Geophysical Fluid Dynamics

Lecture 6

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Motion in the eastward direction

Suppose now that the object is set in motion in the eastward direction by animpulsive force. Axial angular momentum is not conserved in this case, but considering again the centrifugal force will help expose the meridional component of the Coriolis force.

Because the object is now rotating faster than Earth, the centrifugal force on the object will be increased.

The excess of the centrifugal force over that for an object at rest is

$$\left(\Omega+\frac{u}{R}\right)^2\vec{R}=\Omega^2\vec{R}+\frac{2\Omega u\vec{R}}{R}+\cancel{\frac{u^2\vec{R}}{R^2}}$$

total centrifugal force	centrifugal force due to earth's rotation	deflecting forces

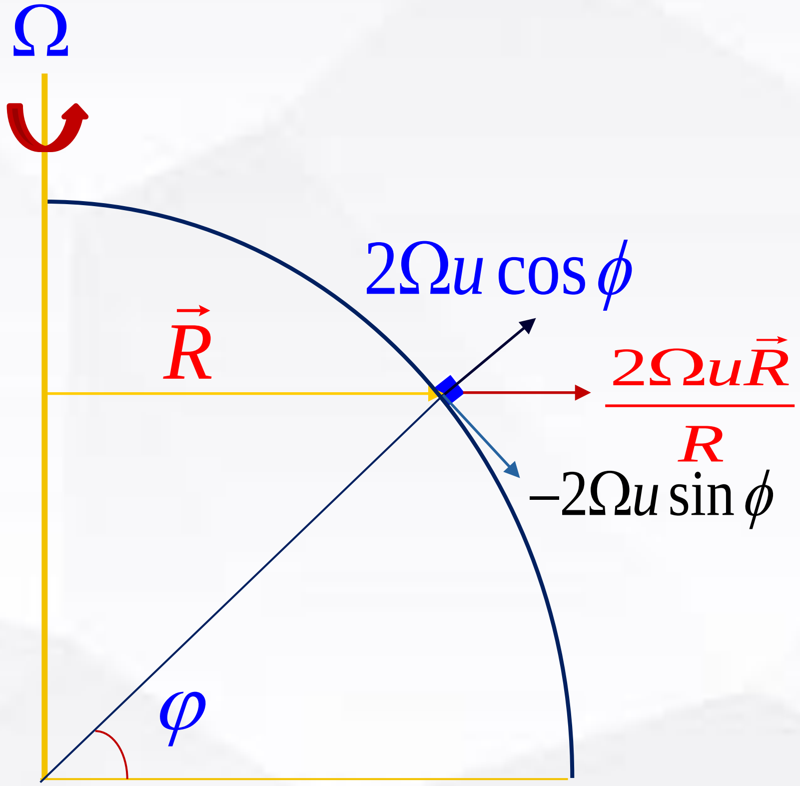
For large-scale atmospheric motions, this term is much smaller than the term to its left and can be neglected.

The deflecting forces act outward in a direction perpendicular to the axis of rotation.

The Coriolis force that results from zonal (i.e., east-west) motion can be divided into components in the vertical and meridional directions.

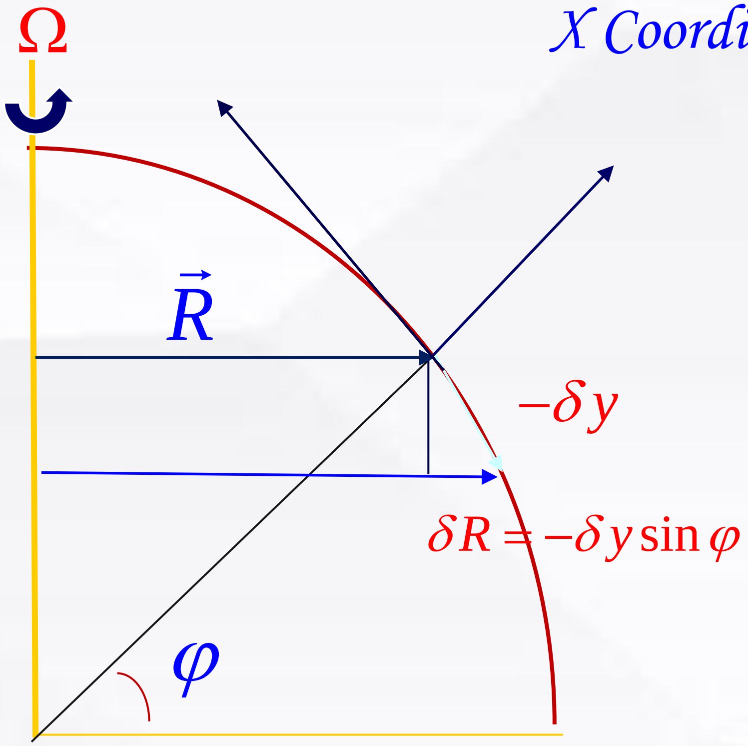
$$\left(\frac{dv}{dt}\right)_{Co} = -2\Omega u \sin\phi$$

