

# *Atmospheric Dynamics*

## Lecture 1

Sahraei

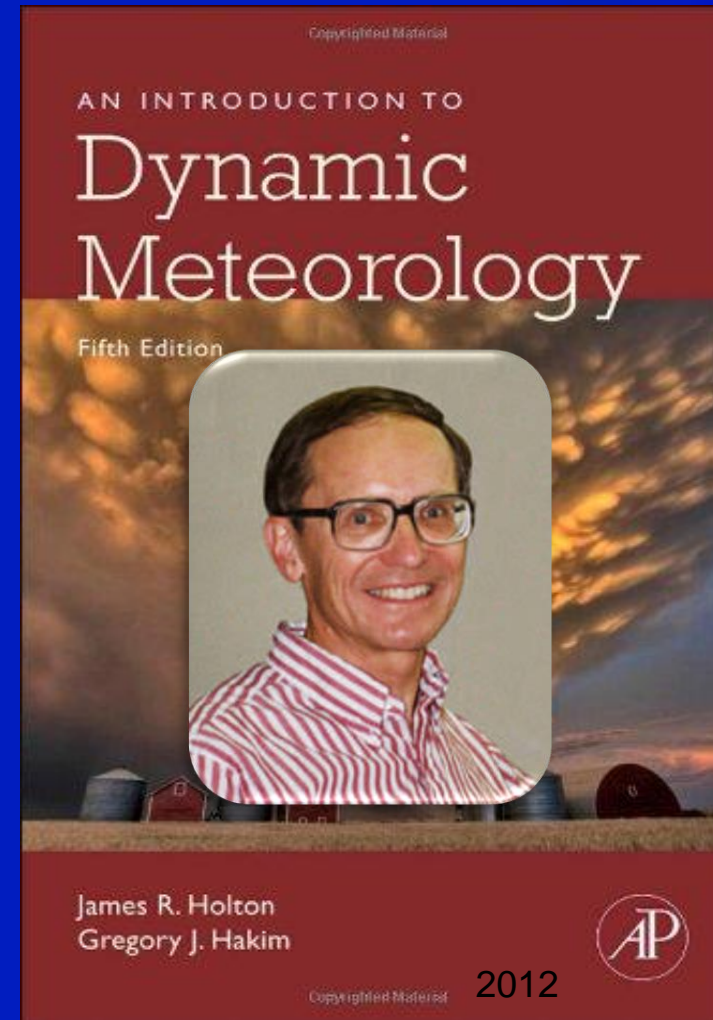
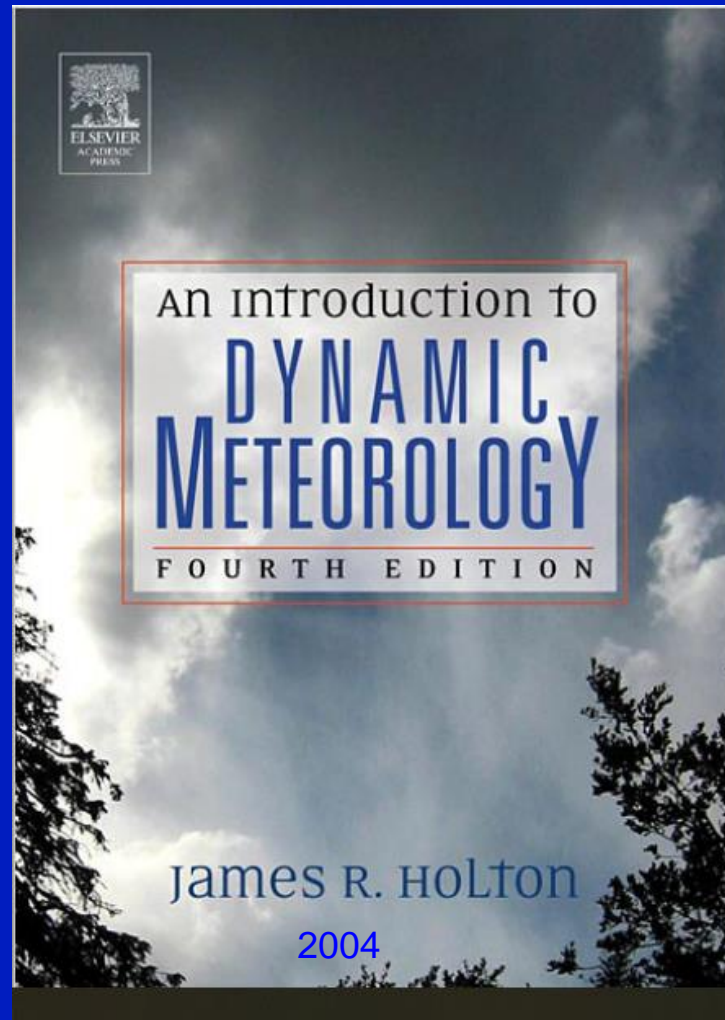
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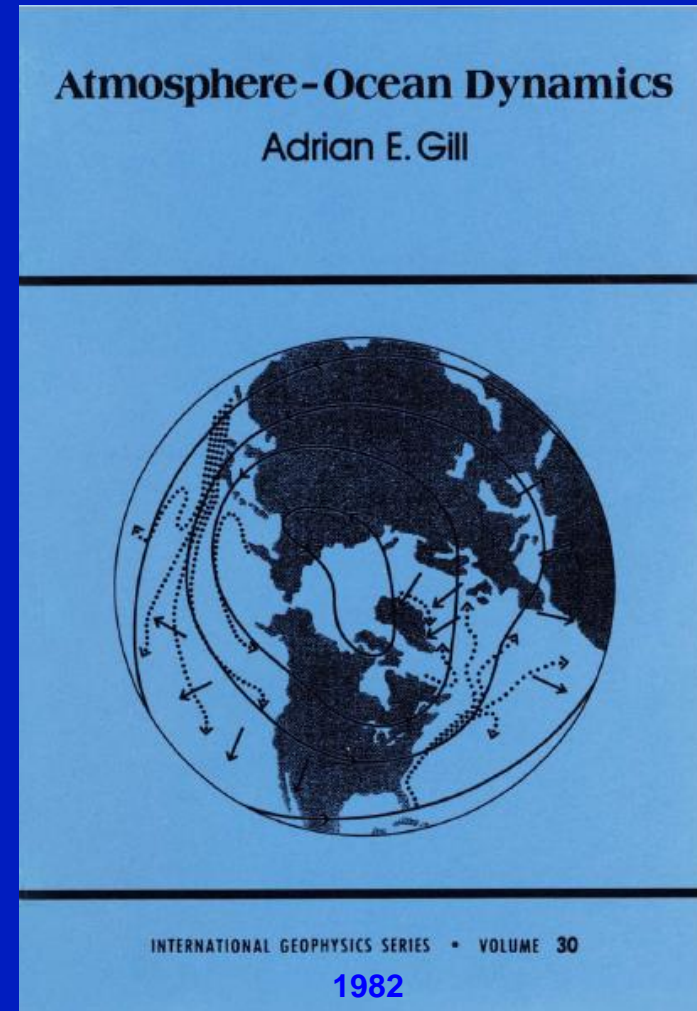
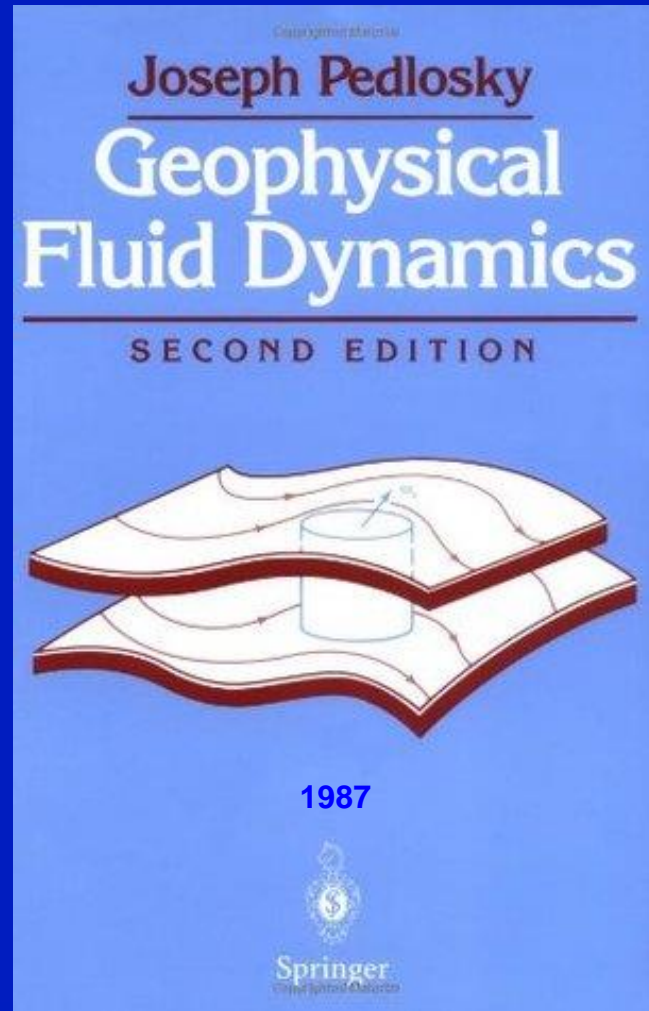


