Atmospheric Dynamics

Lecture 3

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$$\frac{\partial}{\partial x}\text{[y-component momentum equation]} - \frac{\partial}{\partial y}\text{[x-component momentum equation]} =$$

$$\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y}\right]$$

$$-\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x}\right]$$

$$\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+fu=-\frac{1}{\rho}\frac{\partial p}{\partial y}\right]$$

$$\frac{\partial}{\partial x}\frac{\partial v}{\partial t}+u\frac{\partial^2 v}{\partial x^2}+\frac{\partial v}{\partial x}\frac{\partial u}{\partial x}+v\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial v}{\partial y}\frac{\partial v}{\partial x}+w\frac{\partial^2 v}{\partial x\partial z}+\frac{\partial v}{\partial z}\frac{\partial w}{\partial x}+f\frac{\partial u}{\partial x}+\cancel{u\frac{\partial f}{\partial x}}=-\frac{1}{\rho}\cancel{\frac{\partial^2 p}{\partial x\partial y}}+\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}\right)$$

$$-\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv=-\frac{1}{\rho}\frac{\partial p}{\partial x}\right]$$

$-$

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+w\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+f\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$+\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)+v\frac{\partial f}{\partial y}=\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}-\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right)$$

$$\frac{df}{dt}=\frac{\partial f}{\partial t}+u\frac{\partial f}{\partial x}+v\frac{\partial f}{\partial y}+w\frac{\partial f}{\partial z}\qquad\qquad \frac{df}{dt}=v\frac{\partial f}{\partial y}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+u\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+v\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+w\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$+f\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)+v\frac{\partial f}{\partial y}=\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}-\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right)$$

$$\frac{\partial \zeta}{\partial t}+u\frac{\partial \zeta}{\partial x}+v\frac{\partial \zeta}{\partial y}+w\frac{\partial \zeta}{\partial z}+\zeta\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+f\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)+\frac{df}{dt}=\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}-\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right)$$

$$\frac{d\zeta}{dt}+\zeta\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+f\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)+\frac{df}{dt}=\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}-\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right)$$

$$\underbrace{\frac{d}{dt}(\zeta+f)}_{A}=\underbrace{-(\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)}_{B}\underbrace{-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)}_{C}+\underbrace{\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}-\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right)}_{D}$$

vorticity equation

A: Rate of change of absolute vorticity following the fluid motion

B: Effect of horizontal velocity divergence on vorticity (or vortex stretching term)

C: Transfer of vorticity between horizontal and vertical components (“twisting term” or “tilting term”)

D: Effects of baroclinicity (“solenoidal term”)

Term A: Rate of change of absolute vorticity following the fluid motion

$$\frac{d(\zeta + f)}{dt} = \frac{\partial(\zeta + f)}{\partial t} + u\frac{\partial(\zeta + f)}{\partial x} + v\frac{\partial(\zeta + f)}{\partial y} + w\frac{\partial(\zeta + f)}{\partial z}$$

local tendency of absolute vorticity

horizontal advection of absolute vorticity

vertical advection of absolute vorticity

Term B: Effect of horizontal velocity divergence on vorticity

$$-(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

The concentration or dilution of vorticity by the divergence field

This term is the fluid analog of the change in angular velocity resulting from a change in the moment of inertia of a solid body when angular momentum is conserved.

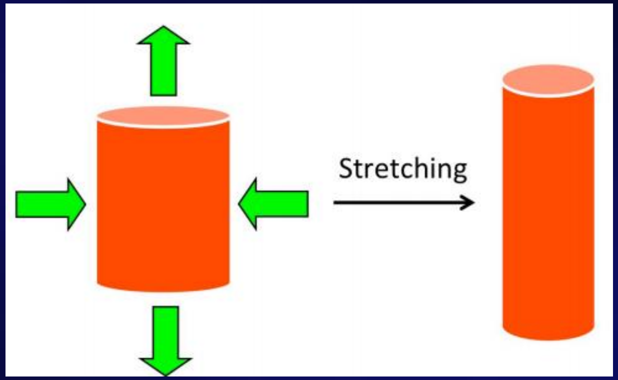
This mechanism for changing vorticity following the motion is very important in synoptic-scale midlatitude systems.

