Atmospheric Dynamics

Lecture 12

Sahraei

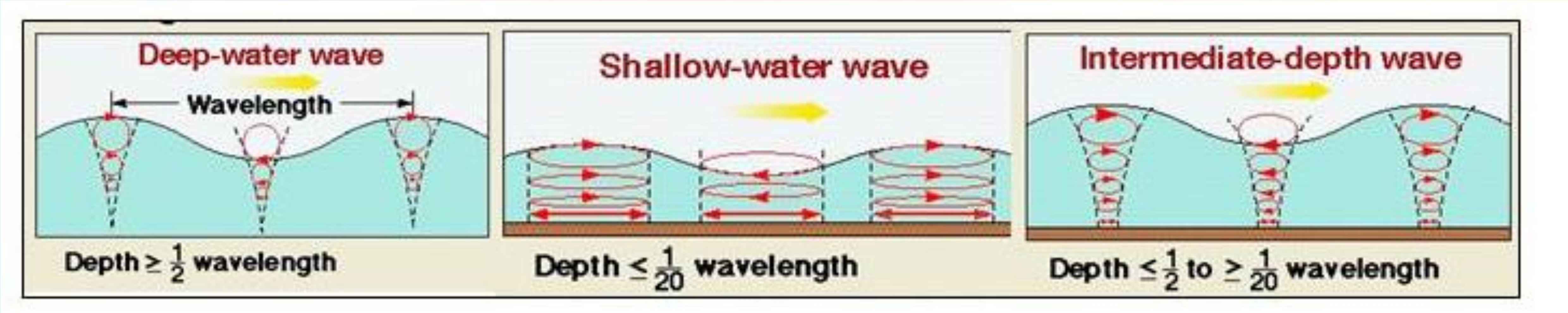
Physics Department

Razi University

https://sci.razi.ac.ir/~sahraei

Progressive waves in

(a) deep waters (b) shallow waters (c) intermediate waters



Notice that the orbital motion changes significantly according with the depth and the relationship between depth and wavelength.

Geostrophic Balance

$$\frac{\partial^3 h'}{\partial t^3} + (f^2 - g\bar{h}\nabla_h^2)\frac{\partial h'}{\partial t} = 0 \quad h' = \mathrm{Re}\{Ae^{i(kx+ly-\omega t)}\} \quad \omega^3 - \omega[f^2 + g\bar{h}(k^2 + l^2)] = 0$$

Turning now to the structure of the waves, consider first the case where ω=0.

Appealing to

$$\frac{\partial u'}{\partial t} = -g\frac{\partial h'}{\partial x} + fv' \qquad \frac{\partial v'}{\partial t} = -g\frac{\partial h'}{\partial y} - fu' \qquad \frac{\partial h'}{\partial t} = -\bar{h}\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right)$$

we see that since the waves are stationary, the left sides are zero.

Clearly, these waves depend on rotation, since when f = 0, the wave amplitude, A, must also be zero.

For non zero f :

$$v' = \frac{g}{f}\frac{\partial h'}{\partial x} \qquad u' = -\frac{g}{f}\frac{\partial h'}{\partial y}$$

The stationary waves are in a state of geostrophic balance and are an example of Rossby waves in the limit of constant Coriolis parameter.

inertia-gravity waves

Solutions for $\omega \neq 0$ are called inertia-gravity waves, since particle oscillations depend on both gravitational and inertial forces.

So that we may focus here on the gravitational aspect.

In the limiting case f=0 , we have simply gravity waves, and from

$$\omega^2 = f^2 + g\bar{h}(k^2 + l^2)$$

The shallow water gravity wave speed becomes

$$c = \sqrt{g\bar{h}}$$

$$c = \sqrt{g\bar{h}}$$

Since the waves all move at the same speed, they are nondispersive.

$$For\ \ \bar{h} = 4\ km\ deep \qquad \rightarrow c \approx 200\ ms^{-1} = 720\ km/h$$

Thus, long waves on the ocean surface travel very rapidly.