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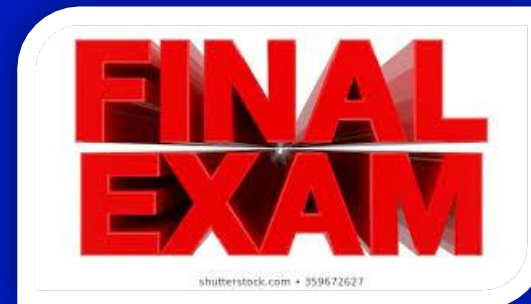
1. Holton, J.R., 2004: *An Introduction to Dynamic Meteorology*. Academic Press, 535 pp.
2. Holton, J.R., and G. J. Hakim, 2013: *An Introduction to Dynamic Meteorology*. Academic Press, 532 pp.
3. Pedlosky, J., 1987: *Geophysical Fluid Dynamics*. Springer Verlag, New York, 624 pp.
4. Gill, A., 1982: *Atmosphere-Ocean Dynamics*. Academic Press, 662 pp.
5. Andrews, D.G., J.R. Holton, and C.B. Leovy, 1987: *Middle Atmosphere Dynamics*. Academic Press, 489 pp.
6. Hoskins, B. J., and I. N. James, 2014: *Fluid Dynamics of the MidLatitude Atmosphere*. John Willey & Sons, Ltd, 408pp.
7. Vallis, G. K., 2006: *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation*. Cambridge University Press, 737pp.
8. Zdunkowski, W., and A. Bott, 2003: *Dynamics of the Atmosphere*. Cambridge University Press, 742pp.

## References





# Assessment



عنوان درس به فارسی: دینامیک جو

عنوان درس به انگلیسی: (Atmospheric Dynamics)

تعداد واحد: ۲

نوع واحد: نظری

نوع درس: تخصصی

تعداد ساعت: ۴۸ ساعت

پیش‌نیاز: دینامیک شاره های ژئوفیزیکی

همینا: -

آموزش تکمیلی عملی: دارد  $O$  ندارد • سفر علمی  $O$  کارگاه  $O$  آزمایشگاه  $O$  سمینار  $O$

اهداف کلی درس: آشنایی با حرکت‌ها و گردش‌های جو و پایداری یا ناپایداری آن‌ها.

سرفصل درس: ۴۸ ساعت نظری

### فصل اول - حرکت‌های مقیاس همدیدی

ساختار مشاهداتی گردش‌های برون‌حاره‌ای، نظریه شبه‌زمینگرد، معادلات شبه‌زمینگرد تکانه، تاوایی، گرایش ارتفاع ژئوپتانسیلی، تاوایی پتانسیلی و سرعت قائم؛ گردش زمینگرد، مدل ایده‌آلی یک آشفتگی کُر فشار.

### فصل دوم - نوسانات و امواج جو

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### فصل چهارم - دینامیک جو میانی

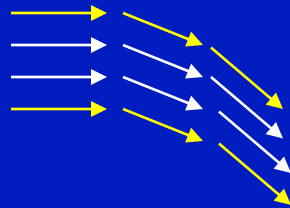
ساختار و گردش جو میانی، گردش میانگین مداری جو میانی، انتشار قائم امواج سیارهای، گرمایش‌های ناگهانی پوشش سپهری، امواج در پوشش سپهر استوایی، نوسان شبه دوسالانه، انتقال ردیاب‌های جو.

# Vorticity

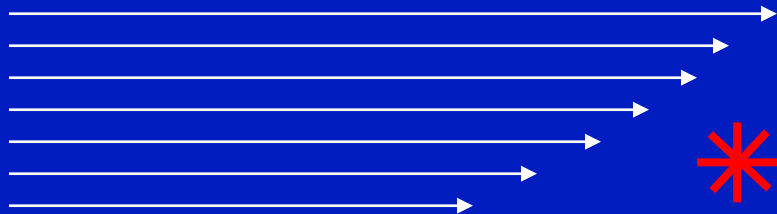
Vorticity is the microscopic measure of spin and rotation in a fluid.

