Atmospheric Dynamics

Lecture 10

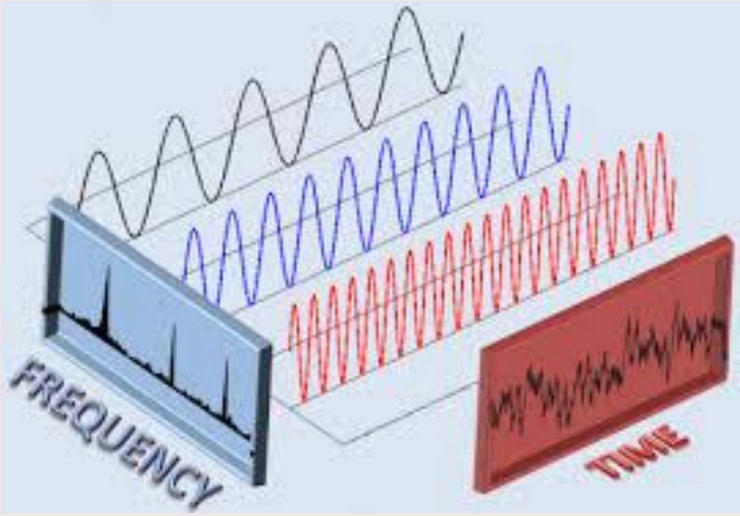
Sahraei

Physics Department
Razi University

https://sci.razi.ac.ir/~sahraei

Fourier Series

Each wave package (disturbance) can be represented as a sum of waves.



Fourier Series

The representation of a perturbation as a simple sinusoidal wave might seem an oversimplification, since disturbances in the atmosphere are never purely sinusoidal.

It can be shown, however, that any reasonably well-behaved function of longitude can be represented in terms of a zonal mean plus a *Fourier series* of sinusoidal components:

$$f(x) = \sum_{s=1}^{\infty} (A_s \sin k_s x + B_s \cos k_s x)$$

$k_s = \frac{2\pi s}{L}$ The zonal wave number (m^{-1})

L is the distance around a latitude circle,

s, the planetary wave number, is an integer des *designating* the number of waves around a latitude circle.

Fourier Series

$$f(x) = \sum_{s=1}^{\infty} (A_s \sin k_s x + B_s \cos k_s x)$$

disturbance

The coefficients A_s are calculated by multiplying both sides of equ. by

$\sin\left(\frac{2\pi nx}{L}\right)$ where *n is an* integer, and integrating around a latitude circle.

Applying the orthogonality relationships

$$\int_0^L \sin\frac{2\pi sx}{L}\sin\frac{2\pi nx}{L}\,dx = \begin{cases} 0 & s \neq n \\ L/2 & s = n \end{cases}$$

$$\therefore A_s = \frac{2}{L}\int_0^L f(x)\sin\frac{2\pi sx}{L}\,dx$$

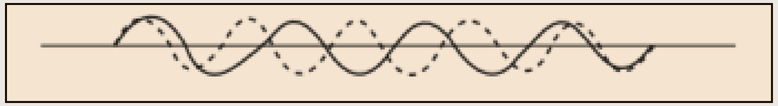
In a similar fashion, multiplying both sides in equ. by $\cos\left(\frac{2\pi nx}{L}\right)$ and integrating gives:

