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Geophysical Fluid Dynamics

Lecture 21

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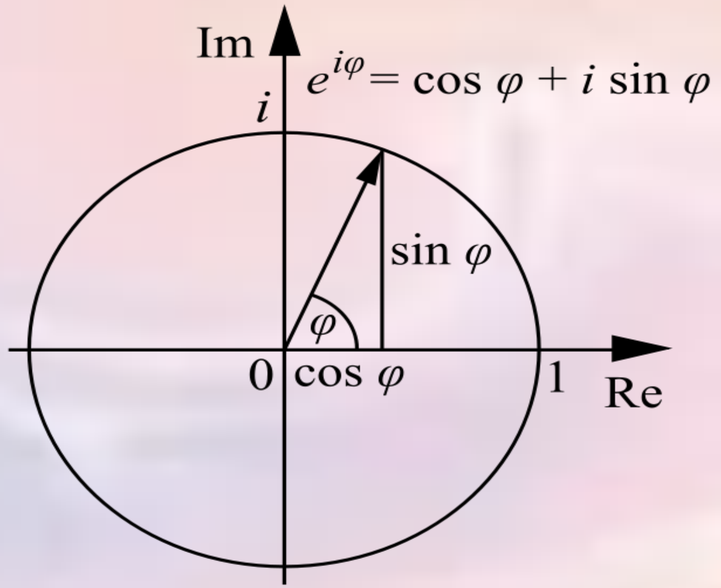
Atmospheric Planetary Boundary Layer

$$u + iv = (u_g + iv_g)(1 - e^{-\gamma z} e^{-i\gamma z})$$

Dividing into the real and imaginary parts

Euler's formula



$$u + iv = (u_g + iv_g)\left[1 - e^{-\gamma z}\left(\cos\gamma z - i\sin\gamma z\right)\right]$$

$$u + iv = (u_g + iv_g)\left[1 - e^{-\gamma z}\cos\gamma z + i\, e^{-\gamma z}\sin\gamma z\right]$$

For simplicity, we will now assume that the geostrophic wind is zonal (west-east) so the $v_g = 0$

$$u + iv = u_g\left[1 - e^{-\gamma z}\cos\gamma z + i\, e^{-\gamma z}\sin\gamma z\right]$$