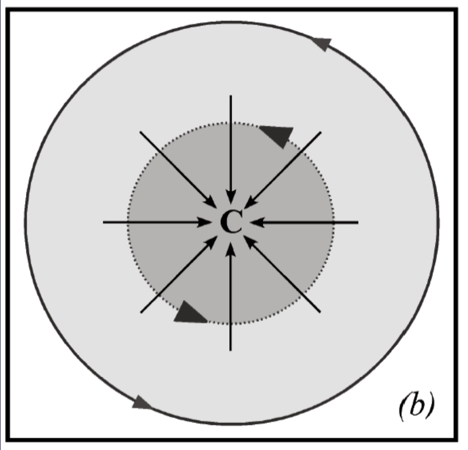
If $\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)<0$ (convergence), $\Rightarrow -(\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)>0$ $\Rightarrow \frac{d(\zeta+f)}{dt}>0$

$\zeta+f$ *must increase*

Then vorticity will increase if absolute vorticity is positive

Vorticity will decrease if absolute vorticity is negative.

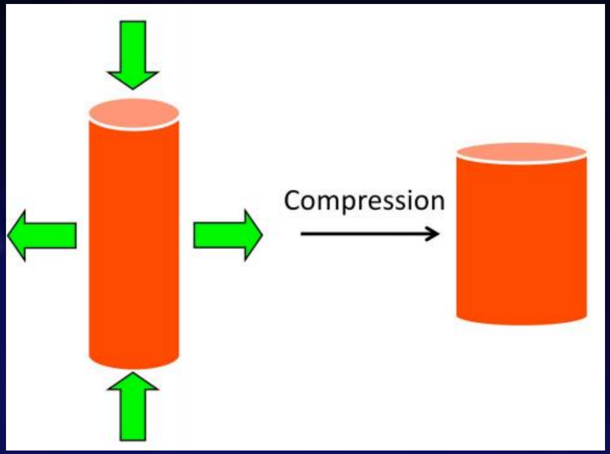


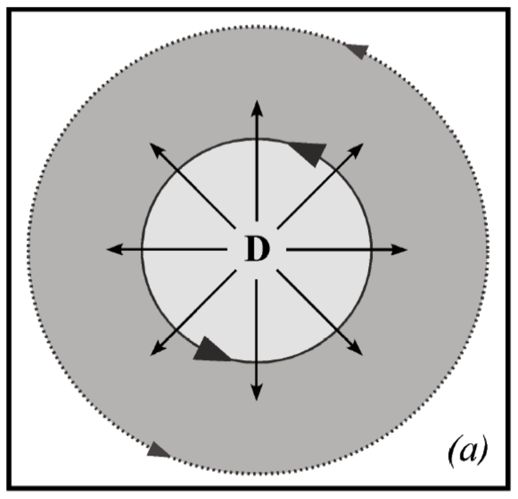


If, however, the flow is convergent, the area enclosed by a chain of fluid parcels will decrease with time and the vorticity will be concentrated.

If $\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)>0$ (divergence), $\Rightarrow -(\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)<0 \Rightarrow \frac{d(\zeta+f)}{dt}<0$

$\zeta+f$ *must decrease*





then vorticity will decrease if absolute vorticity is positive.

Vorticity will increase if absolute vorticity is negative.

