Geophysical Fluid Dynamics

Lecture 4

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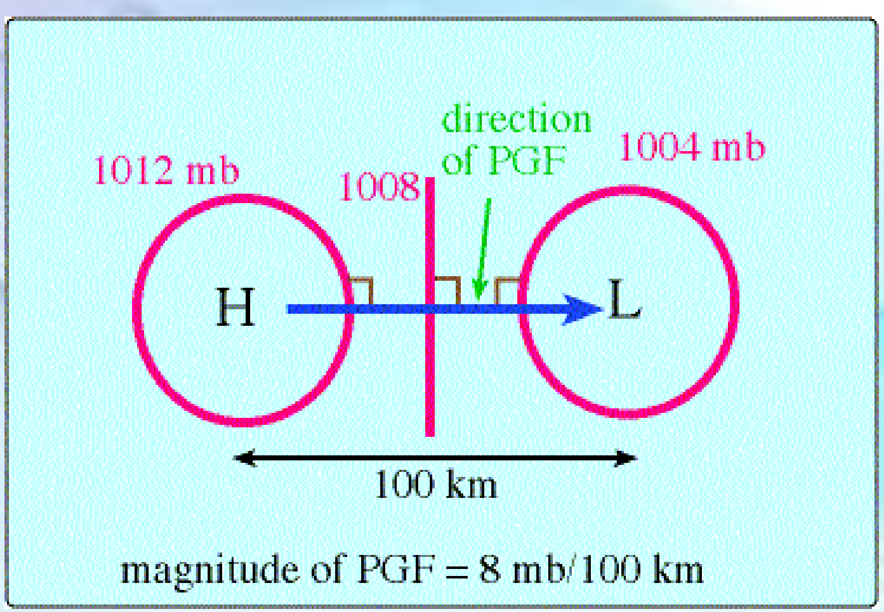
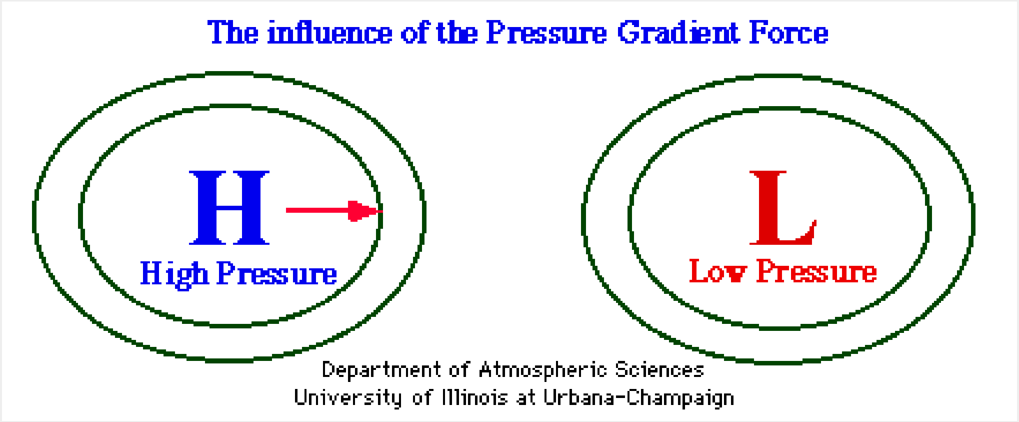
The Pressure Gradient Force (PGF)

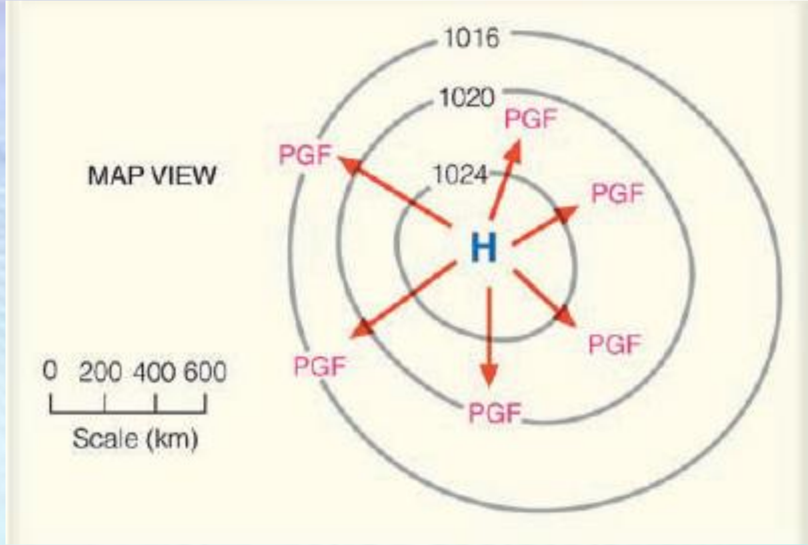
The pressure gradient force, like any other force, has a *magnitude* and a *direction*:

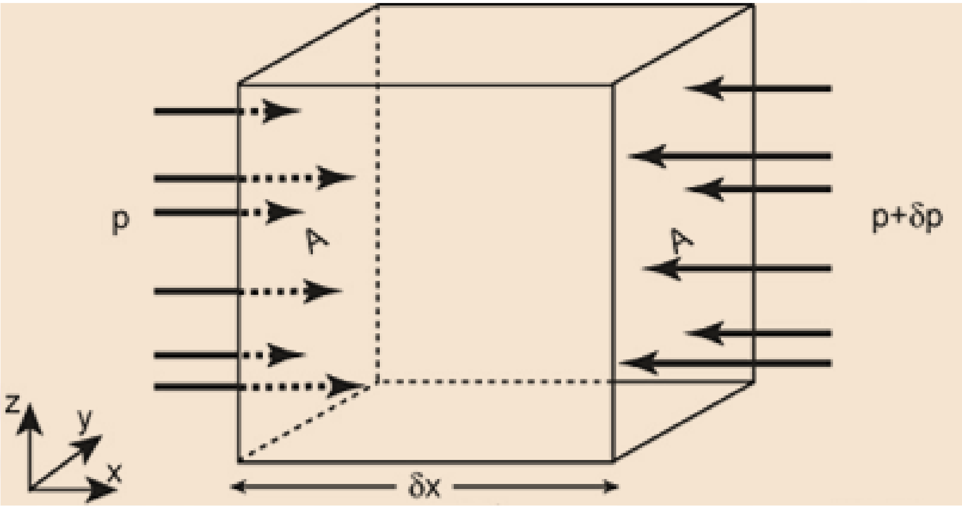
direction - the pressure gradient force direction is ALWAYS directed from high to low pressure and is ALWAYS perpendicular to the isobars

magnitude - is determined by computing the pressure gradient




The influence of the Pressure Gradient Force
H
High Pressure
L
Low Pressure
Department of Atmospheric Sciences
University of Illinois at Urbana-Champaign


MAP VIEW
1016
1020
1024
H
PGF
PGF
PGF
PGF
PGF
PGF
0 200 400 600
Scale (km)



$$pA = p\delta y\delta z$$

$$-(p+\delta p)\delta y\delta z = -(p+\frac{\partial p}{\partial x}\delta x)\delta y\delta z$$ For right side surface

$$p\delta y\delta z$$ For left side surface

$$p\delta y\delta z - (p+\frac{\partial p}{\partial x}\delta x)\delta y\delta z = -\frac{\partial p}{\partial x}\delta x\delta y\delta z$$

$$-\frac{\partial p}{\partial x}\delta x\delta y\delta z \qquad \rho\delta x\delta y\delta z$$

$$\vec{F}_{PG}.\hat{i} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\vec{F}_{PG}.\hat{j} = -\frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$\vec{F}_{PG}.\hat{k} = -\frac{1}{\rho}\frac{\partial P}{\partial z}$$