$$B_s = \frac{2}{L}\int_0^L f(x)\cos\frac{2\pi sx}{L}\,dx$$

A_s and B_s are called the Fourier coefficients

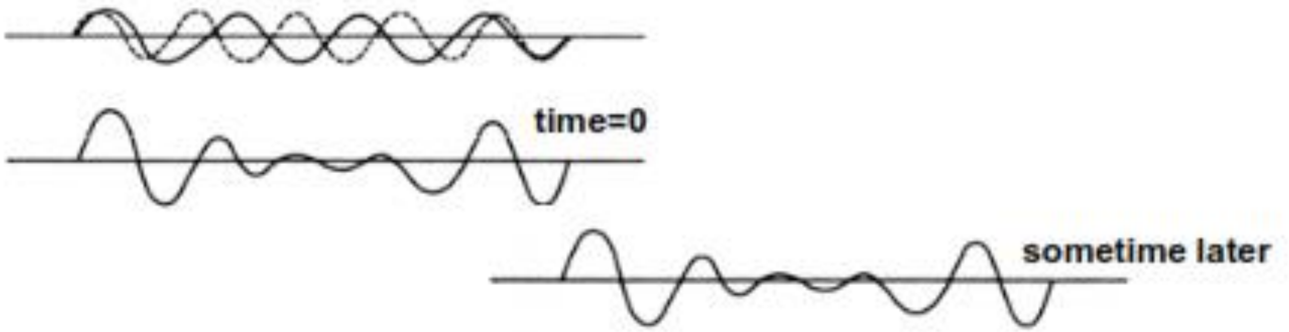
Dispersion and Group Velocity

Wave groups formed from two sinusoidal components of slightly different wavelengths.



For nondispersive waves, propagates without change of shape

Non-Dispersive Waves



- Some types of waves, such as acoustic waves, have phase speeds that are independent of the wave number.
- In such *nondispersive waves a spatially* localized disturbance consisting of a number of Fourier wave components (a *wave group) will preserve its shape as it propagates in space at the phase speed of the* wave.

ESS227
Prof. Jin-Yi Yu

A Wave Solution Strategy

Approximations to the full equations governing atmospheric dynamics will be solved for wave motions many times.

Even though aspects of each individual case are different, a guide to the general approach to solving these problems isas follows:

1. Choose a basic state

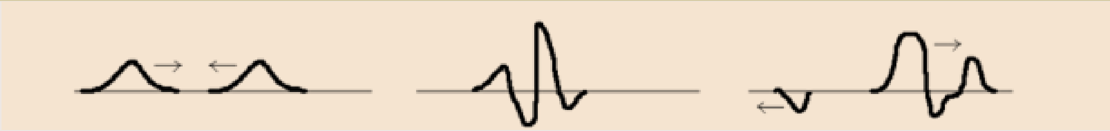
2. Linearize the governing equations

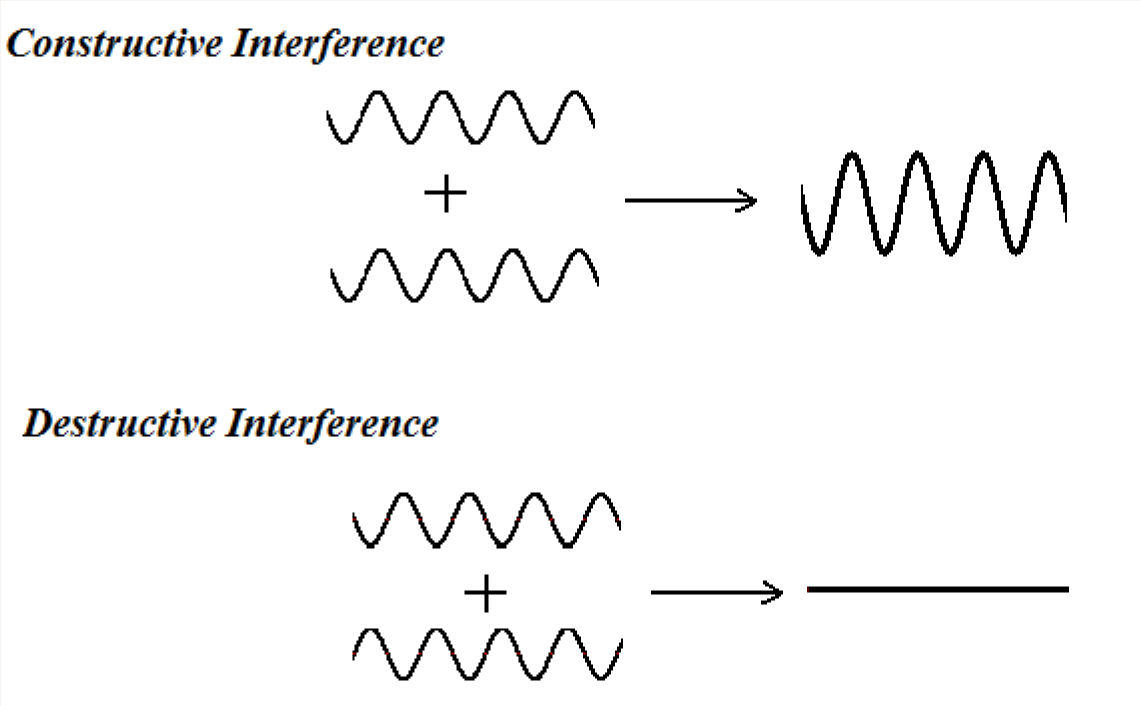
3. Assume wave solutions of the form in equation