If the horizontal flow is divergent, the area enclosed by a chain of fluid parcels will increase with time, and if circulation is to be conserved, the average absolute vorticity of the enclosed fluid must decrease (i.e., the vorticity will be diluted).

vorticity equation

$$\frac{d}{dt}(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x} - \frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right)$$

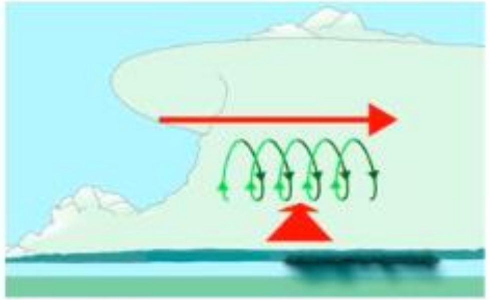
A B C D

A: Rate of change of absolute vorticity following the fluid motion

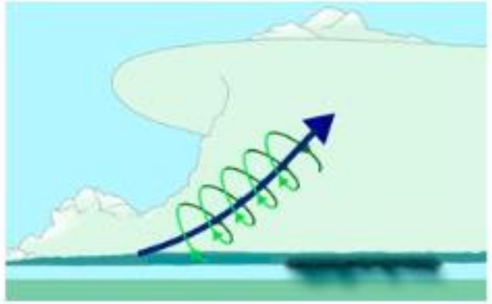
B: Effect of horizontal velocity divergence on vorticity (or vortex stretching term)

C: Transfer of vorticity between horizontal and vertical components ("twisting term" or "tilting term")

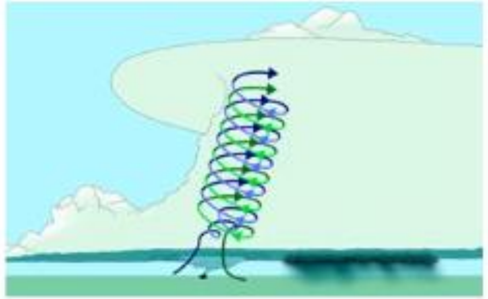
D: Effects of baroclinicity ("solenoidal term")



Wind shear creates horiz vort



Updraft “tilts” the spinning air upright



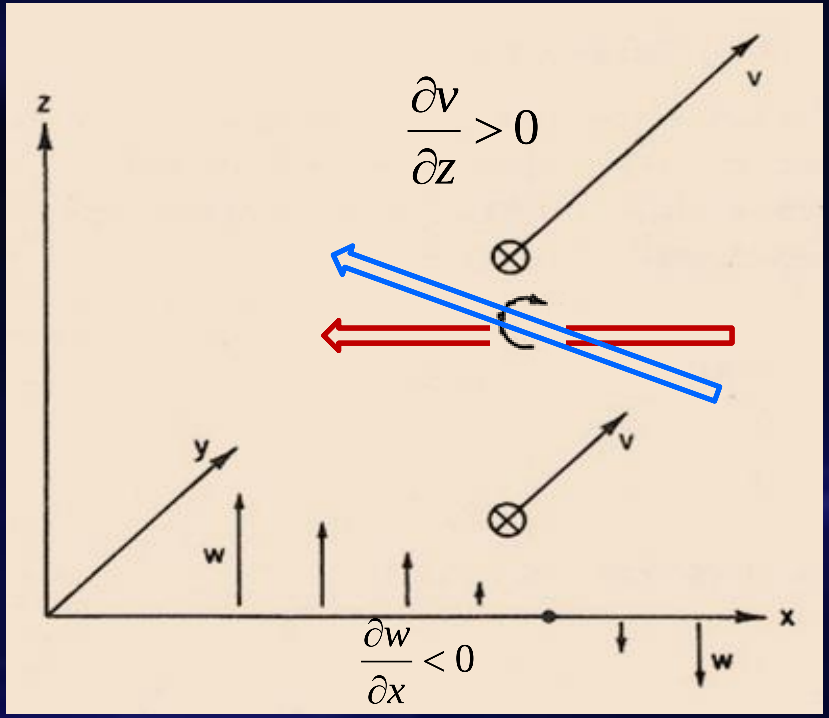
The updraft then starts rotating → supercell

Term C: Transfer of vorticity between horizontal and vertical components ("twisting term" or "tilting term")

$$-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)$$

In this example, vertical shear of v-component wind is producing shear vorticity about an east-west axis (There is a component of shear vorticity oriented in the negative x direction).