$$u = u_g(1 - e^{-\gamma z}\cos\gamma z)$$

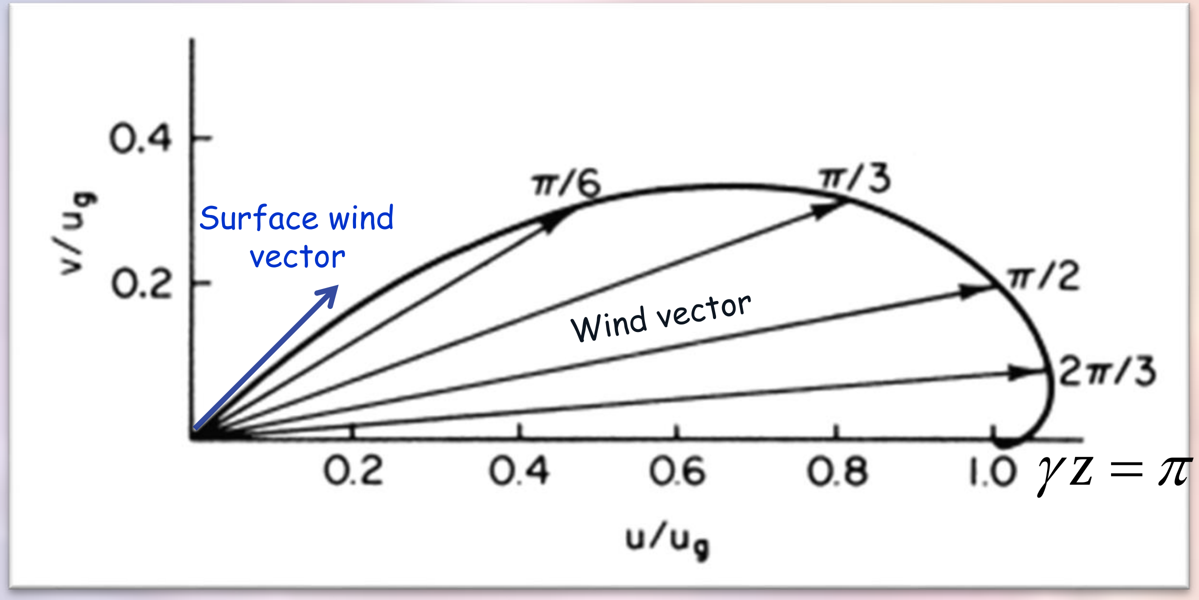
$$\gamma^2 = \frac{f}{2K}$$

$$v = u_g(e^{-\gamma z}\sin\gamma z)$$

These equations describe the Ekman Spiral

Let's look at an example and they we will try to determine what these equations are telling us about the wind in the boundary layer.

$$u = u_g\,(1 - e^{-\gamma z}\cos\gamma z) \qquad v = u_g\,(e^{-\gamma z}\sin\gamma z)$$



When
z = π/γ , the wind is parallel to and nearly equal to the geostrophic value.

Hodograph of wind components in the Ekman spiral solution.

Arrows show velocity vectors for several levels in the Ekman layer, whereas the spiral curve traces out the velocity variation as a function of height.

Points labeled on the spiral show the values of γz, which is a nondimensional measure of height.

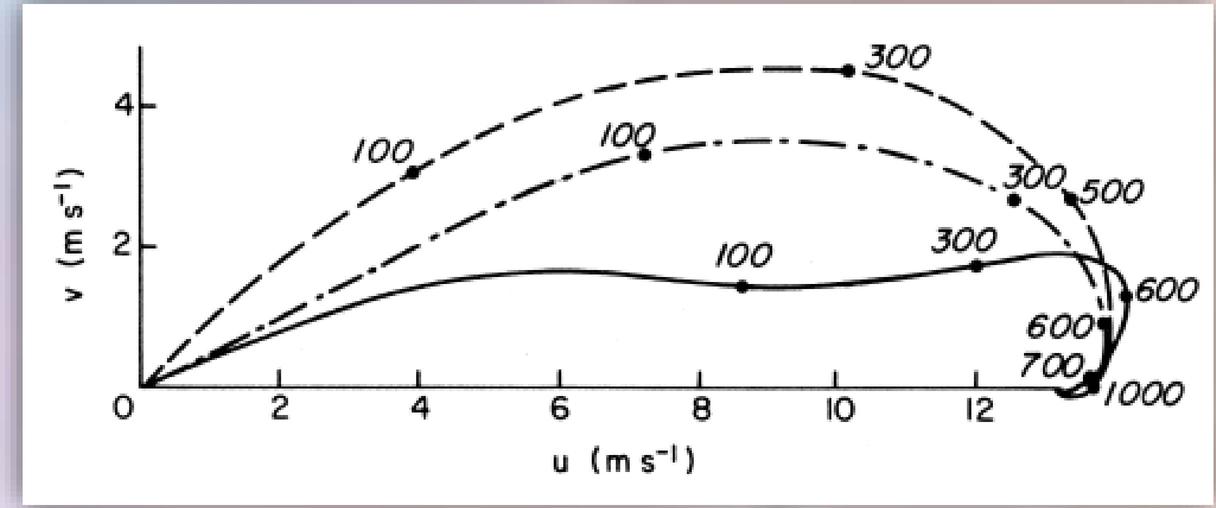
Effective depth of the boundary layer

We assume the values $f = 10^{-4}s^{-1}$ and $K \cong 10\ m^2s^{-1}$

The effective height $z_0 = \pi/\gamma$

$$z_0 = \frac{\pi}{\gamma} = \pi\sqrt{\frac{2K}{f}} = \pi\sqrt{\frac{2\times10}{10^{-4}}} \approx 1400m$$