$$\vec{F}_{PG} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)$$

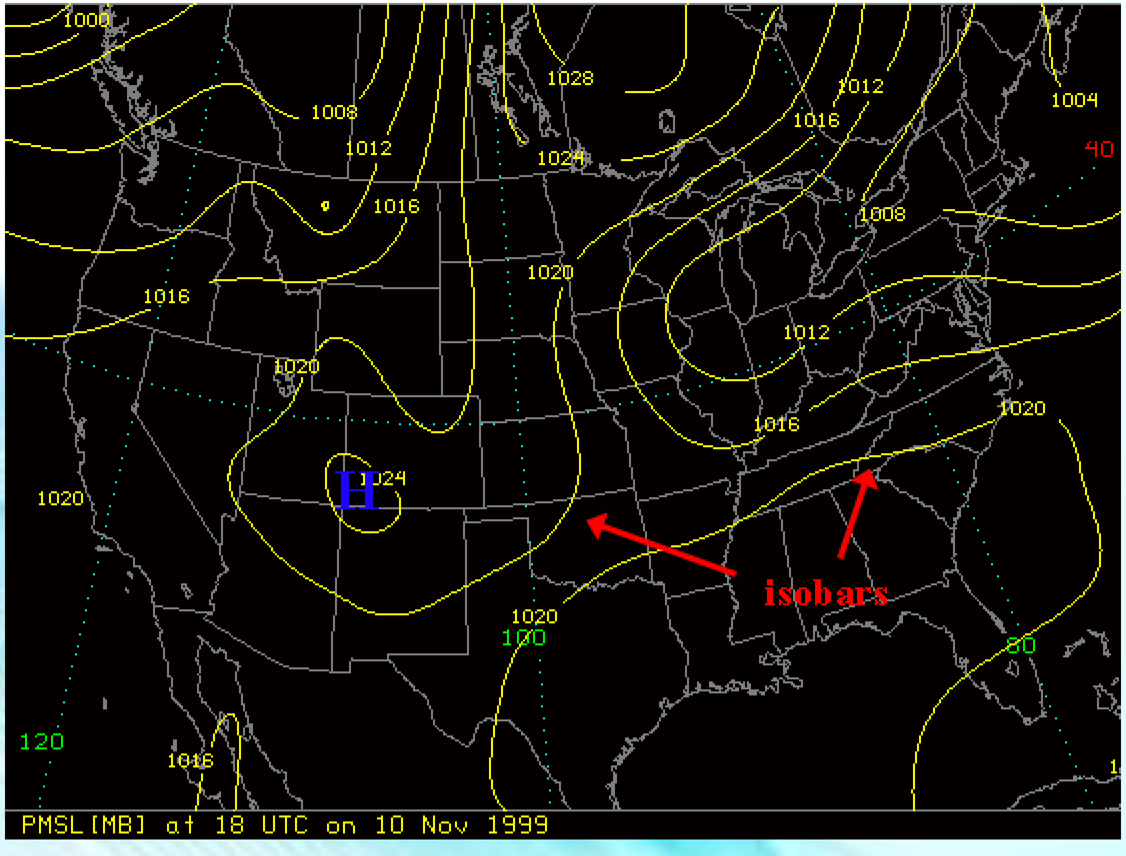
Measuring Pressure at the surface
the surface pressure chart

Surface pressure chart – isobars (lines of constant pressure) are plotted every 4 mb

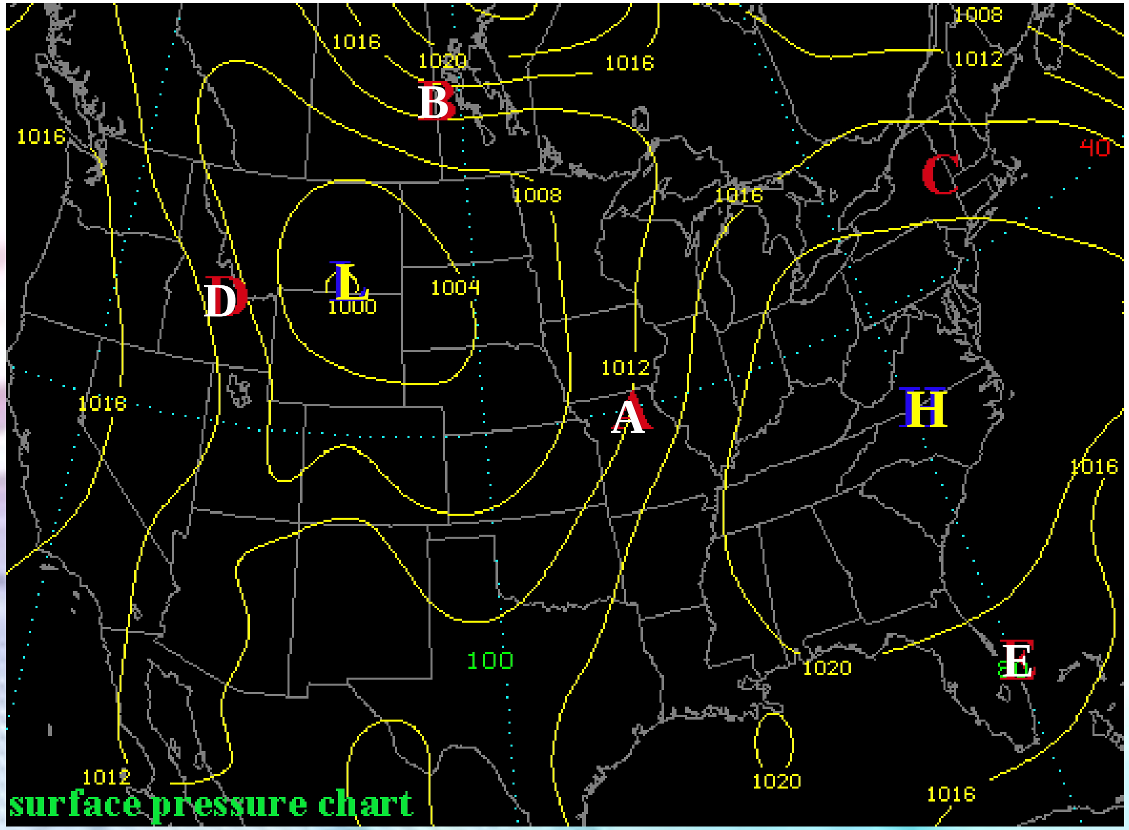
Maps of surface Pressure are very important:

give positions of highs and lows

can give information about the direction and strength of the surface winds



The Pressure Gradient Force - example



Q: What is the direction of the PGF at points A,B,C,D,E ?

Q: At which location is the PGF largest? Location B

Q: At which location is the PGF weakest? Location E

Gravitational force

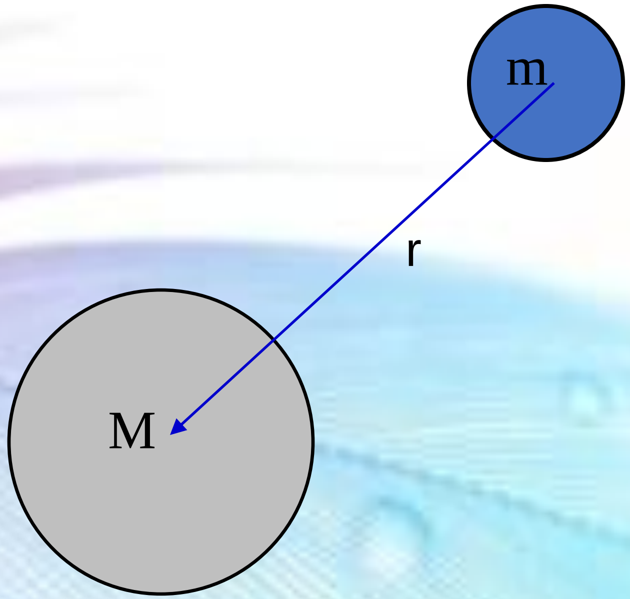
The force of attraction between all masses in the universe; especially the attraction of the earth's mass for bodies near its surface;

“The more remote the body the less the gravity";

The gravitation between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance between them"

Gravitational Force

Newton's law of universal gravitation states that gravitational force exerted by mass M on mass m is:



$$\vec{F}_g = -\frac{GMm}{r^2}\left(\frac{\vec{r}}{r}\right)$$

If the earth is taken to be the mass M and m is taken to be the mass of a fluid parcel or volume element, then we can write the force per unit mass exerted on the fluid by the earth as

$$\frac{\vec{F}_g}{m} \equiv \vec{g}^* = -\frac{GM}{r^2}\left(\frac{\vec{r}}{r}\right) \qquad g^* = -\frac{GM}{(a+z)^2} = -\frac{GM}{a^2\left(1+\frac{z}{a}\right)^2}$$

If *a* is the radius of the earth and *z* is the distance above sea level, then

$$\vec{g}^* = \frac{\vec{g}_0^*}{(1+z/a)^2}, \quad \textit{where} \quad \vec{g}_0^* = -\left(\frac{GM}{a^2}\right)\left(\frac{\vec{r}}{r}\right)$$

Because the depths of the atmosphere and ocean are small compared to the radius of the earth ($z \ll a$) we can treat the gravitational force per unit mass as a constant.

$$\vec{g}^* = \vec{g}_0^*$$

Viscous Force

The surface of the Earth exerts a frictional drag on the air blowing just above it. This friction can act to change the wind's direction and slow it down.