Vorticity is defined as the curl of the velocity:

$$\nabla \times \vec{V}$$



Wind direction varies  $\rightarrow$  clockwise spin



Wind speed varies  $\rightarrow$  clockwise spin

Absolute vorticity (inertial reference frame):

$$\vec{\omega}_a \equiv \nabla \times \vec{V}_a$$

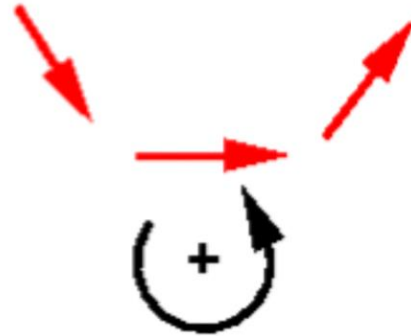
Relative vorticity (relative to rotating earth):

$$\vec{\omega} \equiv \nabla \times \vec{V}$$

# Vorticity

Curvature  
vorticity

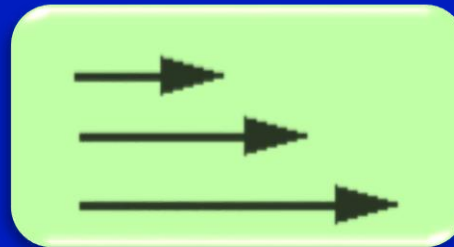
Positive (cyclonic in NH)



Negative (anticyclonic in NH)



Shear  
vorticity



Expansion of relative vorticity into Cartesian components:

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$
$$\nabla \times \vec{V} = \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \hat{i} + \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \hat{j} + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k}$$

For large scale dynamics, the vertical component of vorticity is most important.

The vertical components of absolute and relative vorticity in vector notation are:

$$\zeta = \hat{k} \cdot (\nabla \times \vec{V})$$

relative vorticity

$$\eta = \hat{k} \cdot (\nabla \times \vec{V}_a)$$

absolute vorticity

From now on, vorticity implies the vertical component (unless otherwise stated.)

The absolute vorticity is equal to the relative vorticity plus the earth's vorticity. Since the earth's vorticity is

$$\hat{k} \cdot (\nabla \times \vec{V}_e) = 2\Omega \sin \phi = f$$

then

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad \text{and} \quad \eta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f = \zeta + f$$

For large scale circulations, a typical magnitude for vorticity is

$$\zeta \approx \frac{U}{L} = 10^{-5} \text{ s}^{-1}$$

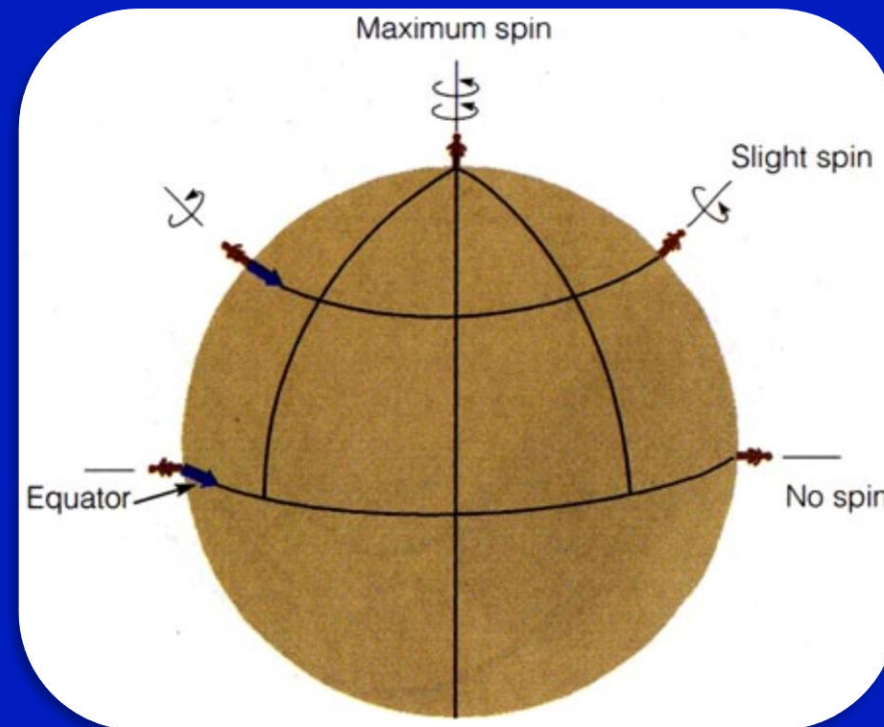
$$\eta = \zeta + f$$

Planetary Vorticity is spin produced by earth's rotation

$$f = 2\Omega \sin \phi$$

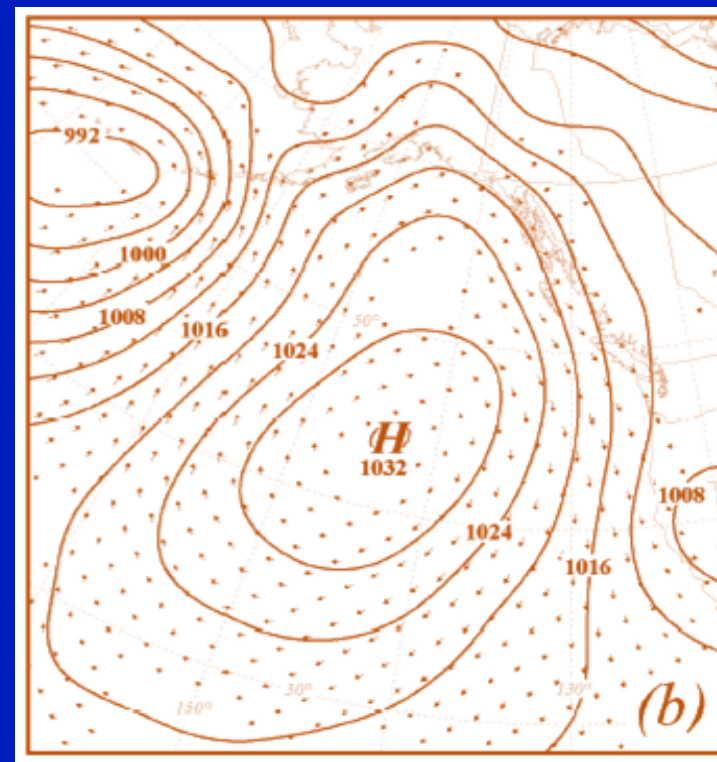
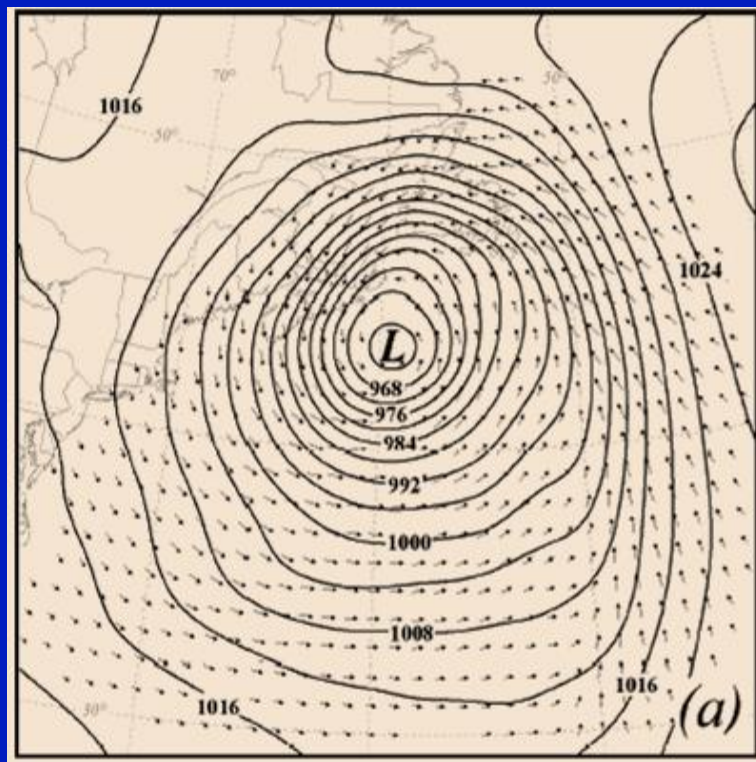
Component of earth's rotation  
oriented around local vertical

Always positive in Northern  
Hemisphere 0 at equator, increases  
northward



# FLUID ROTATION

## Circulation and Vorticity



# Circulation and Vorticity

Two primary measures of rotation in a fluid

By convention, both circulation and vorticity are positive in the counterclockwise direction.