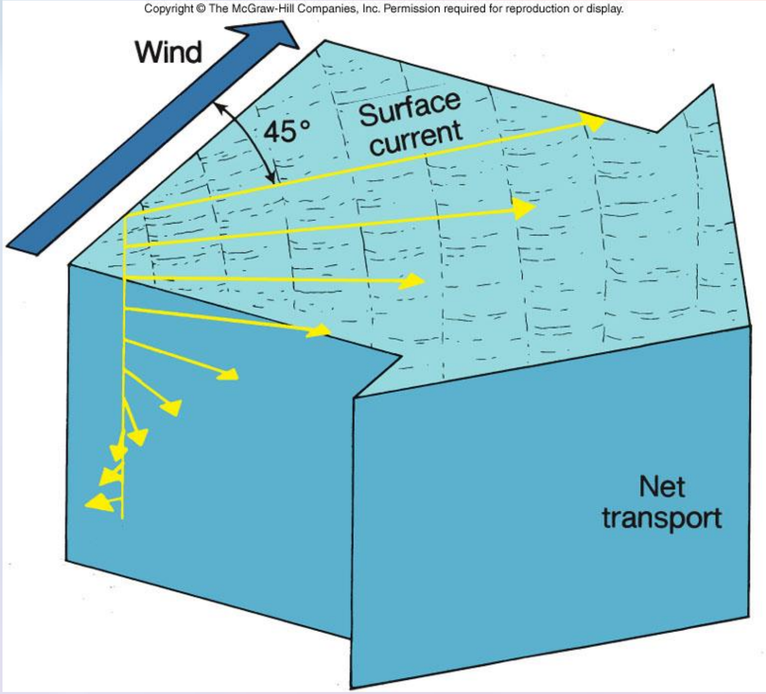
Thus, the effective depth of the Ekman boundary layer is about 1.4 km.

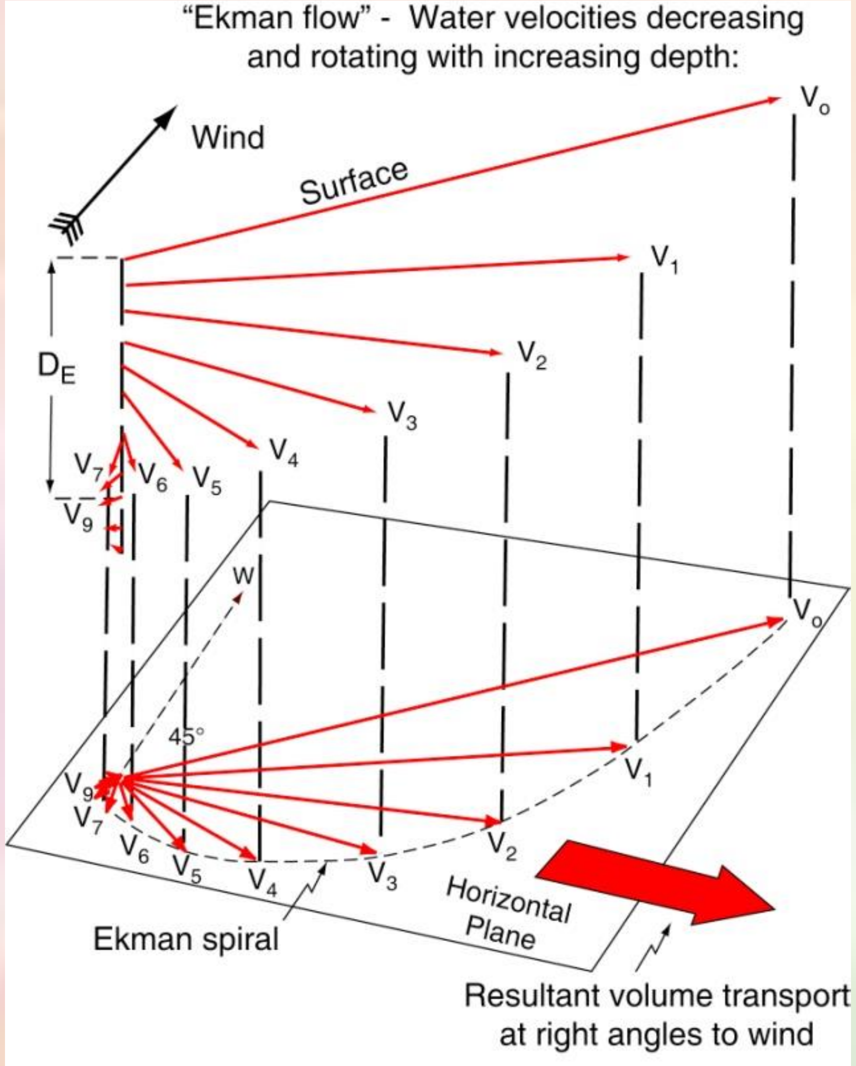


Mean wind hodograph for Jacksonville, Florida (~=30° N), April 4, 1968 (solid line) compared with the Ekman spiral (dashed line) and the modified Ekman spiral (dash-dot line) computed with De~=1200 m. Heights are shown in meters. (Adapted from Brown, 1970).