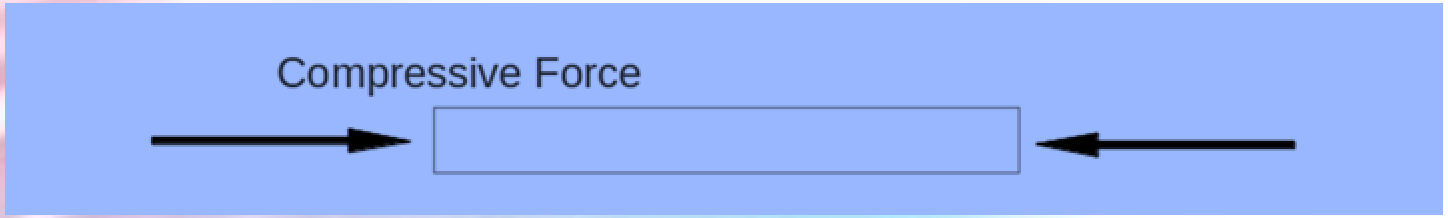
Actually, the difference in terrain conditions directly affects how much friction is exerted.

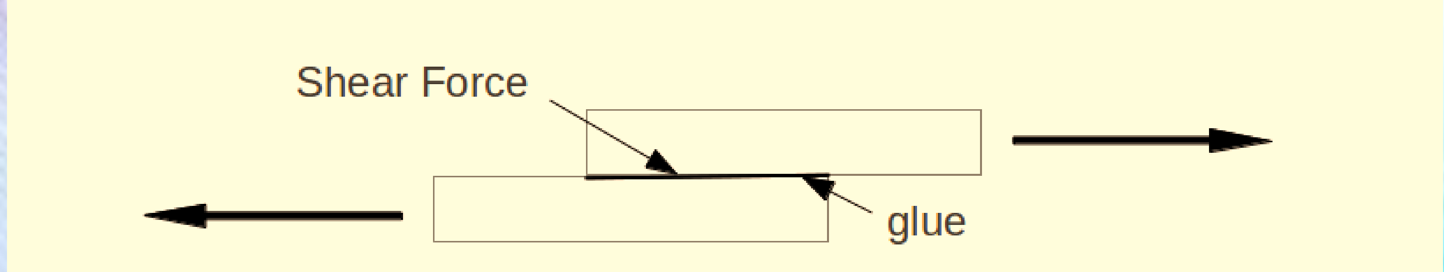
For example, a calm ocean surface is pretty smooth, so the wind blowing over it does not move up, down. By contrast, hills and forests force the wind to slow down and/or change direction much more.

As we move higher, surface features affect the wind less until the wind is indeed geostrophic. This level is considered the top of the boundary (or friction) layer.

The height of the boundary layer can vary depending on the type of terrain, wind, and vertical temperature profile.