$$f(x,y,t) = \mathrm{Re}(Ae^{i(kx+ly-\omega t)}) = \mathrm{Re}(Ae^{i\phi})$$

$$F_y e^{i(kx-\omega t)}$$

4. Solve for the dispersion and polarization relationships




Constructive Interference
Destructive Interference

SIMPLE WAVE TYPES

Waves in fluids result from the action of restoring forces on fluid parcels that have been displaced from their equilibrium positions.

The restoring forces may be due to compressibility, gravity, rotation, or electromagnetic effects.

This section considers the two simplest examples of linear waves in fluids:

1) acoustic waves

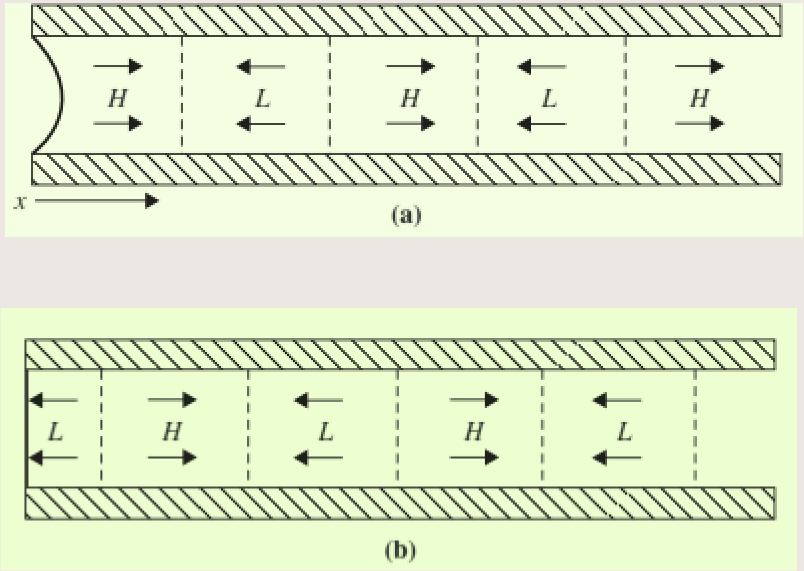
2) shallow water gravity waves

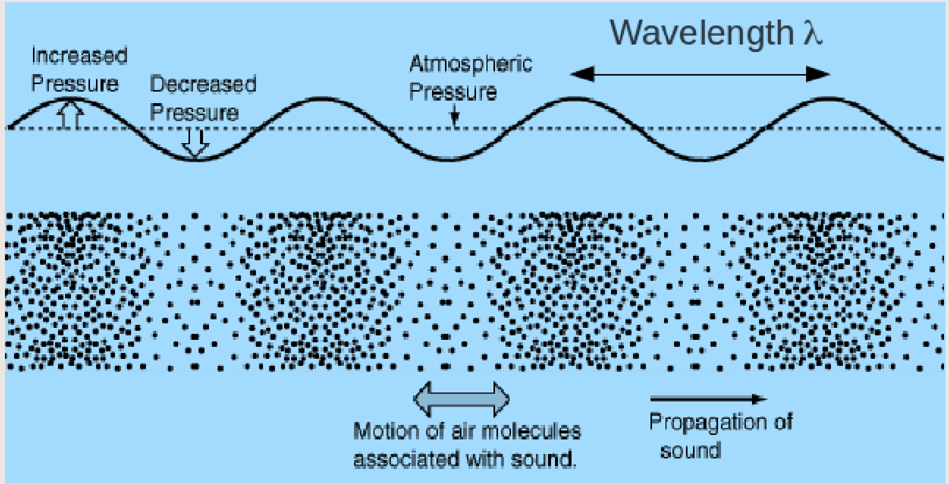
Acoustic (Sound) Waves

1) Sound waves, or acoustic waves, are *longitudinal waves*

2) Sound is propagated by the alternating adiabatic compression and expansion of the medium.

As an example,





Describe mathematiclly

1) the perturbation method

2) adiabatic

3) waves propagating in a straight pipe parallel to the x axis (one-dimensional sound)

4) To exclude the possibility of transverse oscillations (for simplicity) we assume:

$$\vec{V} = (u, 0, 0) \quad and \quad u = u(x, t)$$

With these restrictions,
- the momentum equation,
- thermodynamic energy equation
- continuity equation

for adiabatic motion are

$$\frac{du}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

momentum equation

$$\frac{d\rho}{dt} + \rho\frac{\partial u}{\partial x} = 0$$

continuity equation

$$\frac{d\ln\theta}{dt} = 0$$

thermodynamic energy equation

For adiabatic process