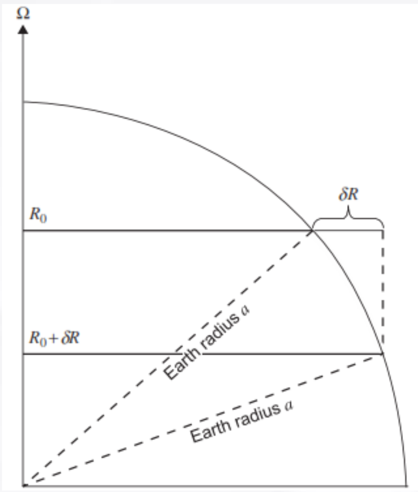
$$\left(\frac{dw}{dt}\right)_{Co} = 2\Omega u \cos\varphi$$



This derivation and result applies only for zonal motion. A more general way to represent apparent forces is required.

X Coordinate of the Coriolis Force





$$\Omega R^2 = (\Omega + \frac{\delta u}{R+\delta R})(R+\delta R)^2$$

$$2\Omega R\delta R = -R^2 \frac{\delta u}{R+\delta R}$$

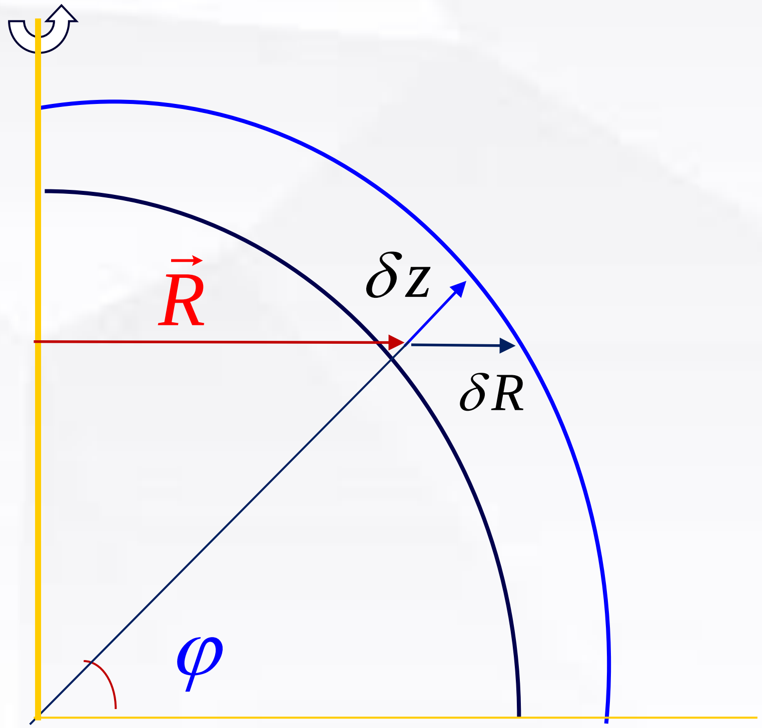
$$\begin{cases} \delta u = -2\Omega\delta R \\ \delta R = -\delta y \sin\varphi \end{cases}$$

$$(\frac{du}{dt})_{co} = 2\Omega v \sin\phi$$

$$\begin{cases} \delta u = -2\Omega\delta R \\ \delta R = \cos\varphi\delta z \end{cases}$$

$$\left(\frac{du}{dt}\right)_{co} = -2\Omega w \cos\phi$$

$$\left(\frac{du}{dt}\right)_{co} = 2\Omega v \sin\varphi - 2\Omega w \cos\varphi$$