(cyclonic in the Northern Hemisphere)

## Circulation:

Macroscopic measure of rotation for a finite area of the fluid = integration of the tangential component of velocity around a closed path

**Vorticity:** The tendency to spin about an axis; Microscopic measure of rotation at any point in the fluid

**Circulation:** The tendency for a group of air parcels to rotate.

If an area of atmosphere is of interest, you compute the circulation.

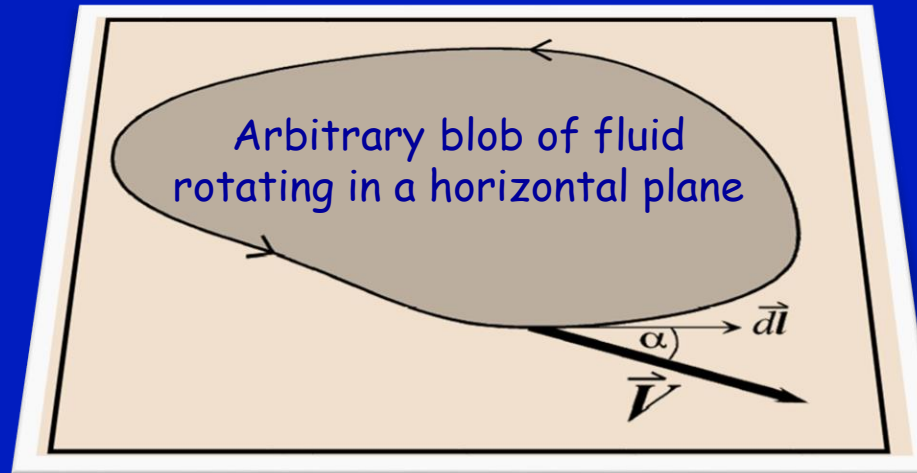
**Vorticity:** The tendency for the wind shear at a given point to induce rotation.

If a point in the atmosphere is of interest, you compute the vorticity

## THE CIRCULATION THEOREM

The circulation,  $C$ , about a closed contour in a fluid is defined as the line integral evaluated along the contour of the component of the velocity vector that is locally tangent to the contour:

$$C = \oint \vec{V} \cdot d\vec{l} = \oint |V| \cos \alpha \, dl$$



Where  $\vec{l}(s)$  is a position vector extending from the origin to the point  $s(x, y, z)$  on the contour  $C$ ,

$d\vec{l}$  represents the limit of  $\delta\vec{l} = \vec{l}(s + \delta s) - \vec{l}(s)$  as  $\delta s \rightarrow 0$ .

Hence, as indicated in Fig.,  $d\vec{l}$  is a displacement vector locally tangent to the contour.

By convention the circulation is taken to be positive if  $C > 0$  for counterclockwise integration around the contour.

That circulation is a measure of rotation is demonstrated readily by considering a circular ring of fluid of radius  $R$  in solid-body rotation at angular velocity  $\Omega$  about the  $z$  axis.

In this case,  $\vec{U} = \vec{\Omega} \times \vec{R}$ , where  $\vec{R}$  is the distance from the axis of rotation to the ring of fluid.

Thus the circulation about the ring is given by

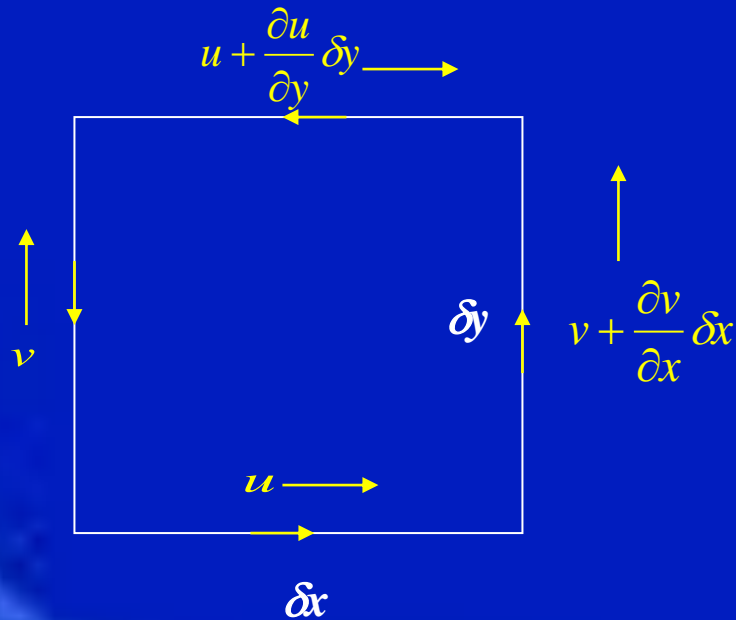
$$C = \oint \vec{V} \cdot d\vec{l} = \int_0^{2\pi} \Omega R^2 d\lambda = 2\Omega\pi R^2$$

In this case the circulation is just  $2\pi$  times the angular momentum of the fluid ring about the axis of rotation.

Alternatively, note that  $C/(\pi R^2) = 2\Omega$  so that the circulation divided by the area enclosed by the loop is just twice the angular speed of rotation of the ring.

## Circulation and Vorticity

The relationship between relative vorticity and circulation can be seen by considering the following expression, in which we will define the relative vorticity as the circulation about a closed contour in the horizontal plane divided by the area enclosed by that contour, in the limit as the area approaches zero.



$$\zeta = \lim_{A \rightarrow 0} \frac{\oint \vec{V} \cdot d\vec{l}}{A} \quad C = \oint u dx + v dy$$

Evaluating  $\vec{V} \cdot d\vec{l}$  for each side of the rectangle yields the circulation:

$$\delta C = u \delta x + \left( v + \frac{\partial v}{\partial x} \delta x \right) \delta y - \left( u + \frac{\partial u}{\partial y} \delta y \right) \delta x - v \delta y$$

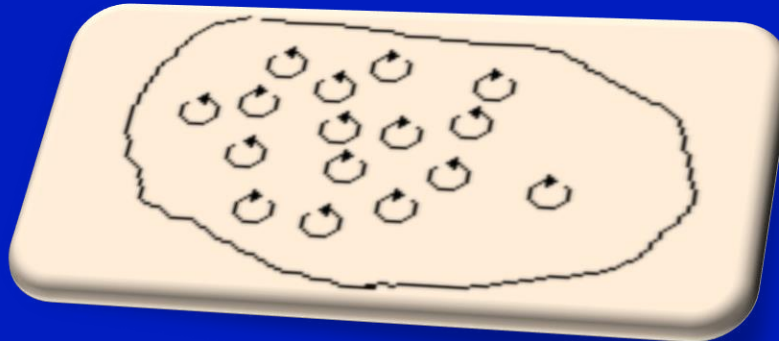
$$\delta C = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \delta x \delta y \quad \delta A = \delta x \delta y$$

$$\frac{\delta C}{\delta A} = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \zeta = \textit{relative vorticity}$$

## *Circulation, Vorticity, and Stokes Theorem*

In more general terms the relationship between vorticity and circulation is given simply by Stokes's theorem applied to the velocity vector:

$$\oint \vec{V} \cdot d\vec{l} = \iint_A (\nabla \times \vec{V}) \cdot \hat{n} dA$$



Here  $A$  is the area enclosed by the contour and  $\hat{n}$  is a unit normal to the area element  $dA$  (positive in the right sense).

