دانشگاه رازی

Geophysical Fluid Dynamics

Lecture 3

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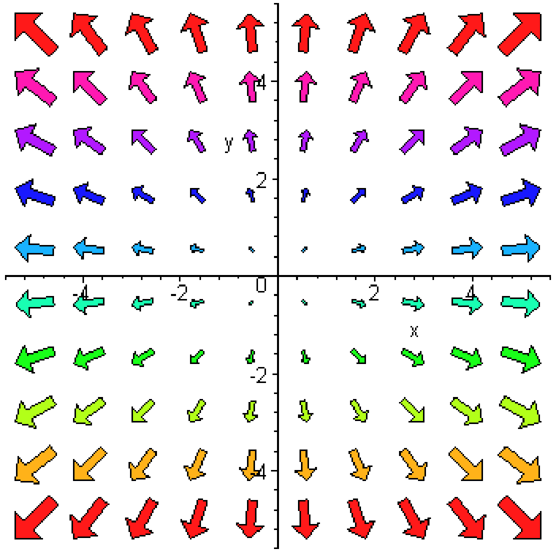
DIVERGENCE

2) عملگر واگرایی

The Divergence of a Vector Field

این عملگر بر روی میدان برداری اثر کرده و از آن یک کمیت نرده ای می سازد.

$$\vec{V} = \hat{i}u + \hat{j}v + \hat{k}w$$

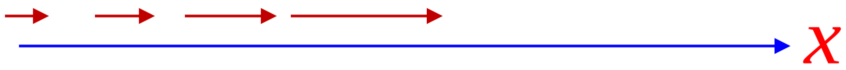


$$\vec{\nabla}.\vec{V} = (\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}).(\hat{i}u + \hat{j}v + \hat{k}w)$$

$$= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

آهنگ انبساط در راستای x, y, z

$$\frac{\partial u}{\partial x} \approx \frac{\delta u}{\delta x} = \frac{u_2 - u_1}{x_2 - x_1}$$



بنا براین حاصل واگرایی سرعت انبساط و انقباض حجم است.

Divergence: The expansion or spreading out of a vector field

$$\vec{\nabla}.\,\vec{V} > 0 \rightarrow Divergence$$

Convergence: The contraction of a vector field

$$\nabla.\,V < 0 \rightarrow Convergence$$

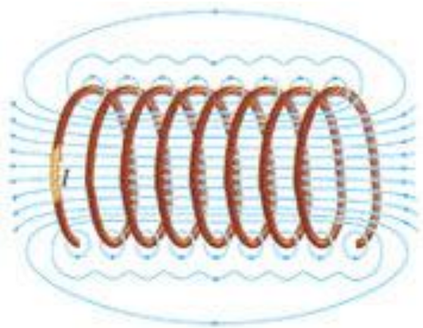
اما واگرایی باد واقعی به ورود و یا خروج هوا از یک نقطه در فضا بستگی دارد .

$\nabla.\left(\rho\vec{V}\right) > 0$ ⇒ خ – ه – ا – ن

$\nabla.\left(\rho\vec{V}\right) < 0$ ⇒ و – ه – ب – ن

$$\nabla . \vec{B} = 0$$

B vector field is solenoidal



In vector calculus a solenoidal vector field (also known as an incompressible vector field or a divergence free vector field) is a vector field **B** with divergence zero at all points in the field.

The Curl of a Vector Field   3) عملگر تاو

عملگر تاو نیز بر روی کمیات برداری اثر می کند

This is a lot harder to visualize than the divergence, but not impossible.

Suppose you are in a boat in a huge river (or Pass) where the current flows mainly in the x direction but where the speed of the current (flux of water) varies with y.

$$\vec{\nabla}\times\vec{F}=(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z})\times(\hat{i}F_x+\hat{j}F_y+\hat{k}F_z)=\hat{i}(\ldots)+\hat{j}(\ldots)+\hat{k}(\ldots)$$

آهنگی که با آن نیرو حول محورها چرخش ایجاد می کند.

مثال: اگراین عملگر روی بردار سرعت اثر نماید خواهیم داشت:

$$\vec{\nabla}\times\vec{V}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ u & v & w\end{vmatrix}=\hat{i}\left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right)+\hat{j}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)+\hat{k}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