The Coriolis force coordinates

$$\begin{cases} \left(\dfrac{du}{dt}\right)_{co} = 2\Omega v \sin\varphi - 2\Omega w \cos\varphi \\ \\ \left(\dfrac{dv}{dt}\right)_{Co} = -2\Omega u \sin\phi \\ \\ \left(\dfrac{dw}{dt}\right)_{Co} = 2\Omega u \cos\varphi \end{cases}$$

مثال (1): فرض می کنیم موشک بالستیکی در عرض جغرافیایی 43^0 شمالی به سمت شرق با سرعت افقی 1000 متر بر ثانیه در حرکت باشد. اگر این موشک 1000 کیلومتر را طی نماید مقدار انحراف از مسیر در جهت شرق توسط نیروی کوریولیس چقدر است؟

$$\left(\frac{dv}{dt}\right)_{Co} = -2\Omega u \sin\phi$$

$$\Omega = 7.3\times10^{-5}\, rad/\sec$$

$$v = -2\Omega u_0 t \sin\phi$$

$$\delta y \approx -50 km$$

$$dy = vdt$$

$$\int_0^{t_0} vdt = \int_{y_0}^{y_0+\delta y} dy = -2\Omega u_0 \sin\phi \int_0^{t_0} dt$$

مثال (2): یک بازیکن بیسبال توپی را در عرض جغرافیایی 30 درجه در امتداد افقی به فاصله 100 متر پرتاب می کند پس از 4 ثانیه انحراف از افق در نتیجه چرخش زمین چقدر است؟

$$\left(\frac{du}{dt}\right)_{co} = 2\Omega v \sin\varphi \qquad \left(\frac{dv}{dt}\right)_{co} = -2\Omega u \sin\varphi$$

$$u = 2\Omega v \sin\varphi t_0$$

$$\frac{dx}{dt} = 2\Omega v \sin\varphi t_0$$

$$\Delta x = \Omega v \sin\varphi t^2 = 1.46cm$$

مثال (3): دو توپ با قطر 4 سانتیمتر در فاصله 100 متر از یکدیگر در فاصله افقی در عرض 43 درجه شمالی قرار دارد. اگر توپها با یک ضربه آنی مستقیما با سرعت مساوی به طرف یکدیگر پرتاب شوند بایستی با چه سرعتی حرکت کنند تا یکدیگر را بطور مماس ترک کنند؟

$$\left(\frac{dv}{dt}\right)_{Co} = -2\Omega u \sin\varphi$$

$$v = -2\Omega u_0 t \sin\varphi$$

$$\frac{dy}{dt} - 2\Omega u_0 t \sin\varphi$$

$$\delta y = -2\Omega u_0 \sin\varphi \frac{t^2}{2}$$

$$t = \frac{x}{u_0} = \frac{50}{u_0}$$

$$u_0 = \frac{-\Omega \sin\varphi x^2}{\delta y = 0.02m} = 6.22ms^{-1}$$

$$\Omega = 7.3\times 10^{-5}\, rad/\sec$$

The Static Atmosphere

Equation of State

The atmosphere at any point can be described by its temperature, pressure, and density.

These variables are related to each other through the Equation of State for dry air:

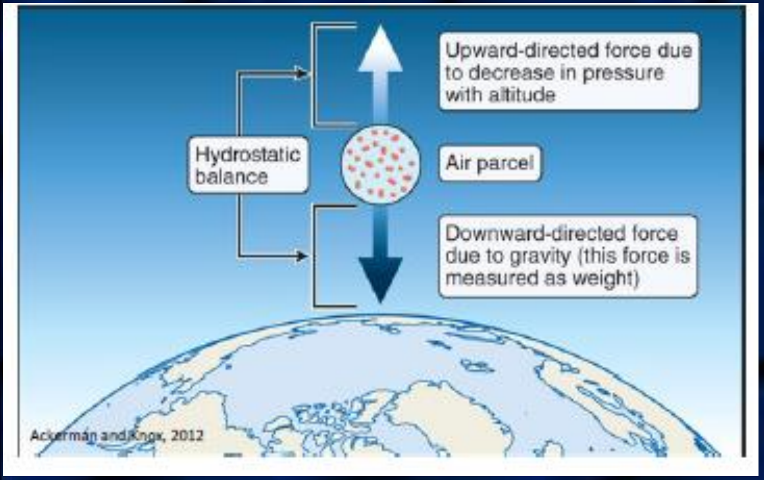
$$P\alpha = RT \qquad P = \rho RT$$

Where R is the gas constant for dry air ($R=287\ Jkg^{-1}K^{-1}$)

