Geophysical Fluid Dynamics

Lecture 8

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Horizontal momentum equation scaled for midlatitude large-scale motions.

$$\frac{d\vec{V}}{dt} = -2\vec{\Omega} \times \vec{V} - \frac{1}{\rho}\nabla p$$

Rate of change of velocity following the fluid motion.

Coriolis acceleration

Pressure gradient force (per unit mass)

To transform to pressure coordinates, we need to transform the pressure gradient term:

$$\cancel{\nabla_p p} = \nabla_z p + \frac{\partial p}{\partial z}\nabla_p z$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$$\nabla_z p = -\frac{\partial p}{\partial z}\nabla_p z$$

$$\nabla_z p = \rho g \nabla_p z$$

$$-\frac{1}{\rho}\nabla_z p = -g\nabla_p z = -\nabla_p \Phi$$

Geopotential gradient

$$\frac{d\vec{V}}{dt} = -2\vec{\Omega}\times\vec{V} - \nabla_p \Phi$$

or

$$\frac{d\vec{V}}{dt} = -2\vec{\Omega}\times\vec{V} - g\nabla_p Z$$

Geopotential height gradient

Characteristics of pressure (isobaric) coordinates:

Vertical velocity is expressed as ω = dp/dt.

Rising air moves from higher to lower pressure, so upward motion occurs when $\omega < 0$.

The geopotential height gradient takes the place of the pressure gradient.

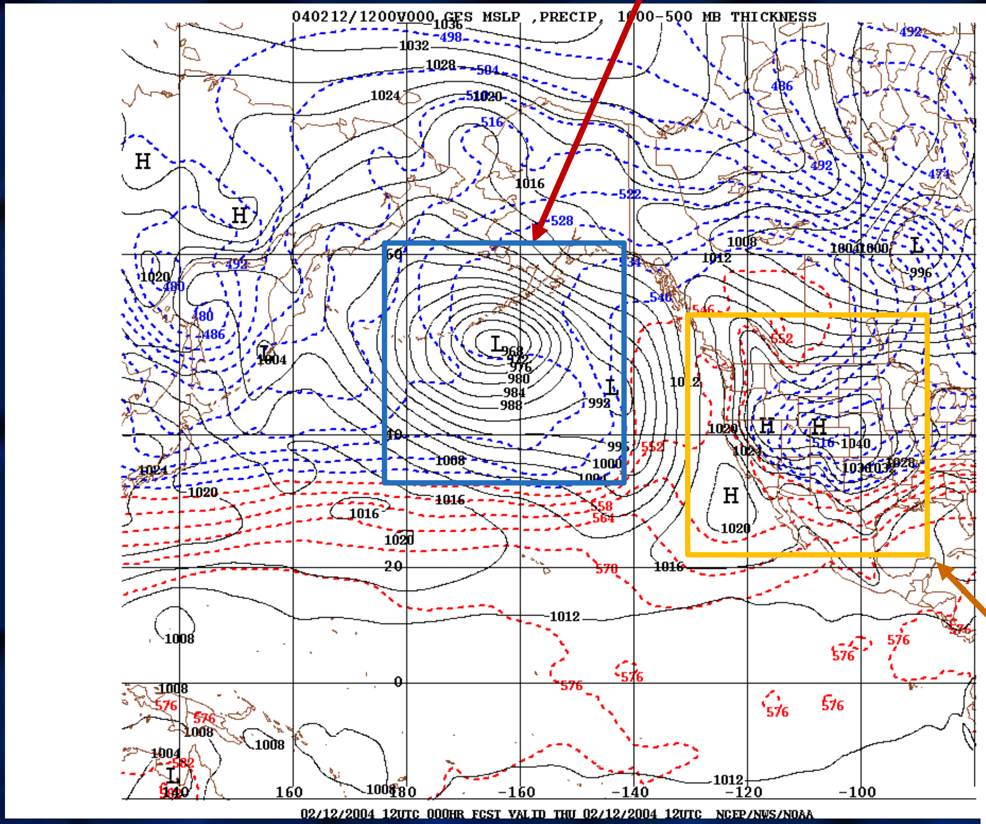
Low geopotential height on an isobaric surface are analogous to low pressure on a surface chart.