*Thus, Stokes's theorem states that the circulation about any closed loop is equal to the integral of the normal component of vorticity over the area enclosed by the contour.*

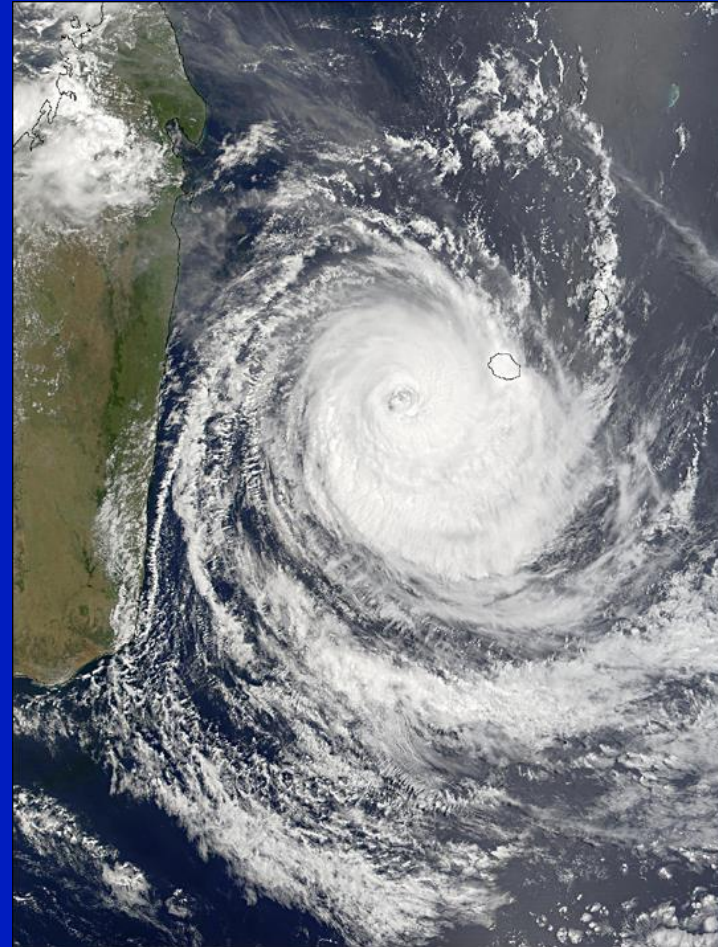
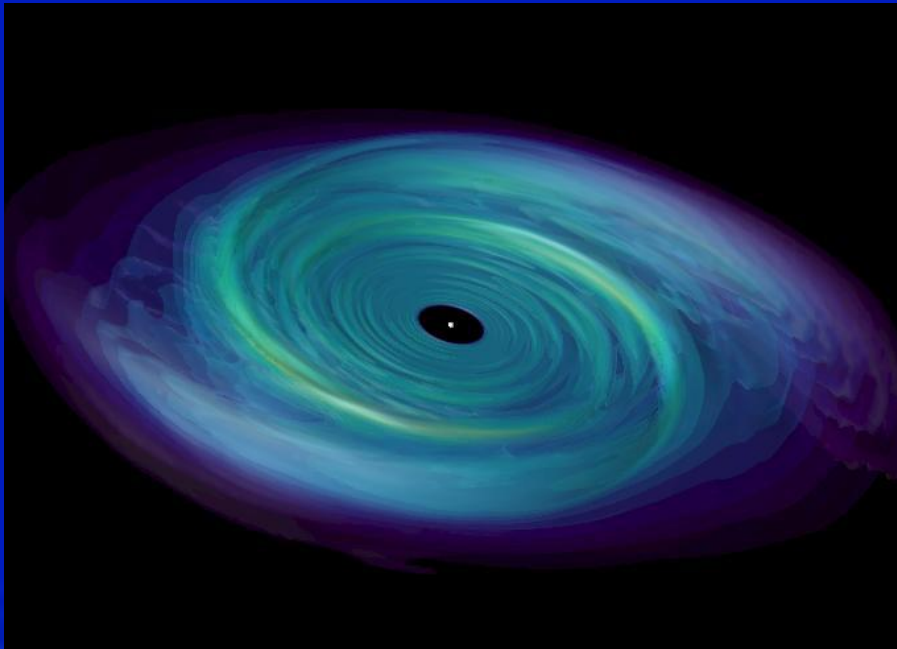
*Hence, for a finite area, circulation divided by area gives the average normal component of vorticity in the region.*

*As a consequence, the vorticity of a fluid in solid-body rotation is just twice the angular velocity of rotation.*

*Vorticity may thus be regarded as a measure of the local angular velocity of the fluid.*

Applications:

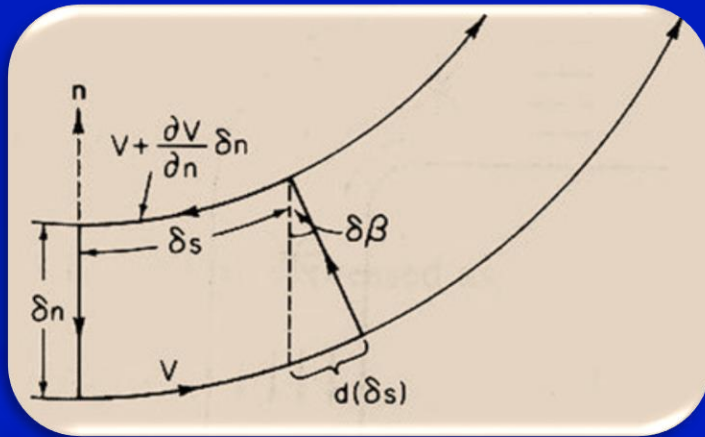
Cyclones & Tornado's



## Vorticity in Natural Coordinates

Using natural coordinates can make it easier to physically interpret the relationship between relative vorticity and the flow.