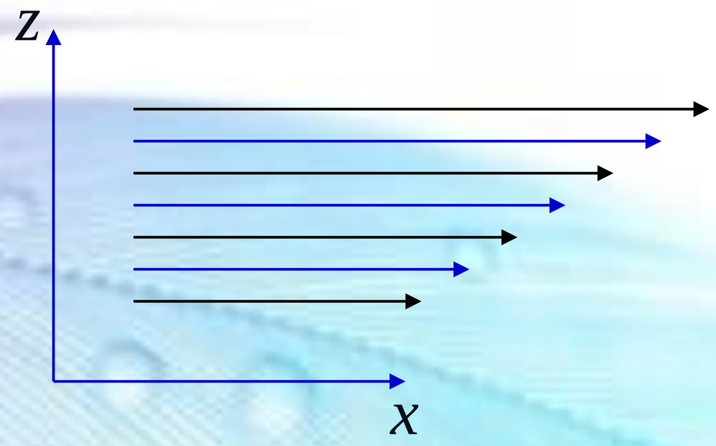

Tensile Force


Compressive Force

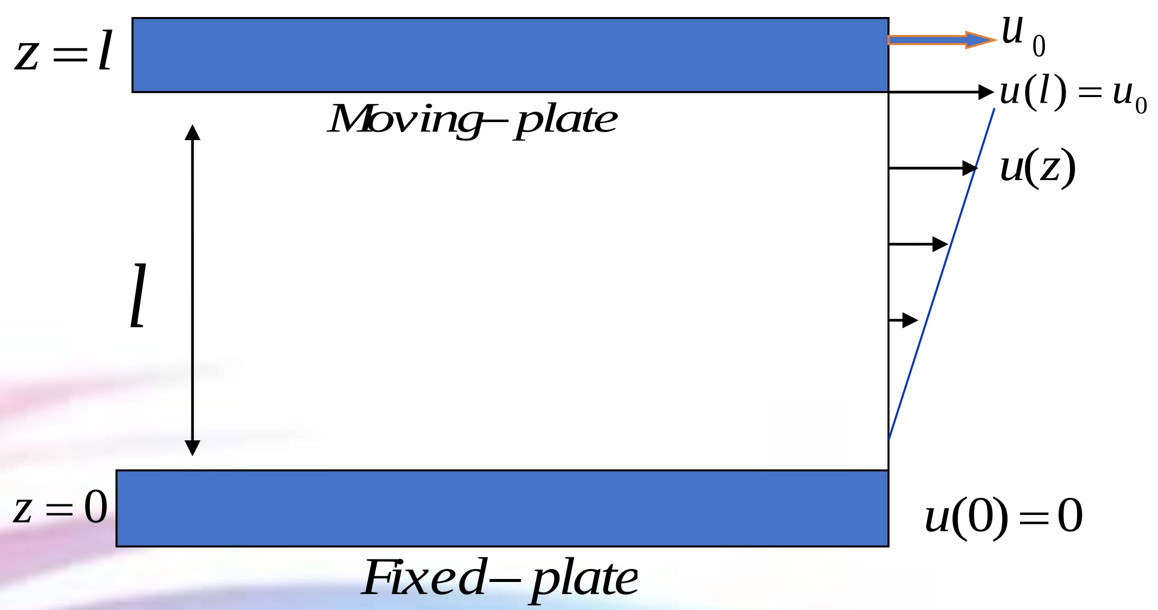

Shear Force
glue

The time of day and season of the year also affect the height of the boundary layer. However, usually the boundary layer exists from the surface to about 1-2 km above it.

If the wind velocity varies with height, random molecular motions will cause momentum to be transferred vertically.



In other words, there is a drag exerted by the layers above and below the level of interest.



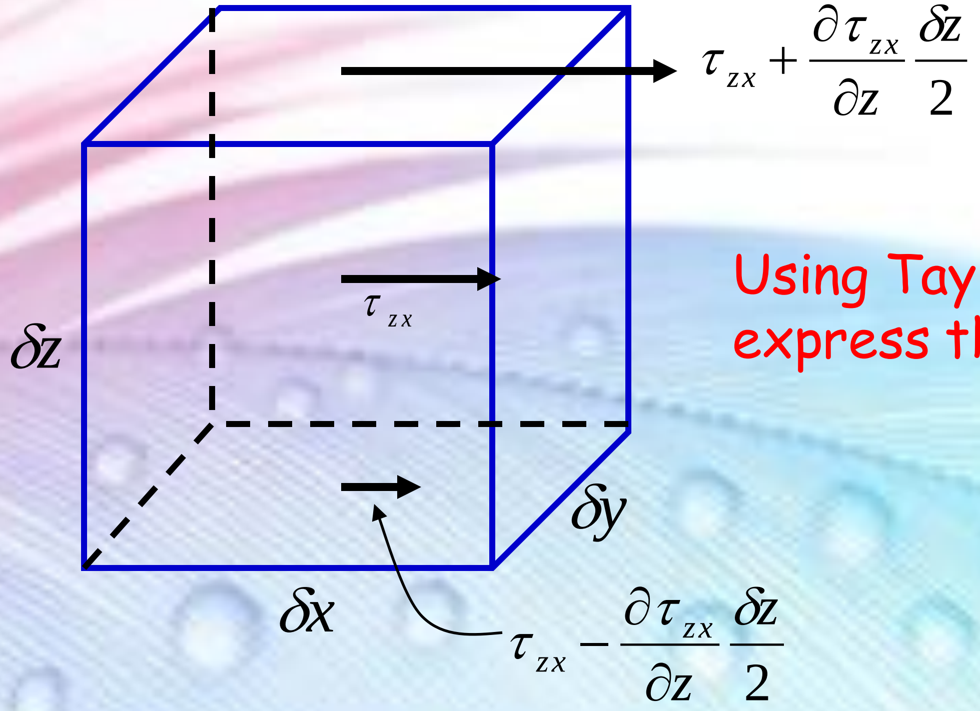
$$F = \mu \frac{Au_0}{l} \qquad F = \mu \frac{A\delta u}{\delta z}$$

$$\delta u = u_0 \frac{\delta z}{l} \qquad \tau_{zx} = \frac{F}{A} = \lim_{\delta z \to 0} \mu \frac{\delta u}{\delta z} = \mu \frac{\partial u}{\partial z}$$