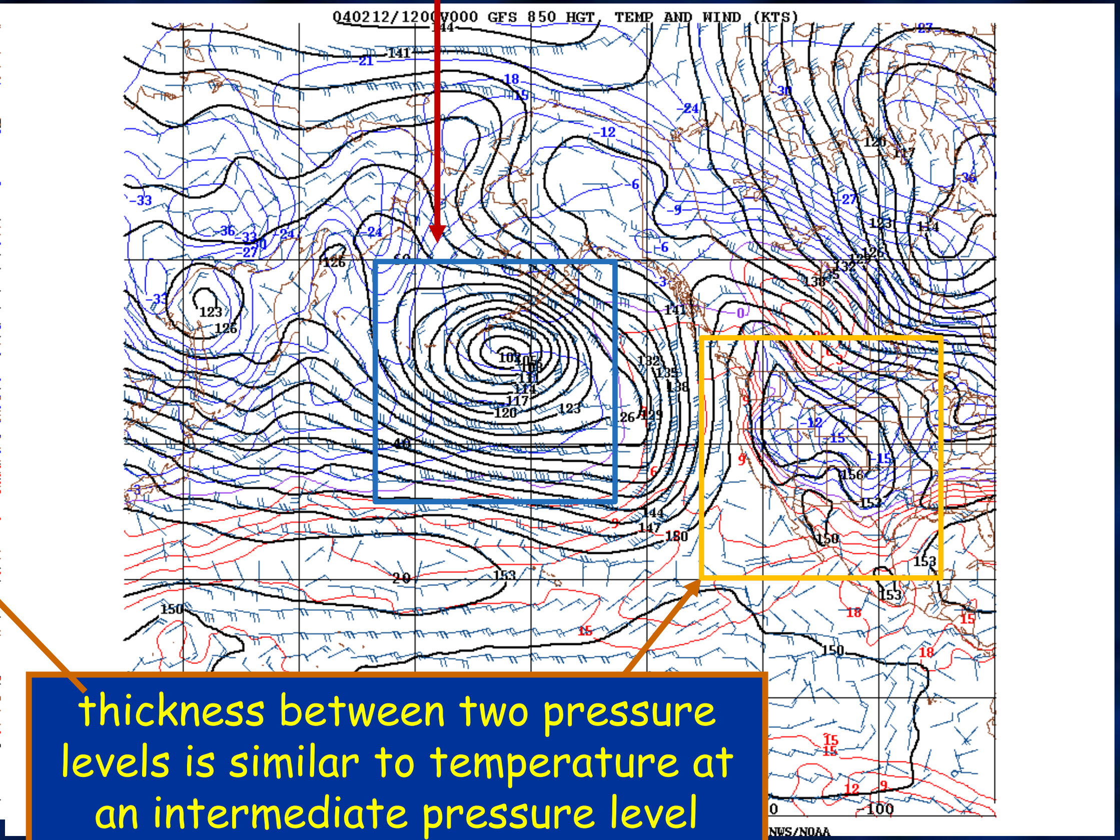
Expansion of the total derivative takes the following form:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial x}{\partial t}\frac{\partial f}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial f}{\partial y} + \frac{\partial p}{\partial t}\frac{\partial f}{\partial p}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + \omega\frac{\partial f}{\partial p}$$

pressure at a constant level is similar to geopotential height at constant pressure





thickness between two pressure levels is similar to temperature at an intermediate pressure level

Surface Map
solid: pressure at sea level
dashed: thickness between 1000 hPa and 500 hPa

850 hPa Map
black: geopotential height at 850 hPa
color: temperature at 850 hPa

The Basic Conservation Laws

Atmospheric motions are governed by three fundamental physical principles:

Conversation of mass

Conversation of momentum

Conversation of energy

The mathematical relations that express these laws may be derived by considering the budgets of mass, momentum, and energy for an infinitesimal control volume in the fluid.

Two types of control volume are commonly used in fluid dynamics:

Eulerian frame

Lagrangian frame

In the Eulerian frame of reference the control volume consists of a parallelepiped of sides δx, δy, δz, whose position is fixed relative to the coordinate axes.

In the Lagrangian frame the control volume consists an infinitesimal mass of “tagged” fluid particle; thus, the control volume moves about following the motion of the fluid, always containing the same fluid particles.

$$f = f(x) \rightarrow \frac{df}{dx}$$

$$f = f(x,t) \rightarrow \frac{\partial f}{\partial x}, \frac{\partial f}{\partial t}$$

$$f = f\left(x, y, z; t\right)$$

Expansion of Total Derivative

If $\quad f = f(x, y, z; t) \quad$ *then*

$$\delta f = \left(\frac{\partial f}{\partial t}\right)\delta t + \left(\frac{\partial f}{\partial x}\right)\delta x + \left(\frac{\partial f}{\partial y}\right)\delta y + \left(\frac{\partial f}{\partial z}\right)\delta z + H.O.T$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

But $\quad u \equiv \frac{dx}{dt}, \quad v \equiv \frac{dy}{dt}, \quad w \equiv \frac{dz}{dt}$

u = west-east component of fluid velocity
v = south-north component of fluid velocity
w = vertical component of fluid velocity

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\overbrace{\frac{dx}{dt}}^{u} + \frac{\partial f}{\partial y}\overbrace{\frac{dy}{dt}}^{v} + \frac{\partial f}{\partial z}\overbrace{\frac{dz}{dt}}^{w}$$

$$\underbrace{\frac{df}{dt}}_{\mathcal{A}} = \underbrace{\frac{\partial f}{\partial t}}_{\mathcal{B}} + \underbrace{u\frac{\partial f}{\partial x}}_{\mathcal{C}} + \underbrace{v\frac{\partial f}{\partial y}}_{\mathcal{D}} + \underbrace{w\frac{\partial f}{\partial z}}_{\mathcal{E}}$$

