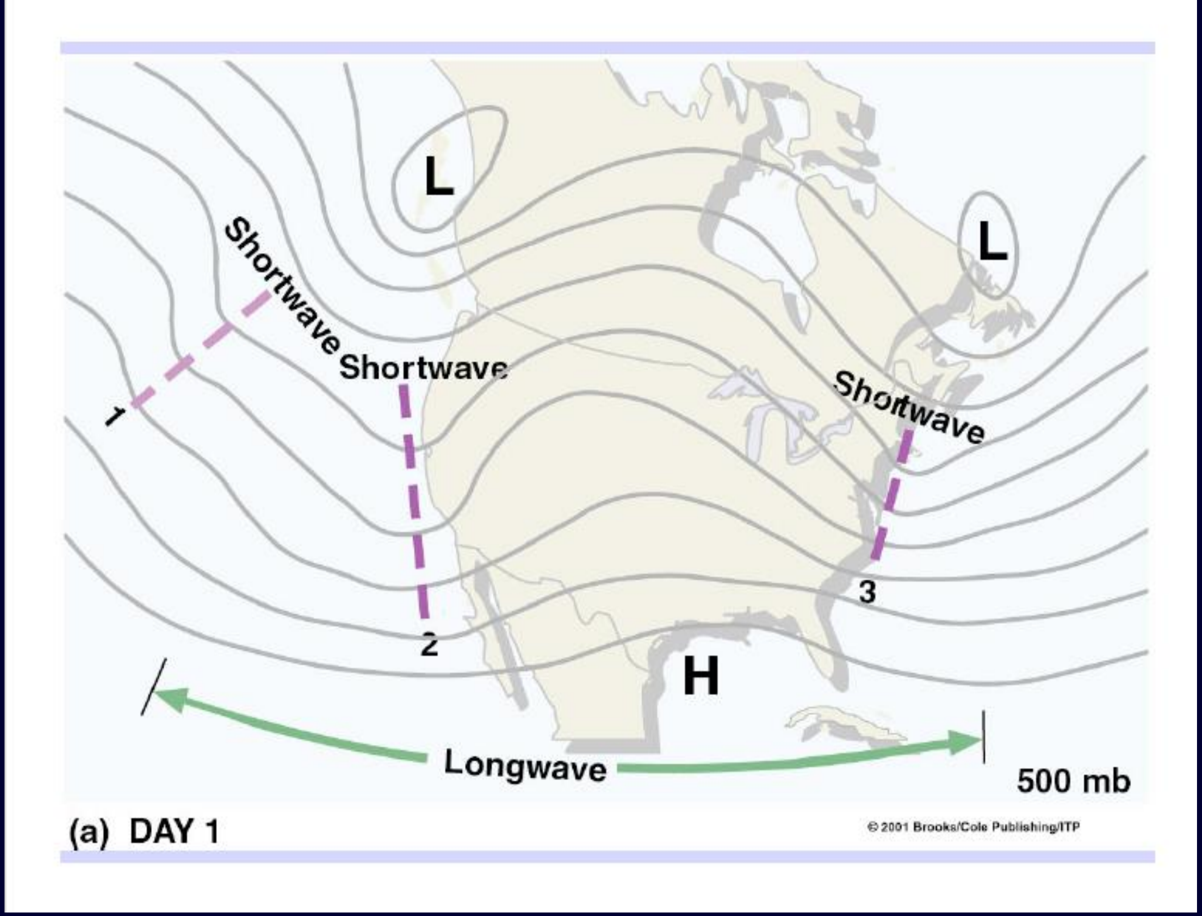
L
L
Shortwave
Shortwave
Shortwave
1
2
3
H
Longwave
500 mb
(a) DAY 1
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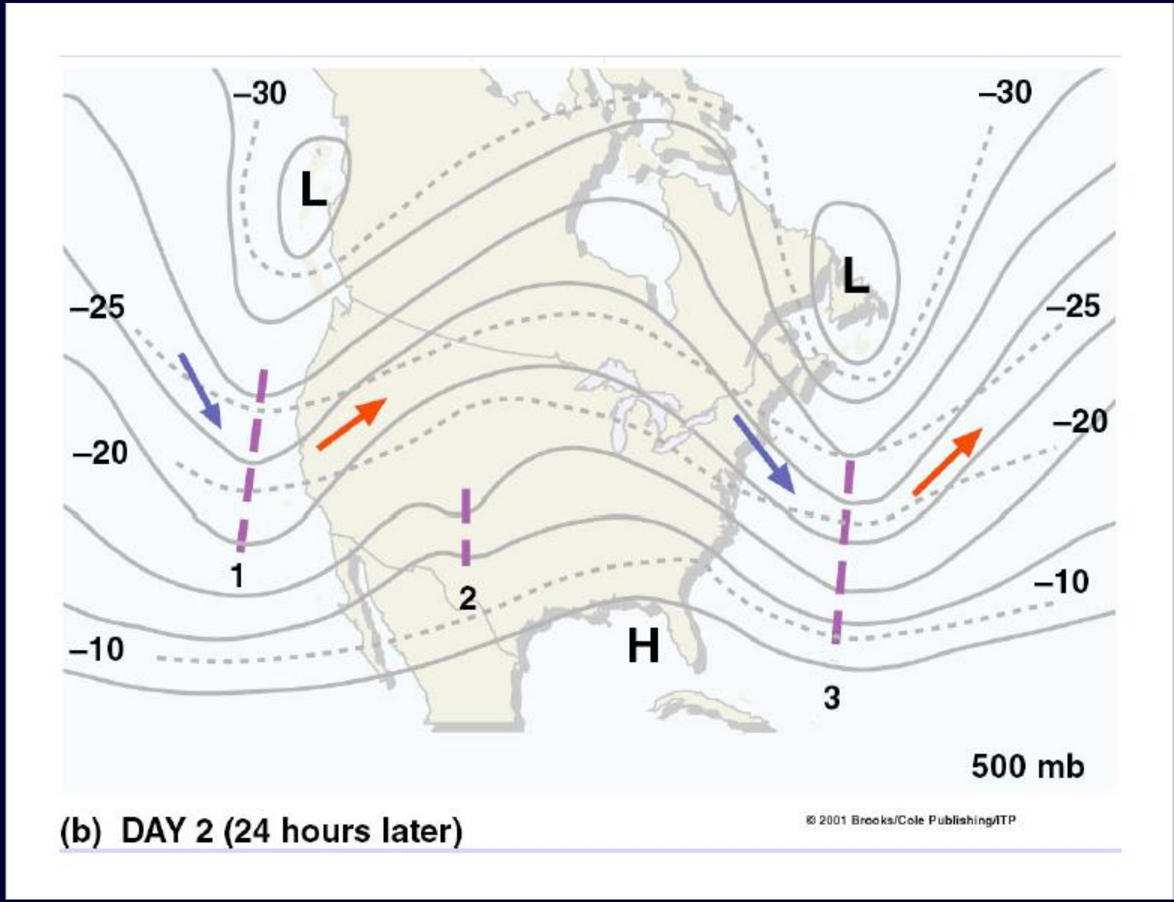
–30
–30
L
L
–25
–25
–20
–20
–10
–10
1
2
3
H
500 mb
(b) DAY 2 (24 hours later)
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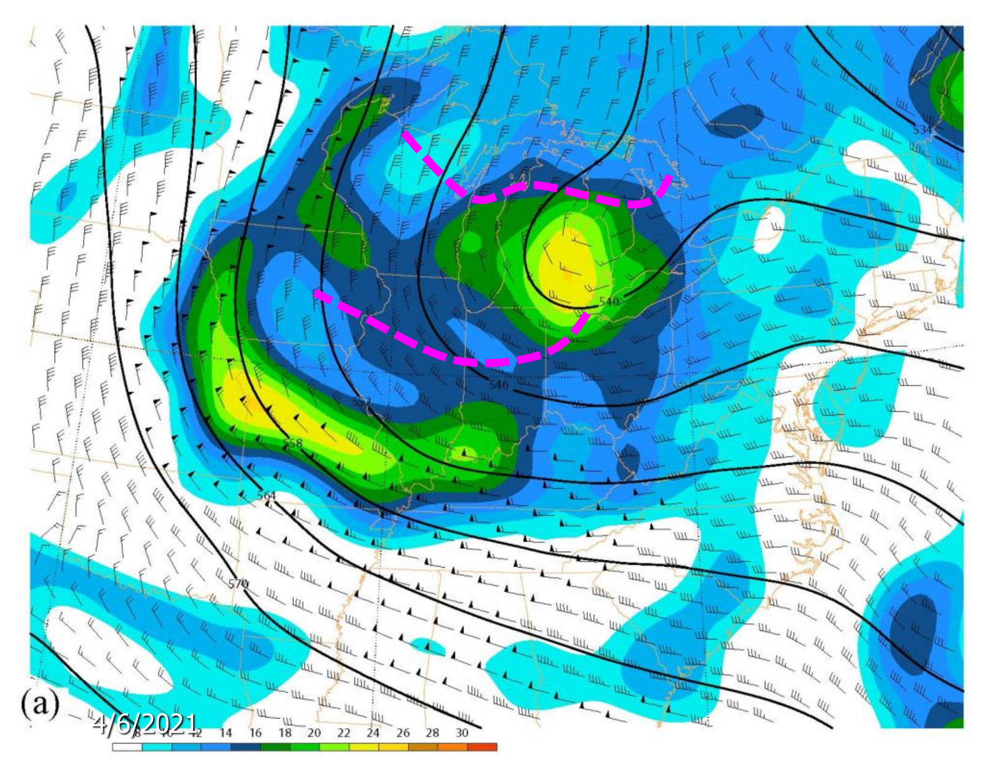
Physical Explanation: Horizontal Advection of Relative Vorticity

$$\frac{\partial \zeta}{\partial t} = -u\frac{\partial \zeta}{\partial x} - v\frac{\partial \zeta}{\partial y} - v\frac{\partial f}{\partial y} - (\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Relative Vorticity (ζ)



Relative Vorticity Advection