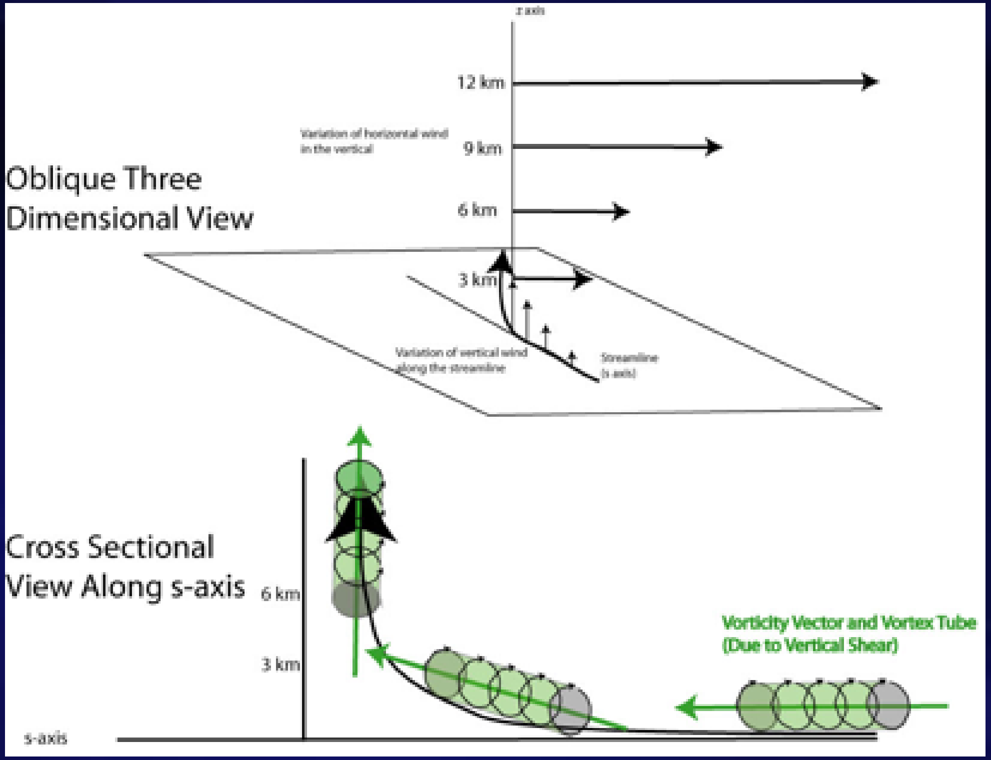
If at the same time there is a vertical motion field in which w decreases with increasing x, East-west variations in the vertical velocity twist or tilt this "vortex tube" toward a more vertical orientation. This gives the vorticity vector a component in the z-direction, indicating a transfer of vorticity from the horizontal to the vertical.



$$\frac{\partial v}{\partial z}\frac{\partial w}{\partial x} < 0 \rightarrow \frac{d(\zeta + f)}{dt} > 0$$

There will be a generation of positive vertical vorticity.