$$=\hat{i}\xi+\hat{j}\eta+\hat{k}\zeta$$

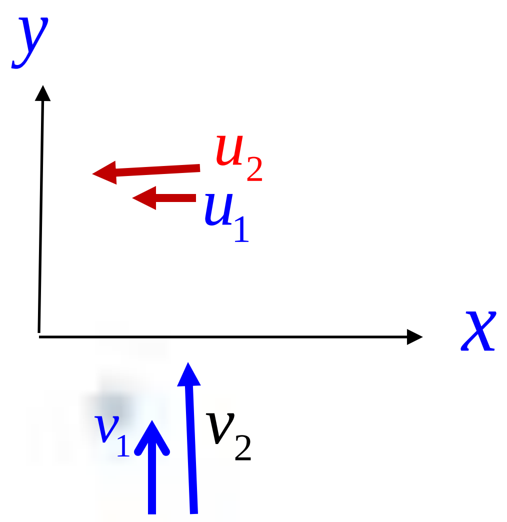
$$\xi,\eta\langle\langle\zeta\Rightarrow\vec{\nabla}\times\vec{V}=\vec{\zeta}$$

$$\therefore\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

مولفه قائم چرخش سرعت

بیانگر تمایل حرکت چرخشی در سطح افق حول محور z است



$$\omega_1 = \frac{\partial v}{\partial x}$$

$$\omega_2 = -\frac{\partial u}{\partial y}$$

4) Advection operator $-\vec{V}.\vec{\nabla}$

Advection: The process of transport of an atmospheric property solely by the velocity field of the atmosphere; Advection may be expressed in vector notation by

$$-\vec{V}.\vec{\nabla}\phi$$

where V is the wind vector, φ the atmospheric property.

In three-dimensional Cartesian coordinate, it is

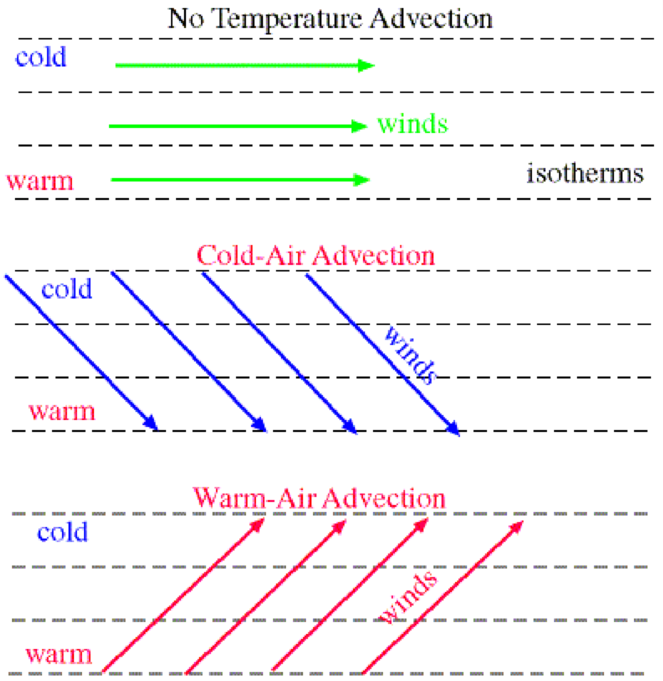
$$-(u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial z})$$

A good example to have in mind would be the transport of pollution in a river: the motion of the water carries the polluted water downstream.

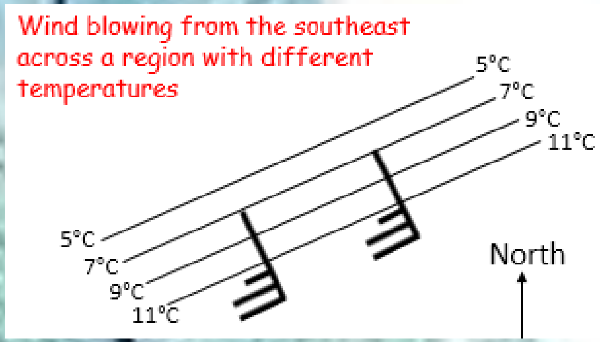
In meteorology, advection usually refers to the predominantly horizontal transport of an atmospheric property or by the wind, e.g. moisture or heat advection.

In this context, the advection operator in P vertical coordinates is:

$$-\vec{V}.\nabla T = -\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \omega\frac{\partial T}{\partial p}\right)$$



the horizontal transport of atmospheric temperature by wind

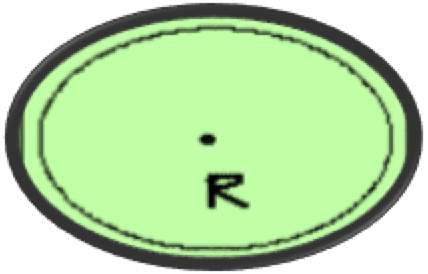


$$-\vec{V}.\nabla T \begin{cases} < 0 \rightarrow cold\ advection \\ > 0 \rightarrow warm\ advection \end{cases}$$

5) The Laplacian Operator

$$\vec{\nabla}.\vec{\nabla}\phi \equiv \nabla^2\phi \equiv \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}$$

Laplacian measures the difference between the average value of φ in a region around R and the value of the φ at a point R



If $\nabla^2\varphi = 0,$ then φ cannot increase or decrease in all directions

Physical meaning of ∇^2

The Laplacian gives the smoothness of a function. It measures the difference between the value of φ at a point and its mean value at surrounding points.