Ekman Motion

Ekman layer velocities (Northern Hemisphere). Water velocity as a function of depth (upper projection) and Ekman spiral (lower projection).

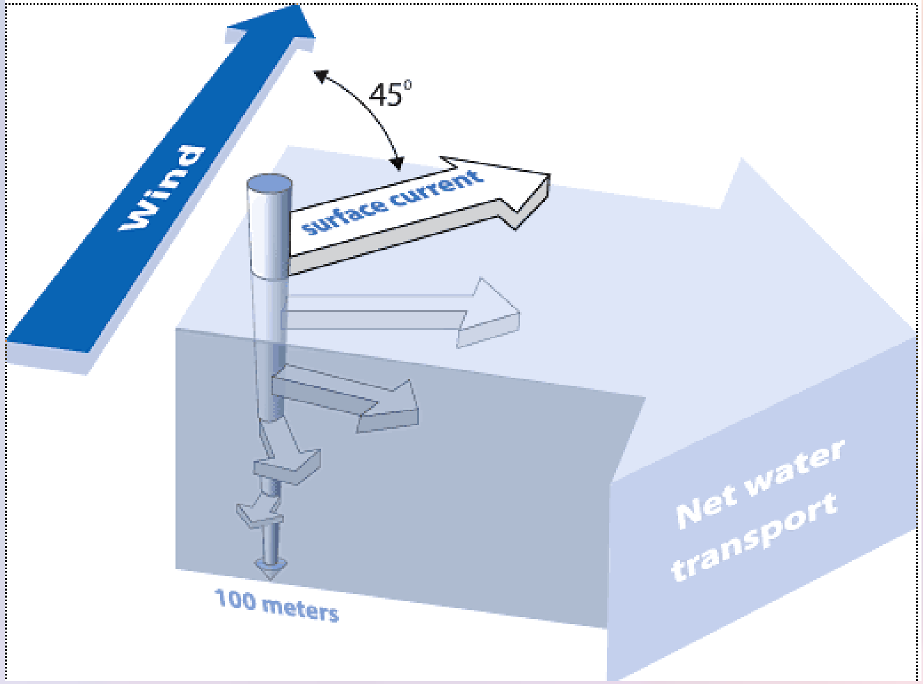


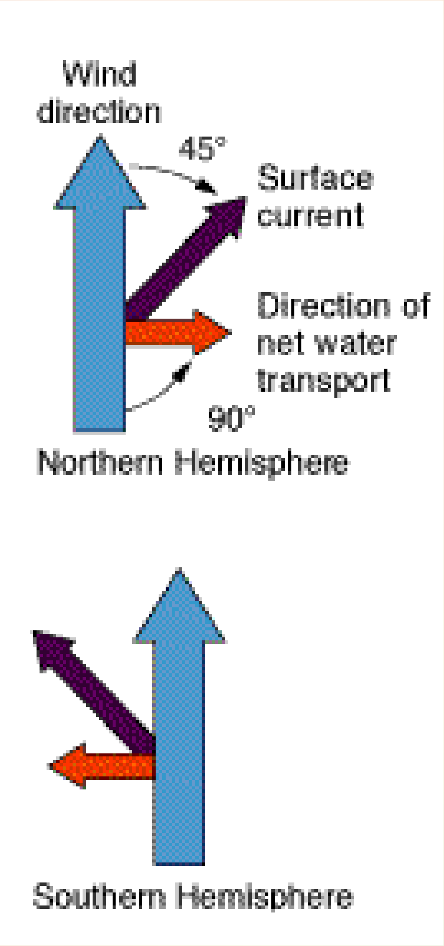


The large open arrow shows the direction of the total Ekman transport, which is perpendicular to the wind.