Gravity waves

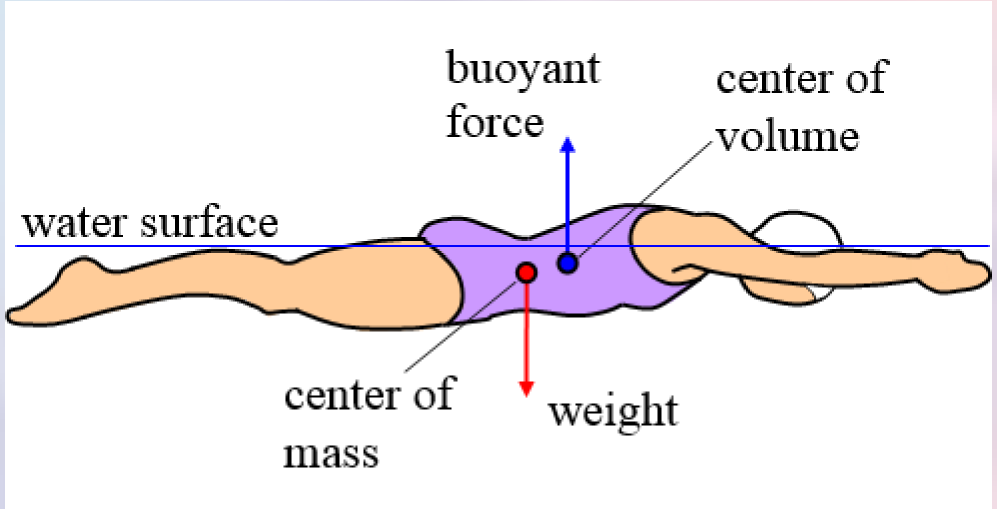
Gravity waves are not something outside your daily experience. Have you ever watched the wake that forms behind a boat? The waves you see are gravity waves.

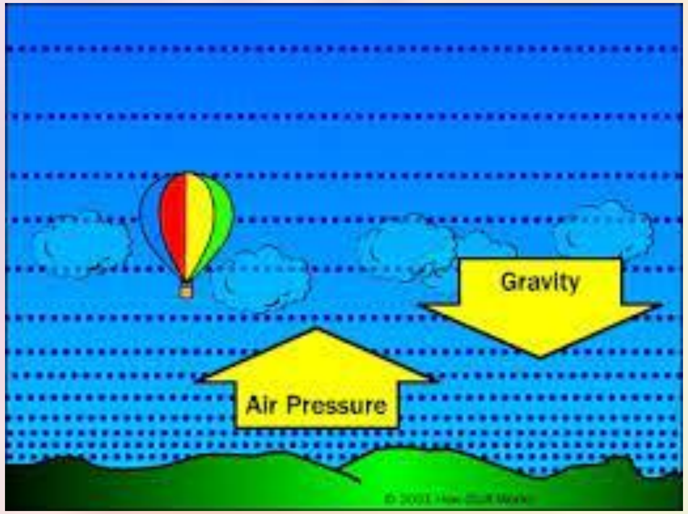
Every noticed the clouds which form in regular bands of cloud and clear sky? These clouds are the result of gravity waves.

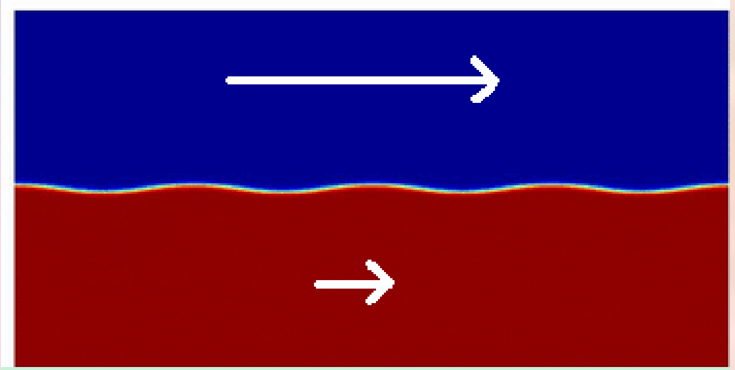
A gravity wave is an oscillation caused by the displacement of an air parcel which is restored to its initial position by gravity.