The Hydrostatic Equilibrium

In the absence of atmospheric motions the gravity force must be exactly balanced by the vertical component of the pressure gradient force.

p-dp .... z+dz



$-\alpha \frac{\partial p}{\partial z}$: *pressure gradient force*

g : *gravity*

p .... z

$$-\alpha \frac{dp}{dz} = g \quad \text{or} \quad \frac{dp}{dz} = -\rho g$$

(one of the best approximations in meteorology)

$$\frac{dp}{dz} = -\rho g \qquad dp = -\rho g\, dz$$

$$p(z) = \int_z^{\infty} \rho g\, dz$$

Pressure at any point is the weight per square meter of the atmospheric column overlying that point.

For average conditions,

$$p(0) = \int_0^{\infty} \rho g\, dz = 1013.25\, hPa$$

This is the mean sea-level pressure.

Geopotential Height

Definition:

The geopotential height (Z) is the actual height normalized by the globally averaged acceleration due to gravity at the Earth's surface (g_0 = 9.81 m s^{-2}), and is defined by:

$$Z = \frac{\Phi(z)}{g_0}$$

Used as the vertical coordinate in most atmospheric applications in which energy plays an important role (i.e. just about everything)

g_o is a constant (9.8 ms^{-2}). In the lower atmosphere, Z is very close to z (called the 'geometric height'). The table below shows how Z, z, and g vary with height at a typical mid-latitude location.

z(km)	Z(km)	g(ms^{-2})
0	0	9.802
1	1.000	9.798
10	9.986	9.771
20	19.941	9.741
30	29.864	9.710
60	59.449	9.620
90	88.758	9.531
120	117.795	9.443
160	156.096	9.327
200	193.928	9.214
300	286.520	8.940
400	376.370	8.677
500	463.597	8.427
600	548.314	8.186

Geopotential Height

- ✓ Earth's gravity has Altitude and Latitude dependences.
- ✓ Overall these are minor (few tenths of a percent under conditions of interest to meteorologist).
- ✓ Geopotential Height allows us a new reference (or coordinate) to accommodate these changes in gravity.
- ✓ Geopotential height is a vertical coordinate referenced to Earth's mean sea level — an adjustment to geometric height (elevation above mean sea level) using the variation of gravity with latitude and elevation.
- ✓ Thus it can be considered a "gravity-adjusted height".