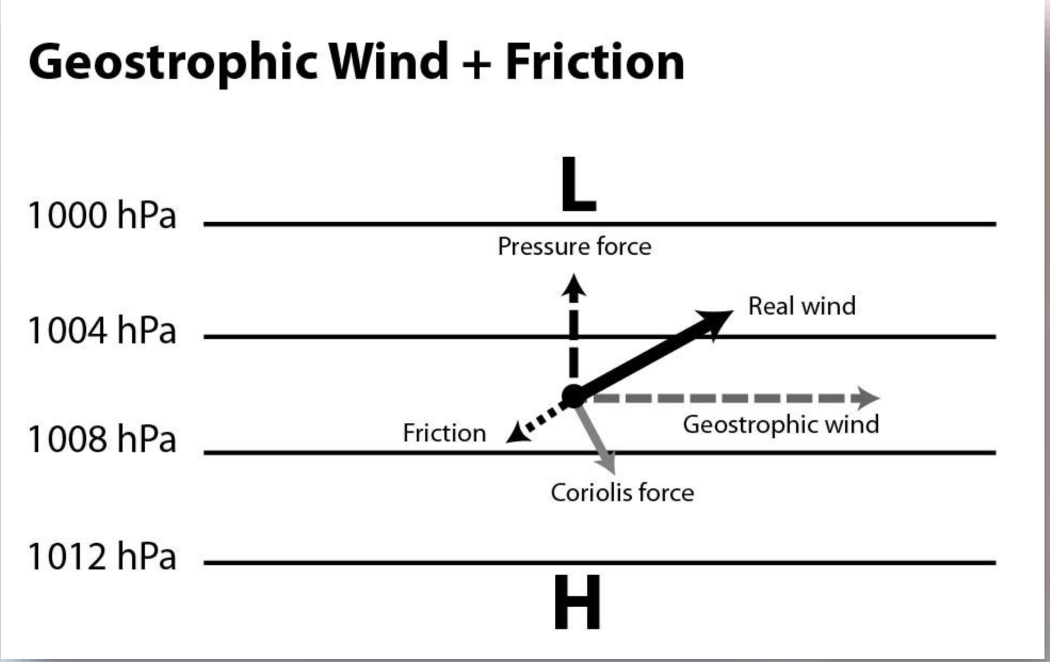
Ekman Spiral

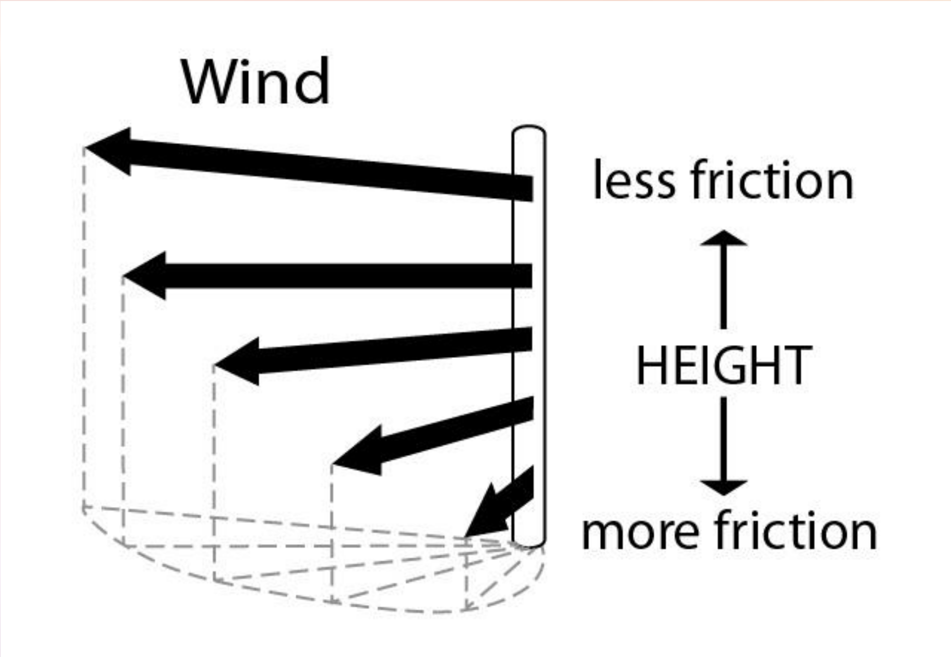
The Ekman spiral occurs as a consequence of the Coriolis effect





How does the atmosphere move?