The lifting force is buoyancy, while the restoring force is gravity, so a few scientists feel they should be called buoyancy waves!

The buoyancy force is proportional to the difference in air temperature inside and outside an air parcel.

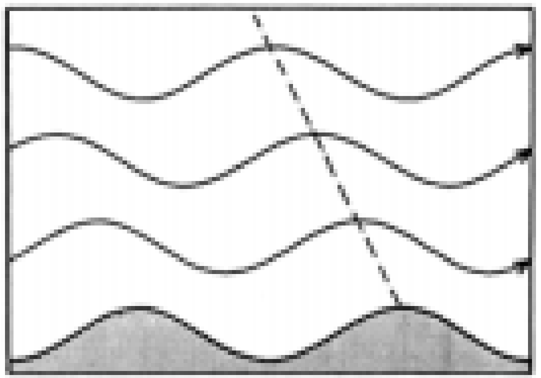






The air in the upper (blue) layer is moving faster than the air in the lower (red) layer, and with enough instability (temperature decreasing with height), these waves form.

Internal Gravity (Buoyancy) Waves

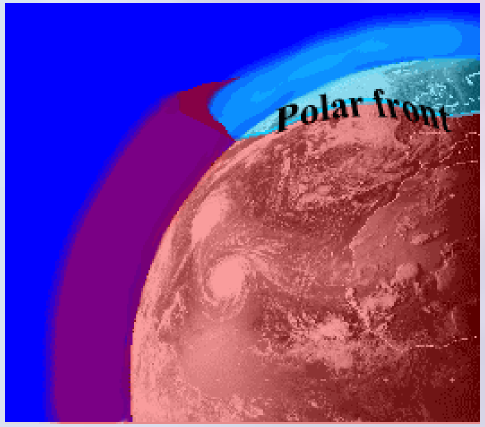


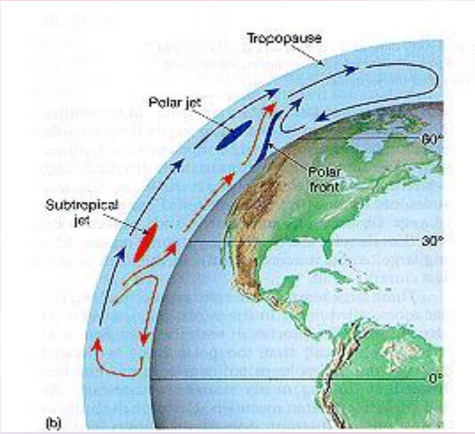
- In a fluid, such as the ocean, which is bounded both above and below, gravity waves propagate primarily in the horizontal plane since vertically traveling waves are reflected from the boundaries to form standing waves.
- In a fluid that has no upper boundary, such as the atmosphere, gravity waves may propagate vertically as well as horizontally. In vertically propagating waves the phase is a function of height. Such waves are referred to as *internal waves*.
- **Gravity waves are generated during the adjustment process toward the geostrophic balance/equilibrium**.
- *Although internal* gravity waves are not generally of great importance for synoptic-scale weather forecasting (and indeed are nonexistent in the filtered quasi-geostrophic models), they can be important in mesoscale motions.