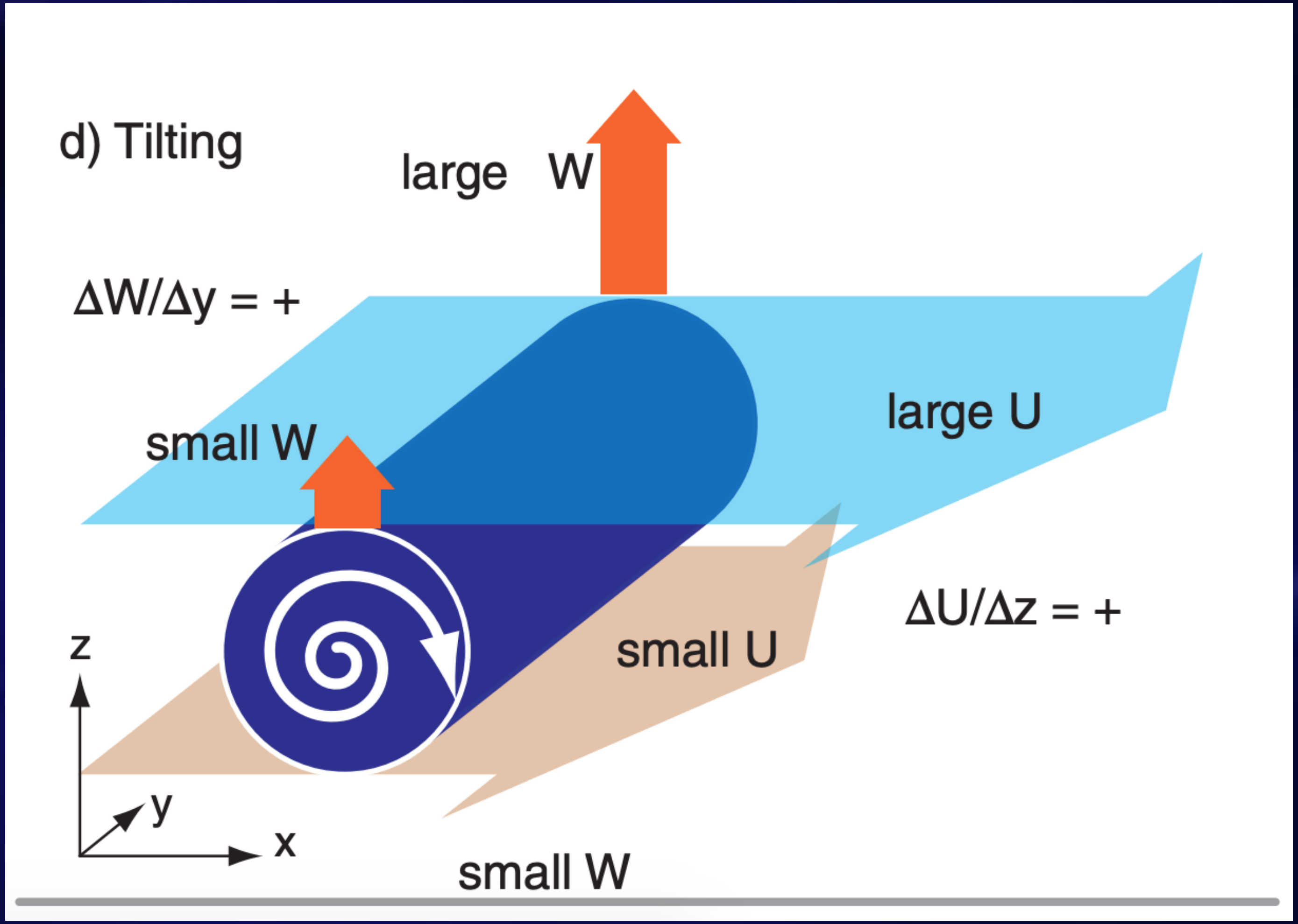
These equations state that vertical vorticity will develop if there is a gradient of the vertical wind along, say, a surface streamline, if that surface streamline is in a region of vertical shear of the horizontal wind.



$$-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)$$

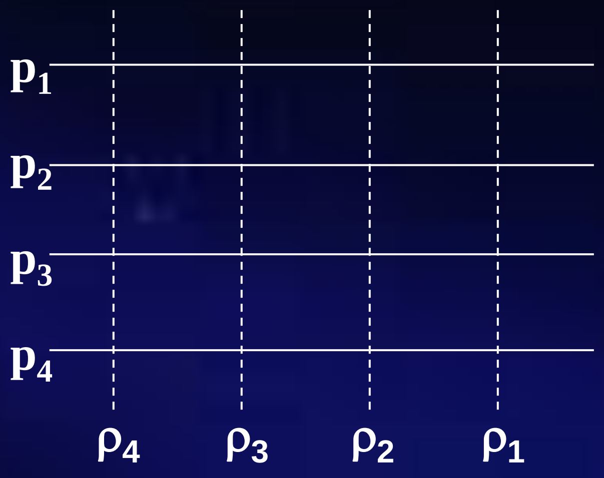
This term represents vertical vorticity generated by the tilting of horizontally oriented components of vorticity into the vertical by a nonuniform vertical motion field.