A little to the left of x $$\varphi(x-a)=\varphi(x)-a\frac{\partial\varphi}{\partial x}+\frac{a^2}{2}\frac{\partial^2\varphi}{\partial x^2}+\ldots$$

A little to the right of x $$\varphi(x+a)=\varphi(x)+a\frac{\partial\varphi}{\partial x}+\frac{a^2}{2}\frac{\partial^2\varphi}{\partial x^2}+\ldots$$

On taking the average $$\bar{\varphi}=\frac{1}{2}\left[\varphi(x-a)+\varphi(x+a)\right]=\varphi(x)+\frac{a^2}{2}\frac{\partial^2\varphi}{\partial x^2}$$

$$\bar{\varphi}-\varphi(x)=\frac{a^2}{2}\frac{\partial^2\varphi}{\partial x^2}$$

The deviation from the value of φ at a point and its mean value in the surrounding region is proportional to Laplacian φ

Fundamental forces

What creates wind?
What are these forces in the atmosphere?

We must analyse the predominant forces in the atmosphere, namely:

pressure and pressure gradients,

gravitation

rotation of the earth

friction

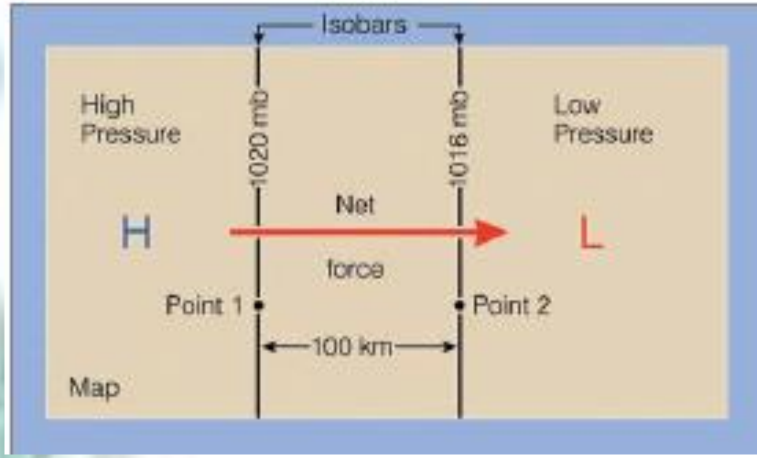
Also keep in mind that all forces have a *magnitude* and a *direction*

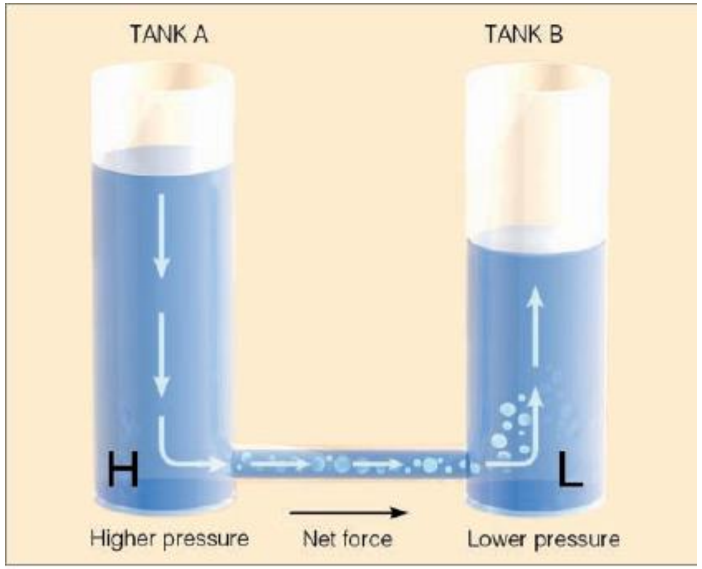
pressure gradient

Notice that when differences in horizontal air pressure exist there is a net force acting on the air.

This force, called the pressure gradient force *(PGF), is directed from higher toward lower pressure at right angles to the isobars.* The magnitude of the force is directly related to the pressure gradient.

Steep pressure gradients correspond to strong pressure gradient forces and vice versa.





The Pressure Gradient Force

The pressure gradient can be defined as a change in pressure over a given distance, i.e.,:

$$\text{Pressure gradient} = \frac{\Delta P}{distance} = \frac{P_{high} - P_{low}}{distance}$$

Isobar spacing and the magnitude of the pressure gradient

If the isobars are close together, the pressure gradient is large

If the isobars are far apart, the pressure gradient is small