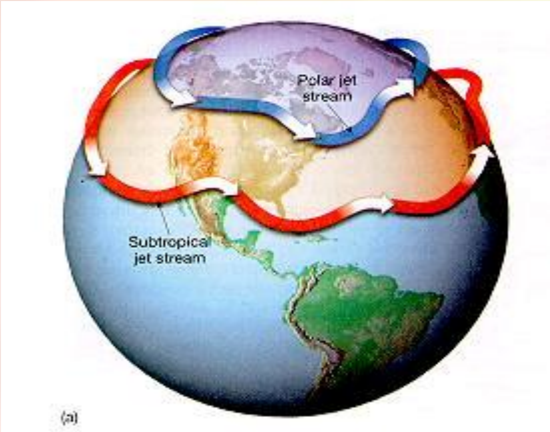
Rossby waves

In meteorology, large horizontal atmospheric undulation that is associated with the polar-front jet stream and separates cold polar air from warm tropical air.

Rossby waves are formed when polar air moves toward the Equator while tropical air is moving poleward.





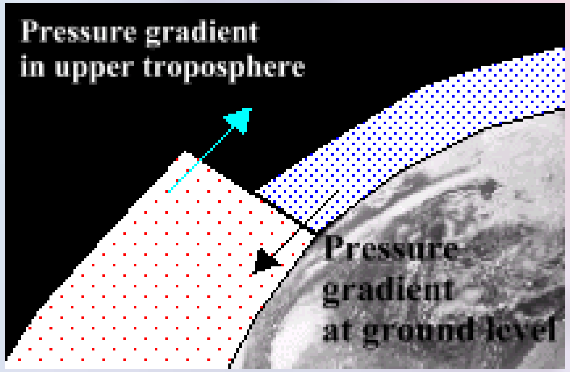


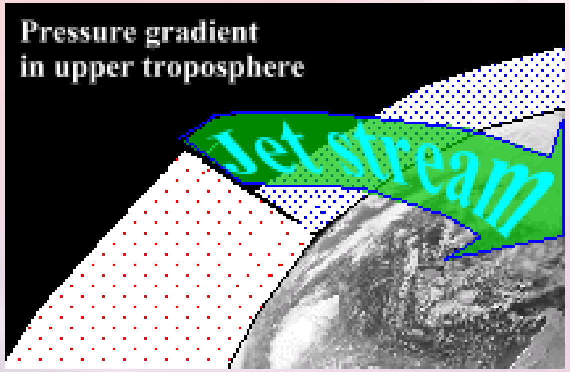
This diagram shows the thicker atmosphere of the tropics converging against the thinner polar atmosphere.

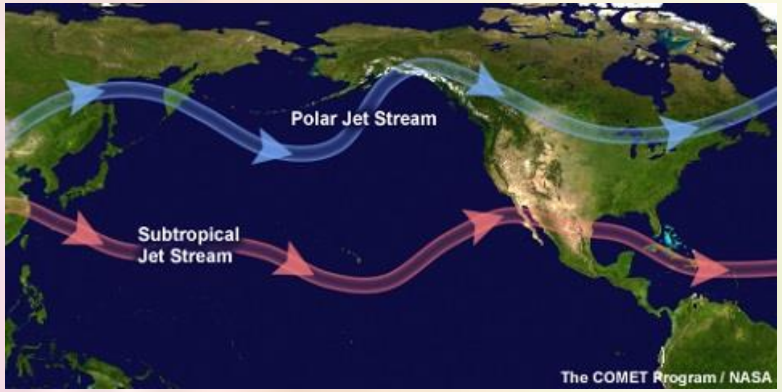
The band on the Earth's surface where this takes place is known as the polar front.

From the diagram, it is clear that this wind would start as a south to north flow but the spin of the earth would deflect it to a westerly wind flow which meanders around the earth at the temperate latitudes.

The net effect is a high speed ribbon of high altitude wind. This flows entirely around the globe and - like all fluid motions - has a tendency to meander.







The meanders in the upper westerly circulation are known as ***Rossby waves***.

Within these upper wind belts particularly strong ribbons of wind form.