$$-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)$$



Term D: Effects of baroclinicity ("solenoidal term")

$$\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}-\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right)$$



$p_4 > p_3 > p_2 > p_1$

$\rho_4 > \rho_3 > \rho_2 > \rho_1$

This term arises because of the horizontal variations in density that occur in a baroclinic atmosphere.

In this example, even though the pressure gradient is uniform, variations in density produce small variations in the pressure gradient force.

The variations in acceleration that result lead to the production of positive vorticity.

$$\frac{\partial p}{\partial y}<0, \quad \frac{\partial \rho}{\partial x}<0 \quad \rightarrow \frac{d(\zeta+f)}{dt}>0$$

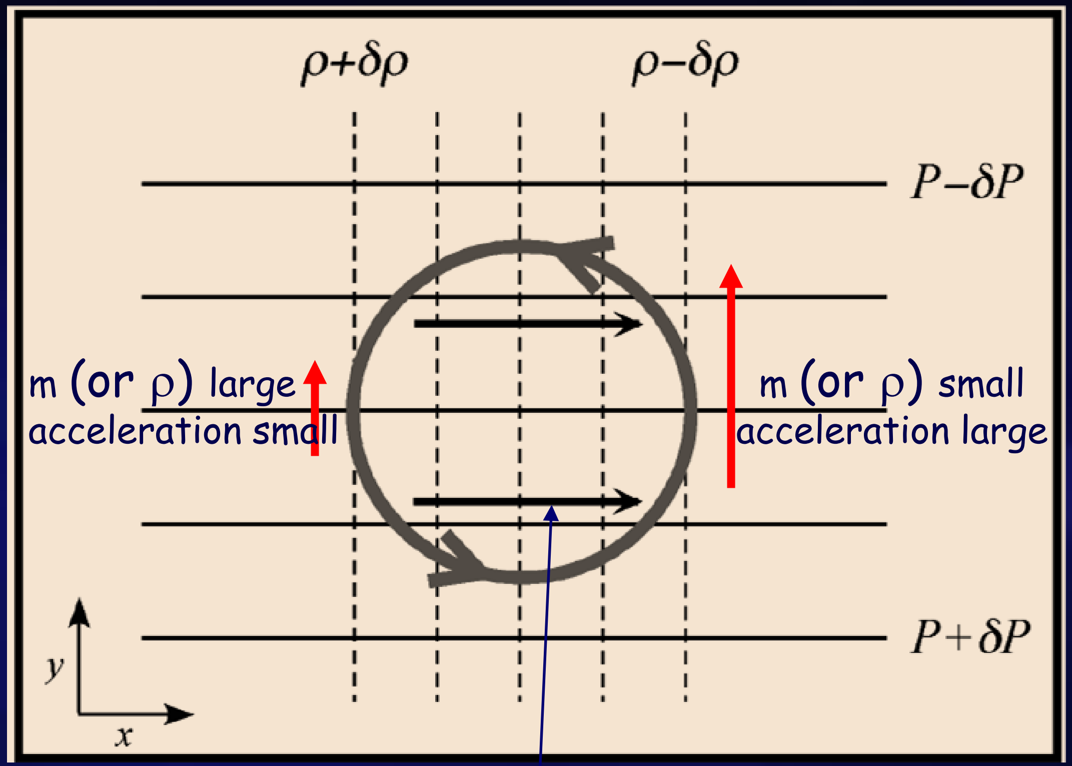
the context of meteorology, a **solenoid** is a tube-shaped region in the atmosphere where isobaric (constant pressure) and isopycnal (constant density) surfaces intersect, causing vertical circulation

a space formed by the intersection of isobaric and isosteric surfaces.