To express vorticity in natural coordinates, we compute the circulation around the infinitesimal contour shown below.



$$\delta C = V[\delta s + d(\delta s)] - \left( V + \frac{\partial V}{\partial n} \delta n \right) \delta s$$

From the diagram,  $d(\delta s) = \delta \beta \delta n$ , where  $d\beta$  is the angular change in wind direction in the distance  $ds$ .

$$\delta C = \left( -\frac{\partial V}{\partial n} + V \frac{\partial \beta}{\partial s} \right) \delta n \delta s$$

Or in the limit  $\delta n \delta s \rightarrow 0$

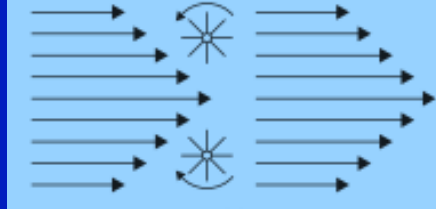
$$\zeta = \lim_{\delta n, \delta s \rightarrow 0} \frac{\delta C}{\delta n \delta s} = -\frac{\partial V}{\partial n} + \frac{V}{R_s} \quad \text{where } R_s \text{ is the radius of curvature of the streamlines}$$

Vorticity in natural coordinates:

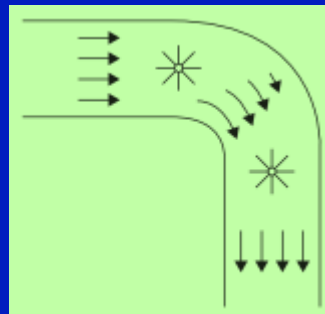
$$\zeta = -\frac{\partial V}{\partial n} + \frac{V}{R_s}$$

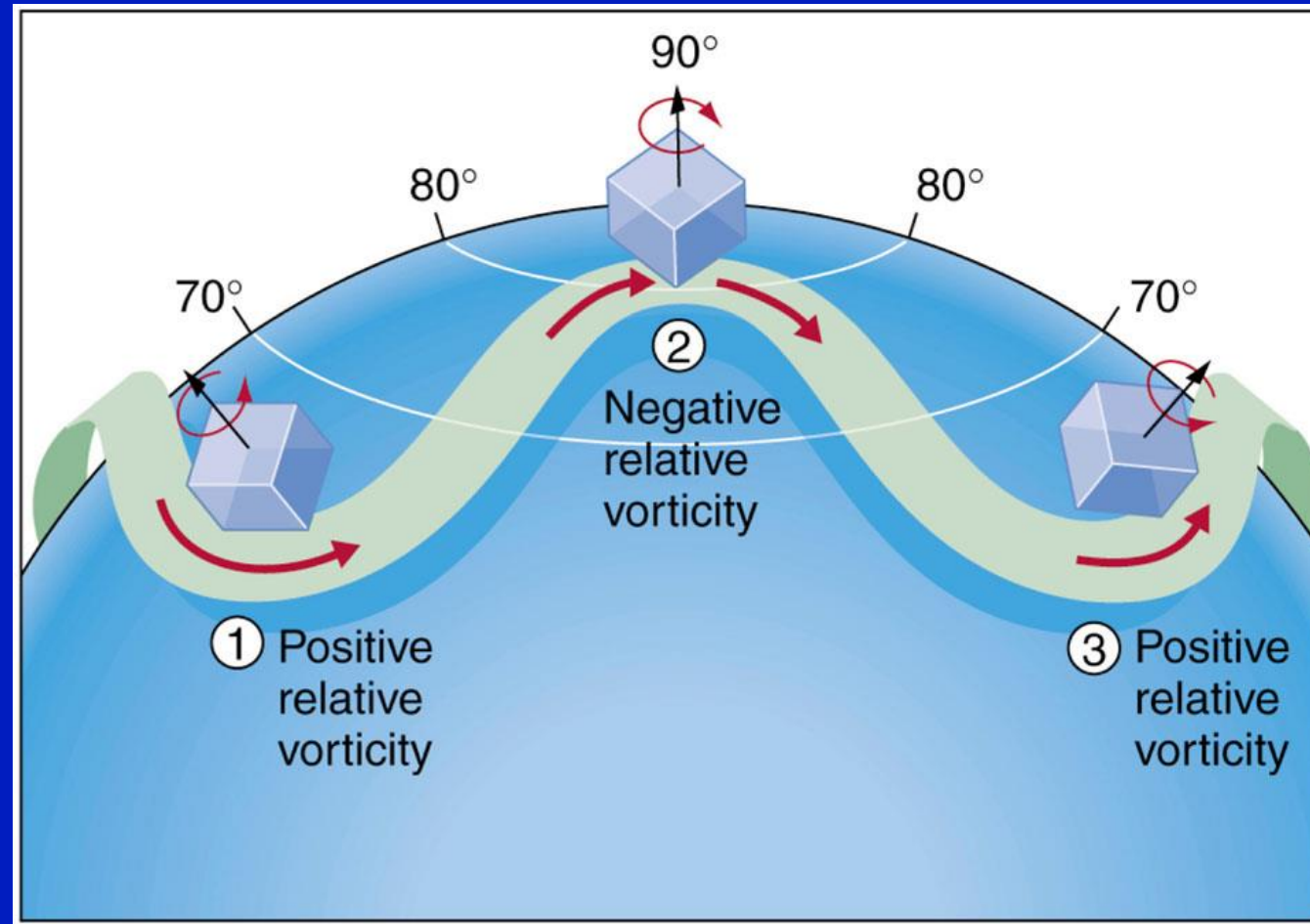
It is now apparent that the net vertical vorticity component is the result of the sum of two parts:

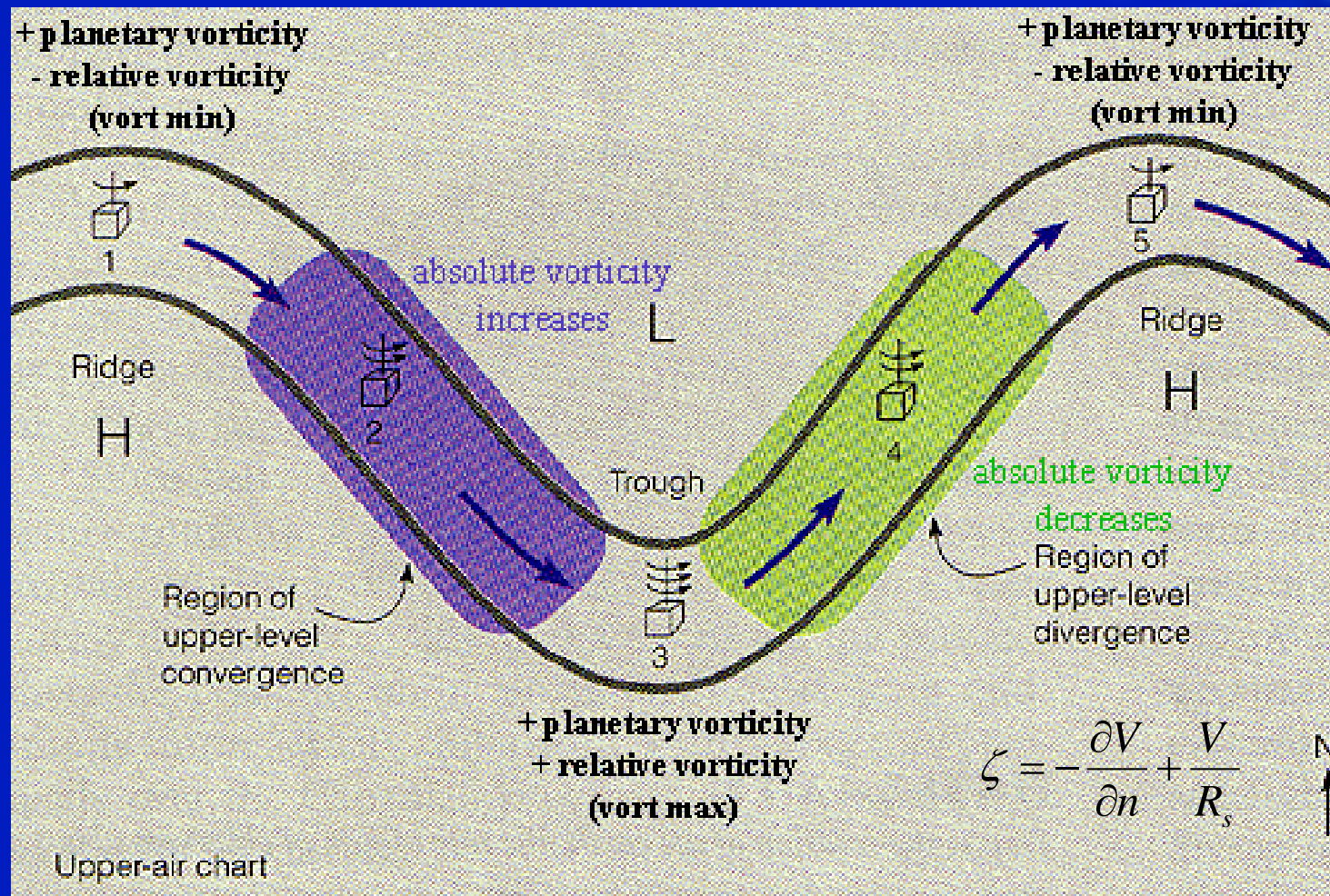
$-\frac{\partial V}{\partial n}$  The rate of change of wind speed normal to the direction of flow, which is called the shear vorticity.



$\frac{V}{R_s}$  The turning of the wind along a streamline, which is called the curvature vorticity.