These are known as jet streams. They are strong, narrow currents of wind thousands of kilometres in length, hundreds of kilometres wide and 2 - 4 kilometres deep.

Rossby Waves

Rossby waves, also known as *planetary waves* owe their origin to the shape and rotation of the earth

Travel from ***east to west***, following latitude

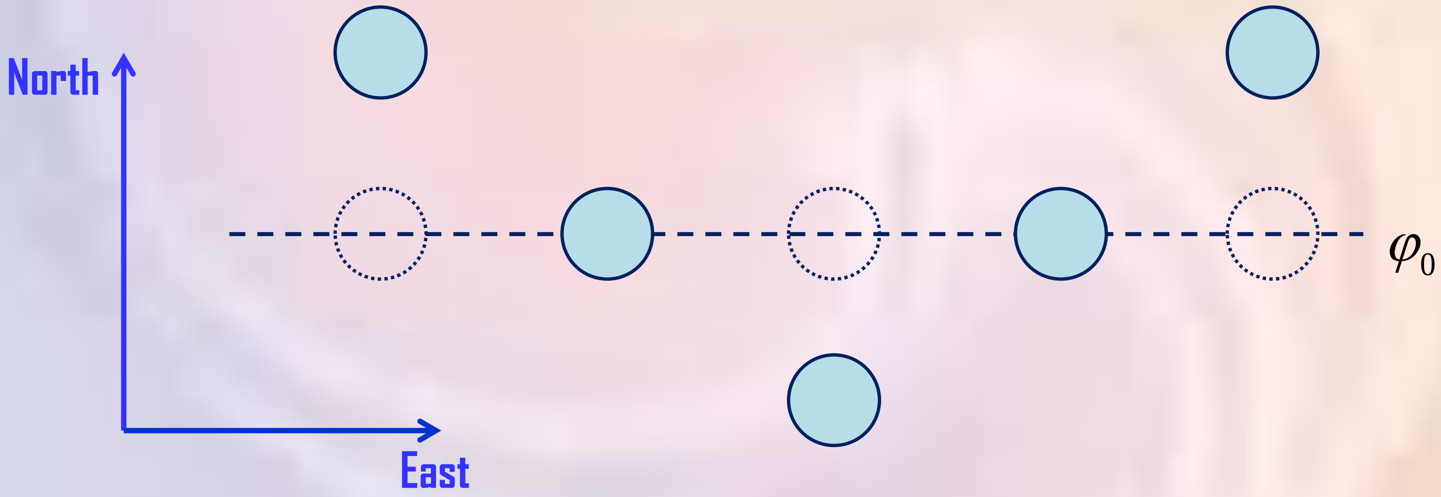
Slow moving

- Speed varies with latitude slower near the pole, faster near the Equator
- On the order of <u>a few cm/s</u> (or a few km/day)