A hypothetical tube formed in space by the intersection of a set of surfaces of constant pressure and a set of surfaces ofconstant specific volume of air. Also known as solenoid.

Solenoid: field loop that converts potential energy to kinetic energy

$$\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}-\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right)$$

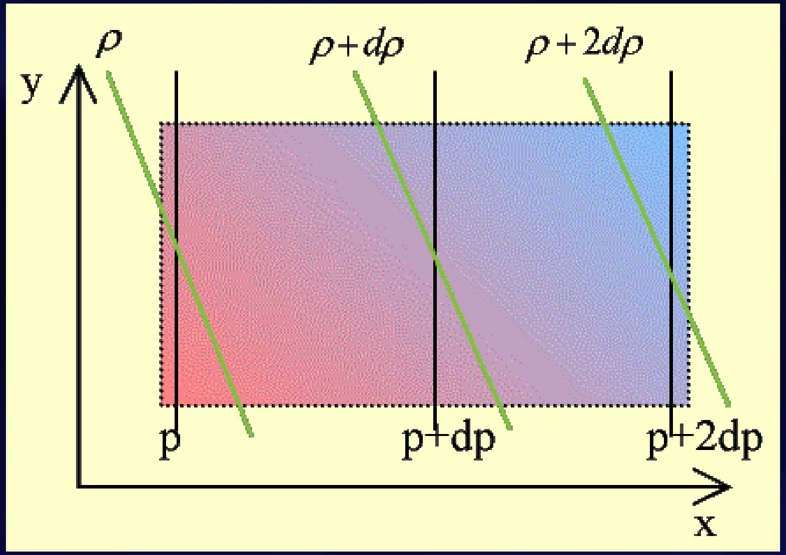


geostrophic wind

The figure below shows the effect of the solenoid term

$$\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x}-\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right)$$

Cold advection pattern



This term states that if the isobaric contours do not coincide with the isosteric contours, vorticity is generated. There is a temperature gradient from the upper right corner (lower temperature) towards the lower left corner (higher temperature). At the same time there is a pressure gradient from left (lower pressure) to right (higher pressure).

This also results in a gradient in density. These do not coincide with the lines of equal pressure. As a result, in the case shown above, negative vorticity is generated.

$$\frac{1}{\rho^2}\left(\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial x} - \frac{\partial p}{\partial x}\frac{\partial \rho}{\partial y}\right) \qquad -\left(\frac{\partial \alpha}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \alpha}{\partial y}\frac{\partial p}{\partial x}\right)$$

However, the solenoidal term in the vorticity equation can be written

$$-\left(\frac{\partial \alpha}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \alpha}{\partial y}\frac{\partial p}{\partial x}\right) = -(\nabla\alpha \times \nabla p)\,.\,\hat{k}$$