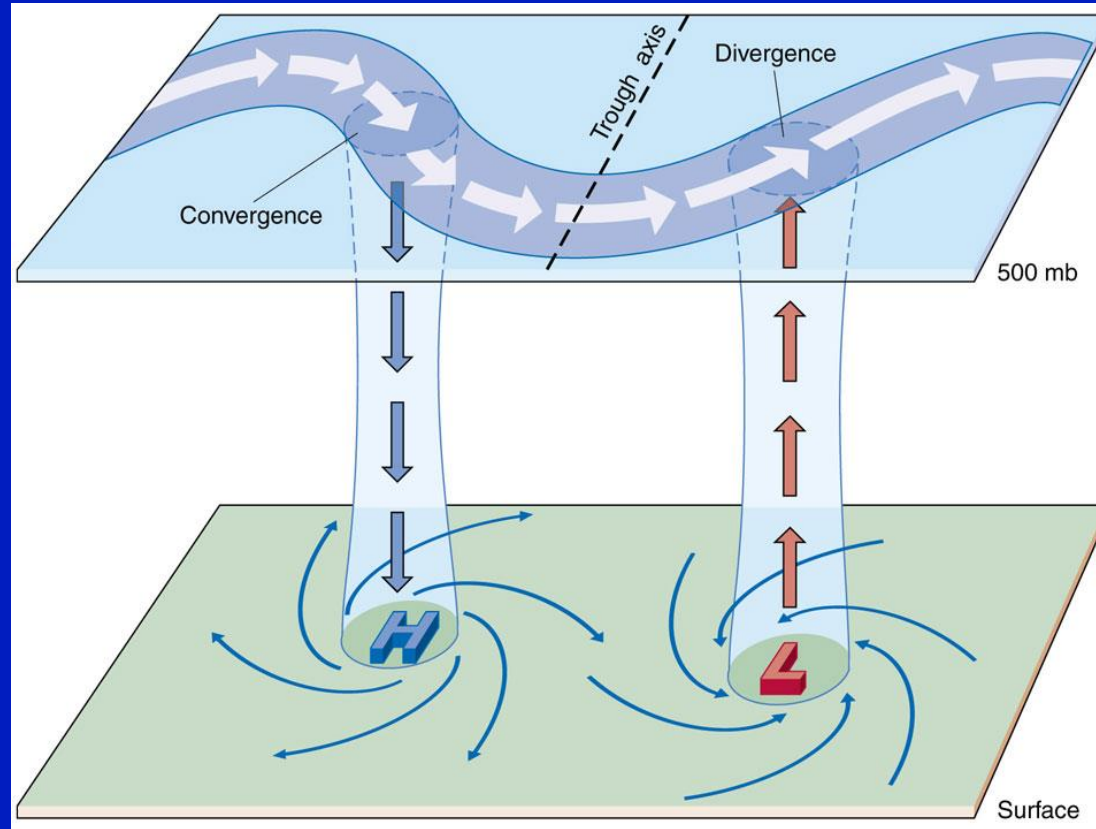


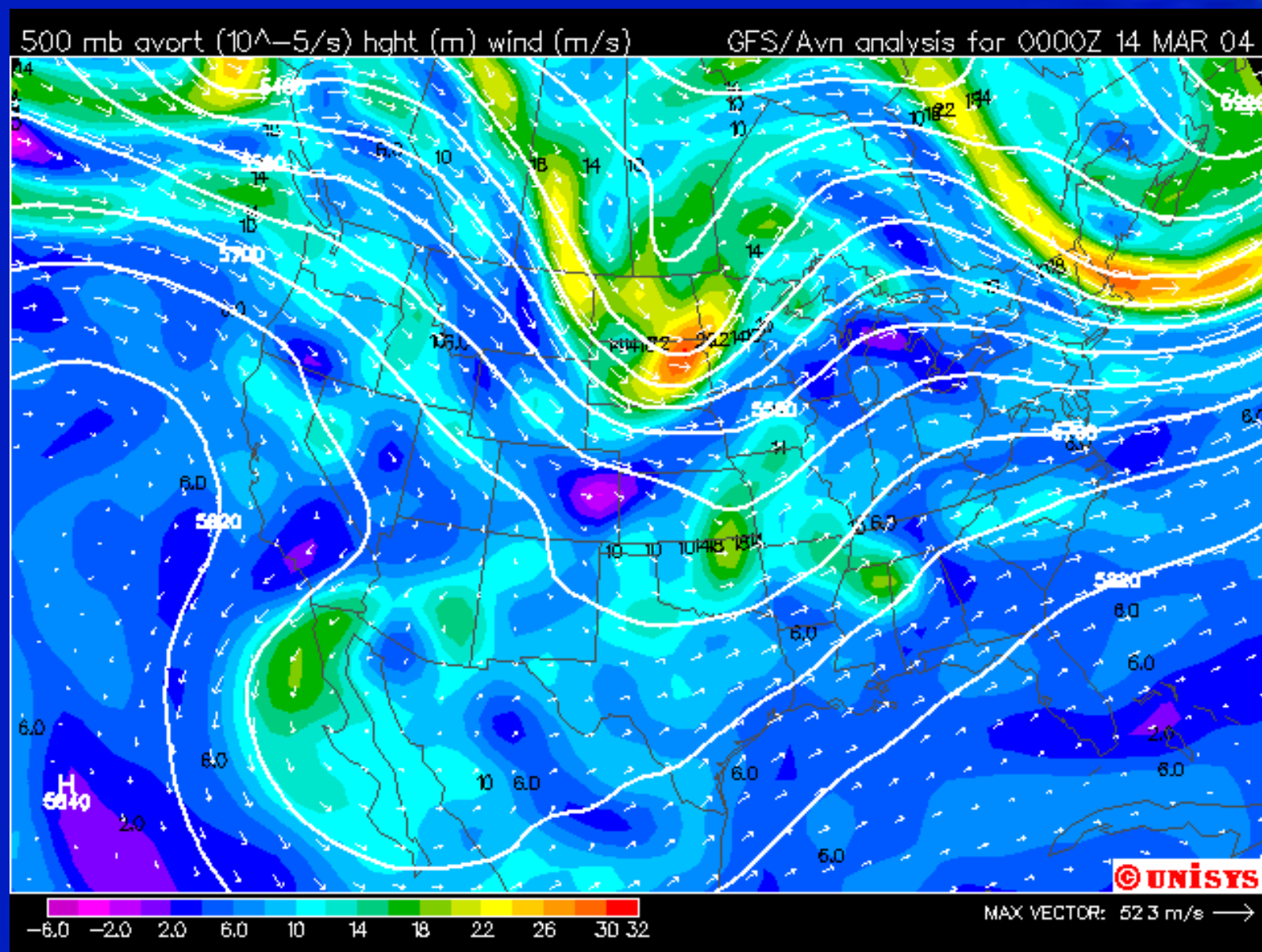




**Vorticity Maximum:** Along the trough axis to left of the strongest flow. Both shear and curvature terms are positive.

**Vorticity Minimum:** Along the ridge axis to right of the strongest flow. Both shear and curvature terms are negative.

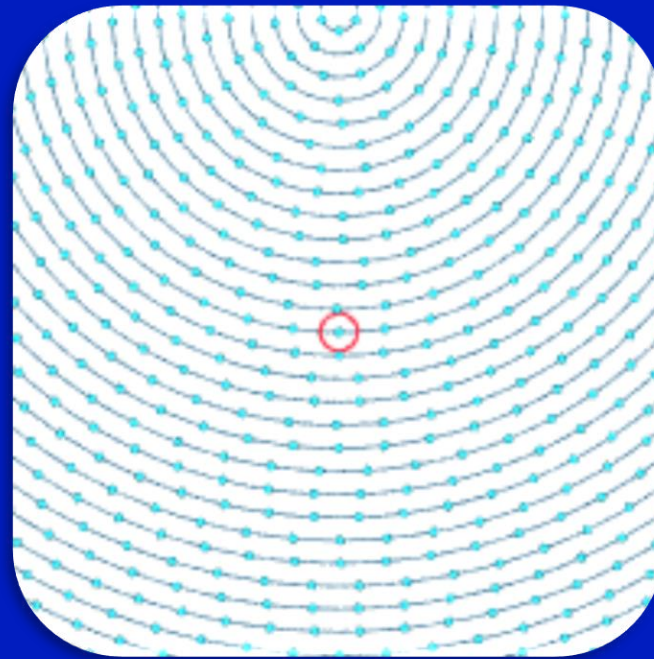




**Absolute vorticity =  $\zeta + f$**

In a mass of continuum that is rotating like a rigid body, the vorticity is twice the angular velocity vector of that rotation.

This is the case, for example, of water in a tank that has been spinning for a while around its vertical axis, at a constant rate.



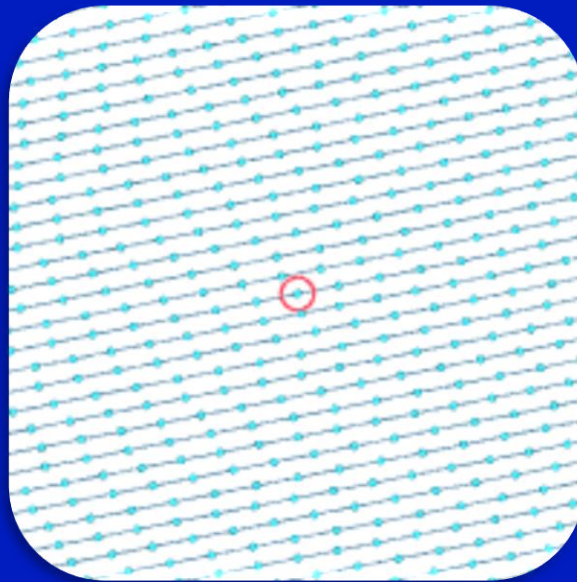
Rigid-body-like vortex

$$v \propto r$$

The vorticity may be nonzero even when all particles are flowing along straight and parallel pathlines, if there is shear (that is, if the flow speed varies across streamlines).

For example, in the laminar flow within a pipe with constant cross section all particles travel parallel to the axis of the pipe; but faster near that axis, and practically stationary next to the walls.

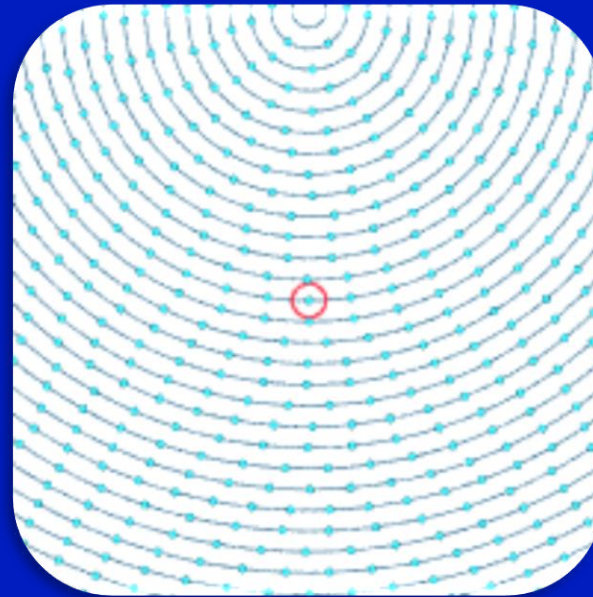
The vorticity will be zero on the axis, and maximum near the walls, where the shear is largest.



Parallel flow with shear

Conversely, a flow may have zero vorticity even though its particles travel along curved trajectories.

An example is the ideal irrotational vortex, where most particles rotate about some straight axis, with speed inversely proportional to their distances to that axis. A small parcel of continuum that does not straddle the axis will be rotated in one sense but sheared in the opposite sense, in such a way that their mean angular velocity *about their center of mass* is zero.



Irrotational vortex  
 $v \propto 1/r$