Mechanism of Rossby Wave Propagation

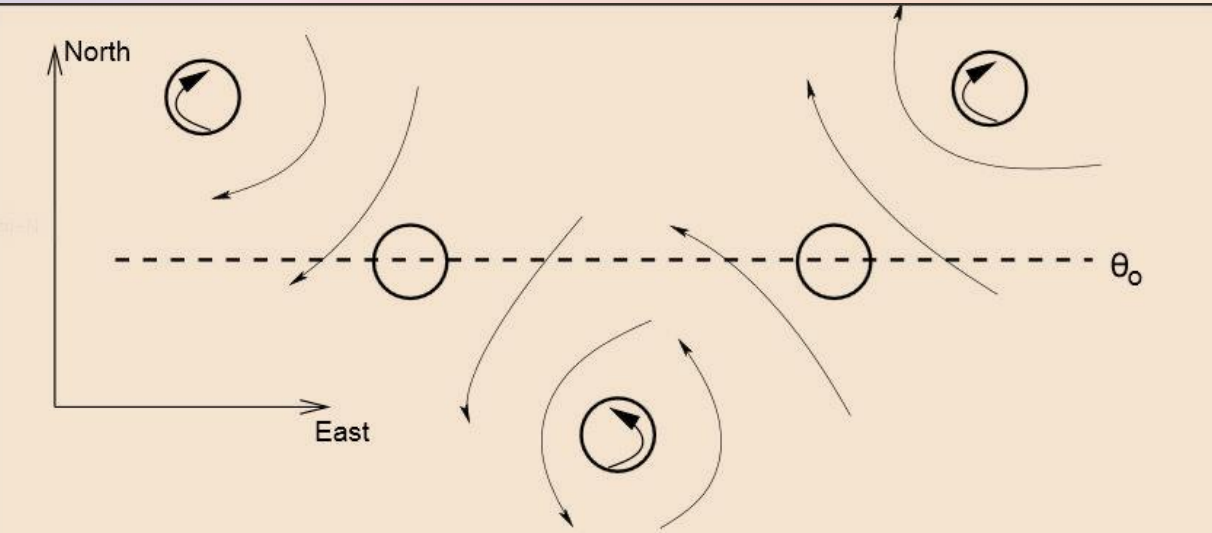
Consider five columns of water all initially at the same latitude on a β-plane. More two the columns north and one to south, conserving potential vorticity.

As we are assuming the water depth is constant, and thus the fluid column lengths remain constant, consevation of potential vorticity is just conservation of total vorticity ξ+f .



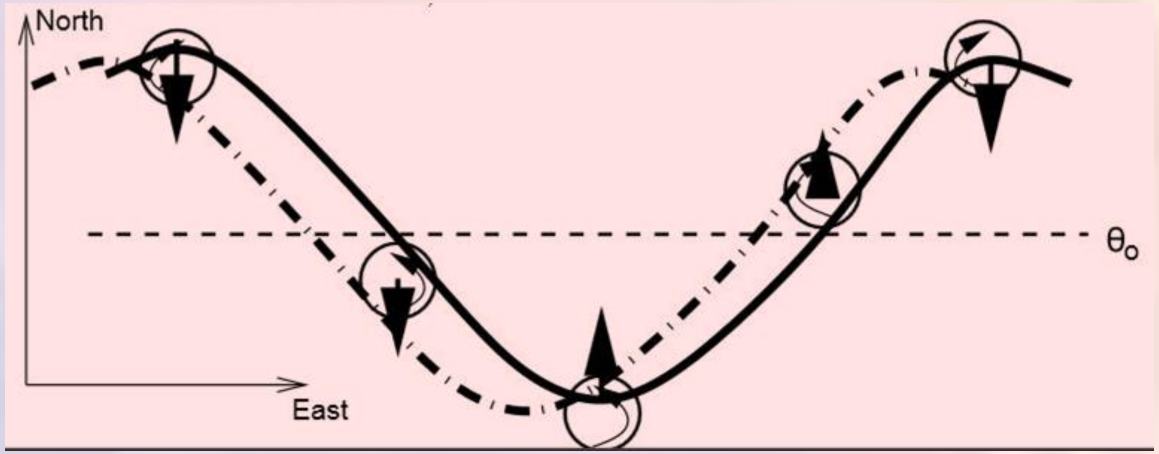
The two columns moved north have increased planetary vorticity and so get anti-cyclonic relative vorticity.

The column moved south has decreased planetary vorticity and thus gets cyclonic relative vorticity. This vorticity field generates as flow field.



This flow field tends to move the other two columns. But as soon as they are displaced, They too have vorticity. This vorticity tends to restore the initial three columns back to their original position.

Also note that the second column from left moves toward deep water whereas the forth column moves toward shallow water. Thus the pattern is moving to the left (from the solid curve to the dash-dot curve).



Propagate zonally from east to west

Results from conservation of potential vorticity, PV = $(\xi + f)/h$