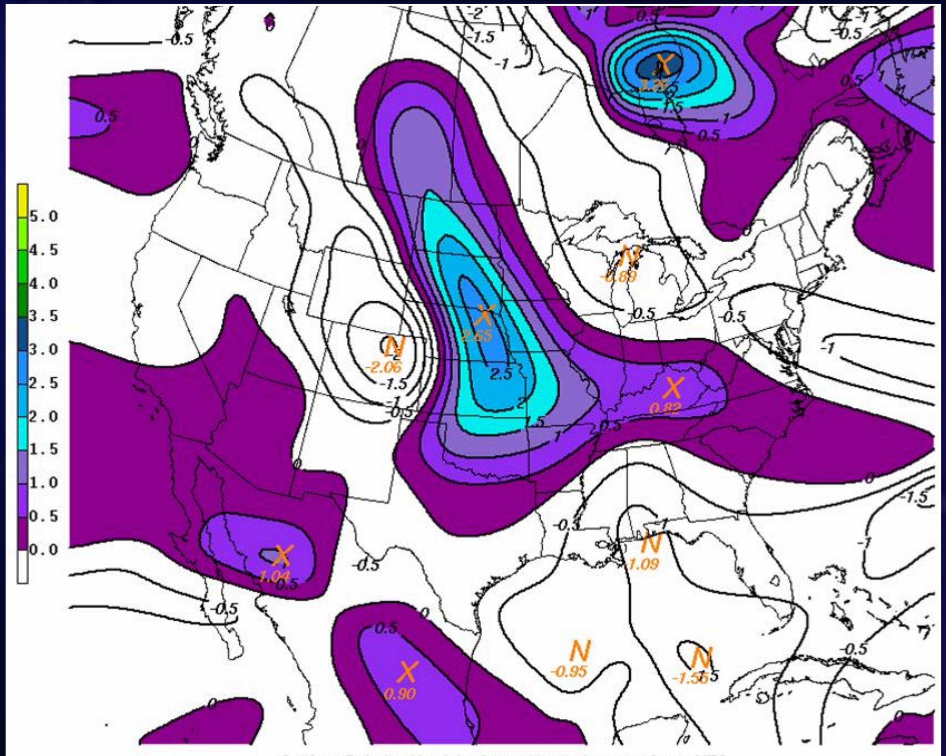
Thus, the solenoidal term in the vorticity equation is just the limit of the solenoidal term in the circulation theorem divided by the area when the area goes to zero.

Circulation and Vorticity

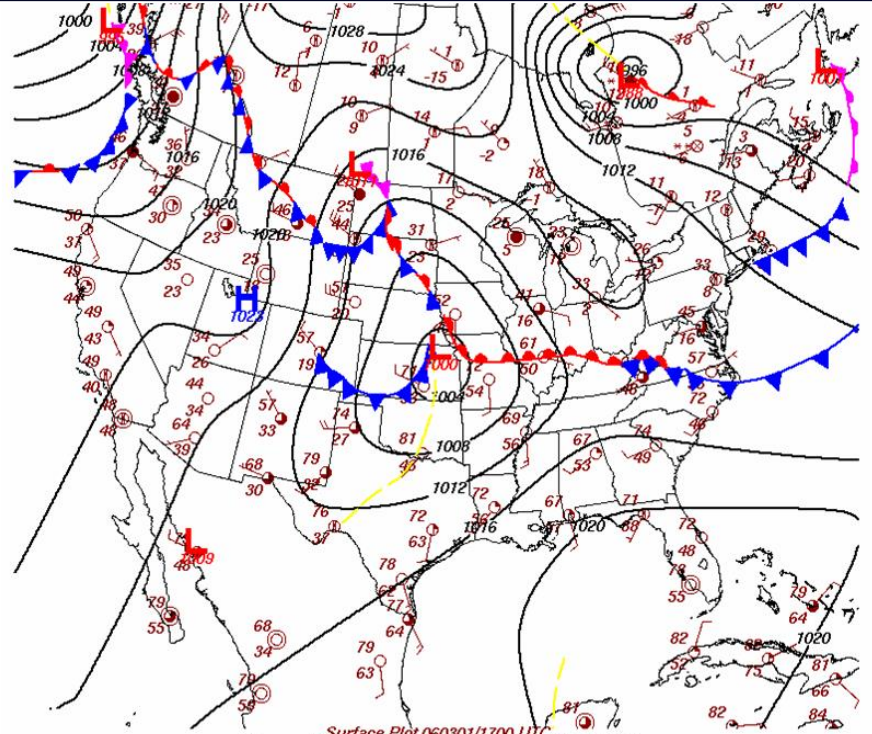
Vorticity Types:

Positive Vorticity: Associated with cyclonic (counterclockwise) circulations in the Northern Hemisphere

Negative Vorticity: Associated with anticyclonic (clockwise) circulations in the Northern Hemisphere



Surface Relative Vorticity (1 x 10**5 s^-1) 060301/1700 UTC



Surface Plot 060301/1700 UTC
Observed Sea Level Pressure (mb) 060301/1700 UTC
FRONTS VALID 030115Z