The stress due to the velocity shear is given by

$$\tau_{zx} = \mu \frac{\partial u}{\partial z}$$

where μ is the dynamic viscosity coefficient.



Using Taylor series expansion to express the net viscous force:

$$\left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}\right)\delta y\,\delta x - \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2}\right)\delta y\,\delta x = \frac{\partial \tau_{zx}}{\partial z}\delta z\,\delta y\,\delta x$$

$$\text{Net viscous force} = \frac{\partial \tau_{zx}}{\partial z} \delta z\, \delta y\, \delta x$$

Dividing the above expression by the mass $\rho\, dx\, dy\, dz$ yields the viscous force per unit mass:

$$\frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right) = \nu \frac{\partial^2 u}{\partial z^2}$$

$\nu = \mu/\rho =$ kinematic viscosity coefficient = 1.46 x 10^{-5} m^2 s^{-1}

Molecular viscosity is too small to be important except very close (cm) to the Earth's surface and above 100 km.

$$F_x = F_{xx} + F_{yx} + F_{zx}$$

$$F_{yx} = \frac{1}{\rho}\frac{\partial \tau_{yx}}{\partial y} \qquad F_{xx} = \frac{1}{\rho}\frac{\partial \tau_{xx}}{\partial x}$$

$$F_x = \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)$$

$$F_x = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$F_x = \nu \nabla^2 u$$

$$\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2} \langle\langle \frac{\partial^2 u}{\partial z^2}$$

$$F_x \cong \nu \frac{\partial^2 u}{\partial z^2} = F_{zx}$$

$$F_y \cong \nu \frac{\partial^2 v}{\partial z^2} = F_{zy}$$

$$F_z \cong \nu \frac{\partial^2 w}{\partial z^2} = F_{zz}$$

$$\vec{F} = \hat{i} F_x + \hat{j} F_y + \hat{k} F_z$$

$$\vec{F} = \nu \nabla^2 \vec{U}$$

$$\vec{F} = \nu (\hat{i} \frac{\partial^2 u}{\partial z^2} + \hat{j} \frac{\partial^2 v}{\partial z^2} + \hat{k} \frac{\partial^2 w}{\partial z^2})$$

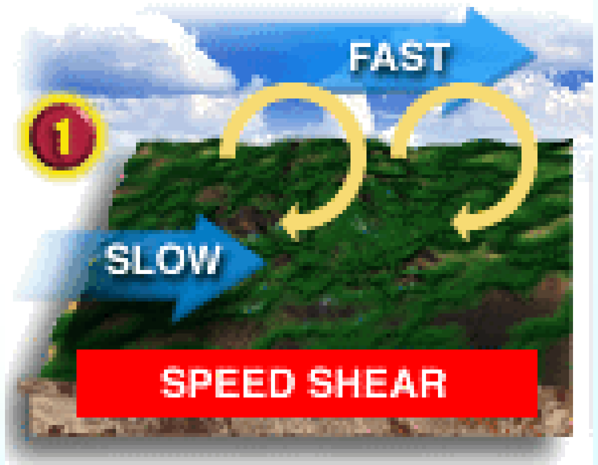
$$F \cong \nu \frac{\partial^2 \vec{U}}{\partial z^2}$$

Wind shear

is a sudden change of direction and/or speed of the air flow.

Wind shear: is the difference between the wind in 2 points divided by the distance between them.

The major cause of the air turbulence that sometimes makes planes bounce up and down in flight is wind shear.



The term wind shear refers to a change in wind speed or direction, or both, over a short distance.

Such changes help create eddies, or swirls of air, that cause turbulence.

Wind shear can be both vertical and horizontal and can cause anything from minor turbulence to tornadoes, depending on the scale of shear.