$$c_p\frac{dT}{dt} - \alpha\frac{dp}{dt} = \dot{q}$$

Dividing through by T and again using the equation of state, we obtain the entropy form of the first law of thermodynamics:

$$c_p\frac{d\ln T}{dt} - R\frac{d\ln p}{dt} = \frac{\dot{q}}{T} \equiv \frac{ds}{dt} \quad *$$

the rate of change of entropy per unit mass following the thermodynamically *reversible process.*

$$\theta = T\left(\frac{p_0}{p}\right)^{R/c_p} \qquad c_p \frac{d\ln\theta}{dt} = c_p \frac{d\ln T}{dt} - R\frac{d\ln p}{dt}$$

Comparing *, we ontain $$c_p \frac{d\ln\theta}{dt} = \frac{\dot{q}}{T} = \frac{ds}{dt}$$

where for this case $$\frac{d}{dt} = \frac{\partial}{\partial t} - u\frac{\partial}{\partial x}$$

$$\theta = \left(\frac{p}{\rho R}\right)\left(\frac{p_0}{p}\right)^{R/c_p} \rightarrow \ln\theta = \ln\frac{p_0^{\frac{R}{c_p}}}{R} + \ln p^{\frac{1-R}{c_p}} - \ln\rho$$

$$\frac{d\ln\theta}{dt} = -\frac{d\ln\rho}{dt} + \frac{1}{\gamma}\frac{d\ln p}{dt} = 0 \qquad where \quad \frac{1-R}{c_p} = \frac{c_v}{c_p} = \frac{1}{\gamma}$$

where p_0 =1000 hPa, we may eliminate θ in $$\frac{d \ln \theta}{dt} = 0$$

$$\frac{1}{\gamma}\frac{d \ln p}{dt} - \frac{d \ln \rho}{dt} = 0$$

$$\frac{d\rho}{dt} + \rho\frac{\partial u}{\partial x} = 0$$

Eliminating ρ

$$\frac{1}{\gamma}\frac{d \ln p}{dt} + \frac{\partial u}{\partial x} = 0$$

The perturbation method

$$u(x,t) = \bar{u} + u'(x,t)$$

$$p(x,t) = \bar{p} + p'(x,t)$$

$$\rho(x,t) = \bar{\rho} + \rho'(x,t)$$

Substituting into $\frac{du}{dt}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0$ and $\frac{1}{\gamma}\frac{d\ln p}{dt}+\frac{\partial u}{\partial x}=0$