Geometric vs. Geopotential Meters

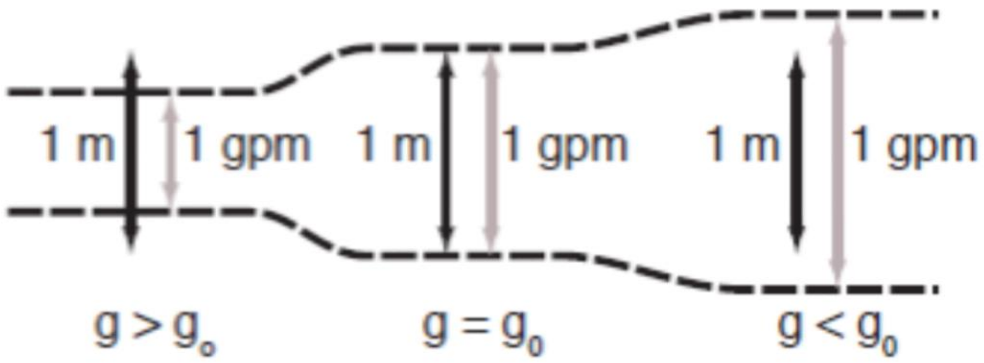
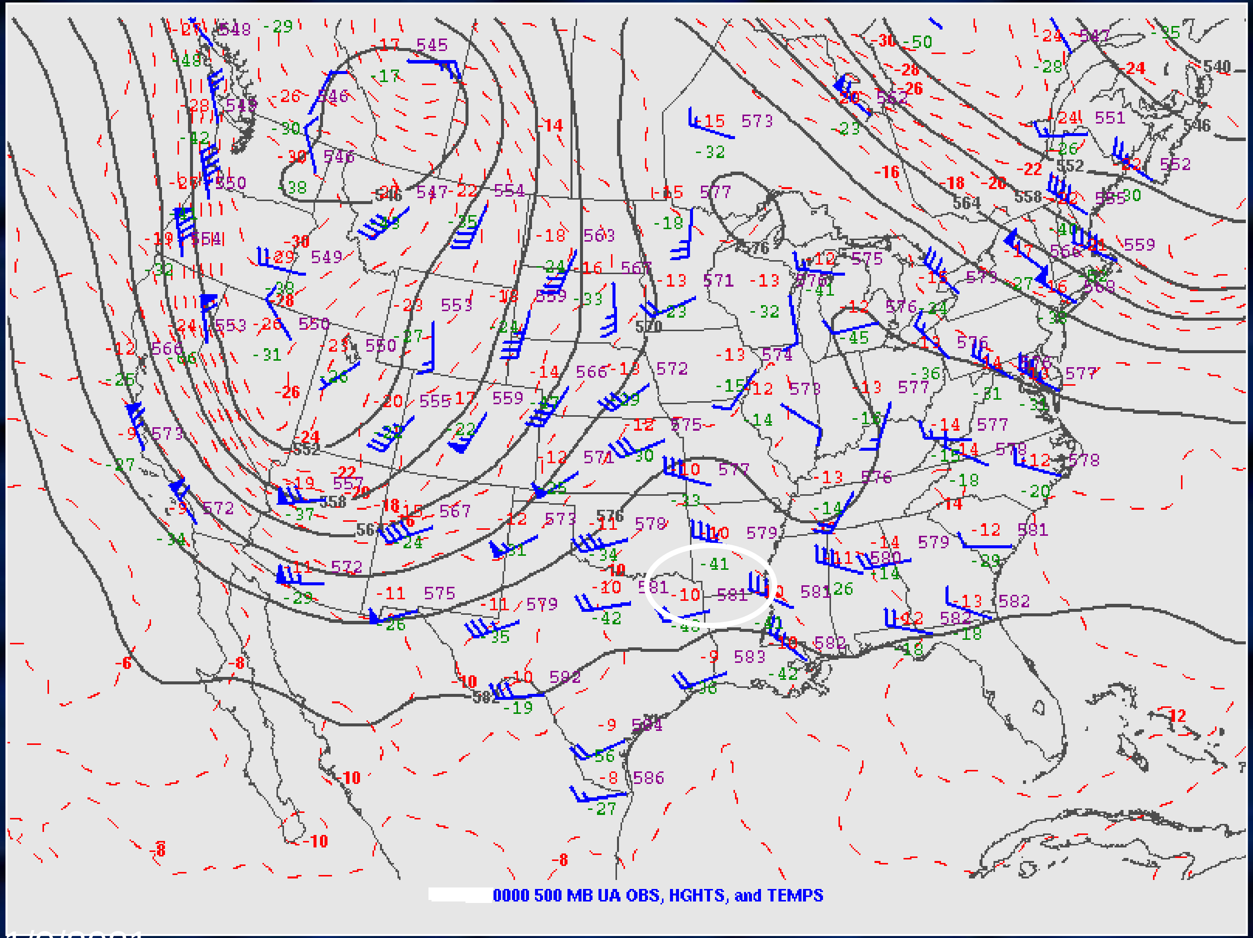


Fig. 4.2

Geopotential => Potential energy per unit mass

Application: Geopotential Height

The geopotential height (Z) is the standard "height" parameter plotted on isobaric charts constructed from daily soundings:



500 mb

Geopotential heights (Z) are solid black contours (Ex: Z = 5790 meters)

Air temperatures (T) are red dashed contours (Ex: T = -11°C)

Winds are shown as barbs