Term A: Total rate of change of *f* following the fluid motion

Term B: Local rate of change of *f* at a fixed location

Term C: Advection of *f* in *x* direction by the *x*-component flow

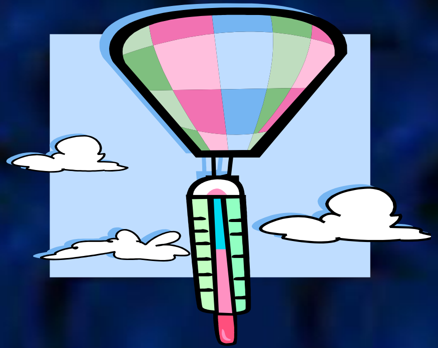
Term D: Advection of *f* in *y* direction by the *y*-component flow

Term E: Advection of *f* in *z* direction by the *z*-component flow

Total Derivative vs. Local Derivative

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{U}.\nabla f \qquad \frac{\partial f}{\partial t} = \frac{df}{dt} - \vec{U}.\nabla f$$

Total derivative is the temporal rate of change following the fluid motion.



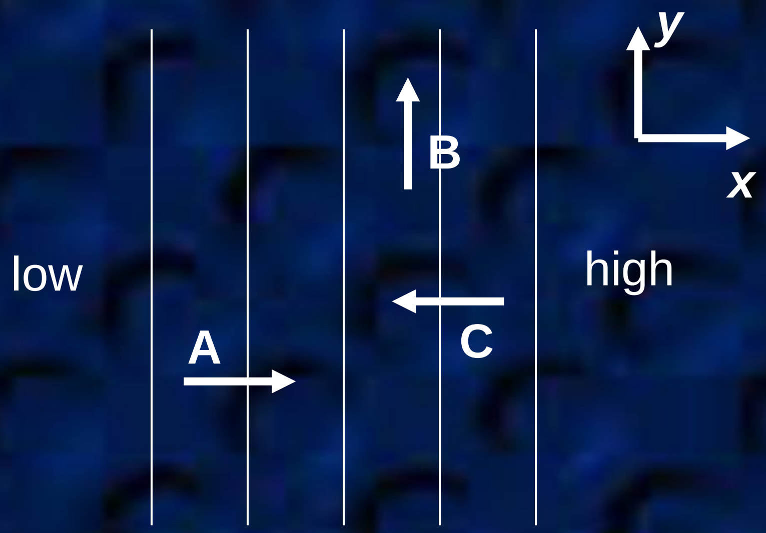
Example: A thermometer measuring changes as a balloon floats through the atmosphere.

Local derivative is the temporal rate of Change at a fixed point.

Example: An observer measures changes in temperature at a weather station.

Advection Terms

Assume that thin lines are contours of a scalar quantity *f* and thick arrows indicate the fluid motion.



We wish to evaluate the advection term

$$-u\frac{\partial f}{\partial x}$$

At point A: $u > 0, \frac{\partial f}{\partial x} > 0 \rightarrow u\frac{\partial f}{\partial x} > 0$ → Transport from low to high: "negative advection of *f*

At point B: $u = 0, \frac{\partial f}{\partial x} > 0 \rightarrow u\frac{\partial f}{\partial x} = 0$ → "neutral advection of *f*

At point C: $u < 0, \frac{\partial f}{\partial x} > 0 \rightarrow u\frac{\partial f}{\partial x} < 0$ → Transport from high to low: "positive advection of *f*

مثال: فرض می کنیم که در جهت شرق (غرب به شرق) یک کاهش فشار برابر 0.3 کیلو پاسکال به ازای هر 180 کیلومتر مسافت داشته باشیم.یک کشتی با سرعت 10 کیلومتر بر ساعت به سمت شرق در حرکت است و افت فشاری که کشتی اندازه می گیرد 0.1 کیلو پاسکال بر سه ساعت است. تغییرات فشار در خشکی که کشتی از مجاور آن می گذرد چقدر است؟

$$\frac{\partial p}{\partial x} = -0.3kpa/180km$$

$$u = 10km/hr$$

$$\frac{dp}{dt} = -0.1kpa/3hr$$

$$\frac{\partial p}{\partial t} = ?$$

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial t} = -0.1kpa/3hr - (10km/hr)\left(\frac{-0.3kpa}{180km}\right) = -0.1kpa/6hr$$

Taylor Series

A function *f(x)* can be computed by Taylor expansion given the values of the function and its derivatives at a point x_0:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \ldots$$

$$f(x) = f(x_0) + \sum_{n=1}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

A truncated Taylor series can be used to approximate *f(x)*.