$$\frac{\partial}{\partial t}(\bar{u}+u')+(\bar{u}+u')\frac{\partial}{\partial x}(\bar{u}+u')+\frac{1}{(\bar{\rho}+\rho')}\frac{\partial}{\partial x}(\bar{p}+p')=0$$

$$\frac{\partial}{\partial t}(\bar{p}+p')+(\bar{u}+u')\frac{\partial}{\partial x}(\bar{p}+p')+\gamma(\bar{p}+p')\frac{\partial}{\partial x}(\bar{u}+u')=0$$

$|\rho'/\bar{\rho}|\ll 1$, we can use the binomial expansion to approximate the density term as

$$\frac{1}{(\bar{\rho}+\rho')}=\frac{1}{\bar{\rho}}\left(1+\frac{\rho'}{\bar{\rho}}\right)^{-1}\approx\frac{1}{\bar{\rho}}\left(1-\frac{\rho'}{\bar{\rho}}\right)$$

we obtain the linear perturbation equations

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)u'+\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x}=0 \qquad \left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)p'+\gamma\bar{p}\frac{\partial u'}{\partial x}=0$$

Eliminate u' by applying $\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)$ on *

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)u' = -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x}$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)p' + \gamma\bar{p}\frac{\partial u'}{\partial x} = 0 \quad *$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)^2 p' - \frac{\gamma\bar{p}}{\bar{\rho}}\frac{\partial^2 p'}{\partial x^2} = 0$$

which is a form of the standard wave equation familiar from electromagnetic theory. A simple solution representing a plane sinusoidal wave propagating in x is

$$p' = A\,\mathrm{e}^{ik(x-ct)}$$

the assumed solution

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)^2 p' - \frac{\gamma\bar{p}}{\bar{\rho}}\frac{\partial^2 p'}{\partial x^2} = 0$$

$$p' = A\,e^{ik(x-ct)}$$

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)^2 p' = \left(\frac{\partial^2}{\partial t^2} + 2\bar{u}\frac{\partial^2}{\partial x \partial t} + \bar{u}^2\frac{\partial^2}{\partial x^2}\right)p' = (-ikc + ik\bar{u})^2 p'$$

$$\frac{\gamma\bar{p}}{\bar{\rho}}\frac{\partial^2 p'}{\partial x^2} = \frac{\gamma\bar{p}}{\bar{\rho}}(ik)^2 p'$$

$$(-ikc + ik\bar{u})^2 - \frac{\gamma\bar{p}}{\bar{\rho}}(ik)^2 = 0$$

Dispersion relation

Solving for c gives

$$c = \bar{u} + \sqrt{\frac{\gamma\bar{p}}{\bar{\rho}}} = \bar{u} + \sqrt{\gamma R\bar{T}}$$

the phase speed

the speed of wave propagation relative to the zonal current is

$$c - \bar{u} = \pm c_s$$

where $c_s = \sqrt{\gamma R \bar{T}}$ is called the *adiabatic speed of sound*

The mean zonal velocity here plays only a role of Doppler shifting the sound wave so that the frequency relative to the ground corresponding to a given wave number k is:

$$\omega = kc = k(\bar{u} \pm c_s)$$

Thus, in the presence of a wind, the frequency as heard by a fixed observer depends on the location of the observer relative to the source.

$$if \quad \bar{u} > 0$$

The frequency of a stationary source will appear to be higher for an observer to the east (downstream) of the source

$$c = \bar{u} + c_s$$

than for an observer to the west (upstream) of the source

$$c = \bar{u} - c_s$$