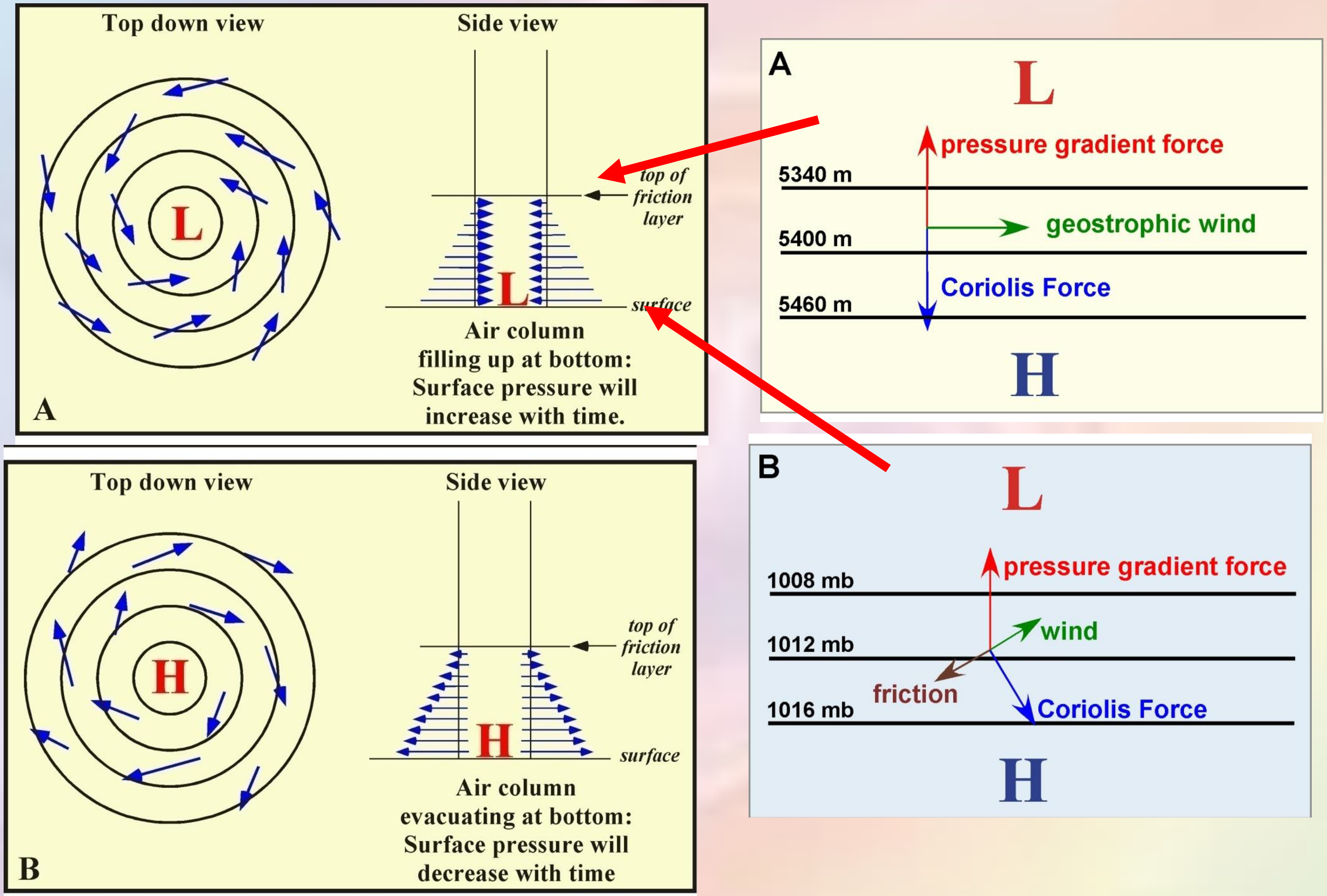




Free atmosphere

Planetary boundary layer

Implication of Ekman theory for synoptic scale systems



Wind Hodograph

A wind hodograph displays the change of wind speed and direction with height (vertical wind shear) in a simple polar diagram.

Wind speed and direction are plotted as arrows (vectors) with their tails at the origin and the point in the direction toward which the wind is blowing.

This is backward from our station model.

The length of the arrows is proportional to the wind speed. The larger the wind speed, the longer the arrow.

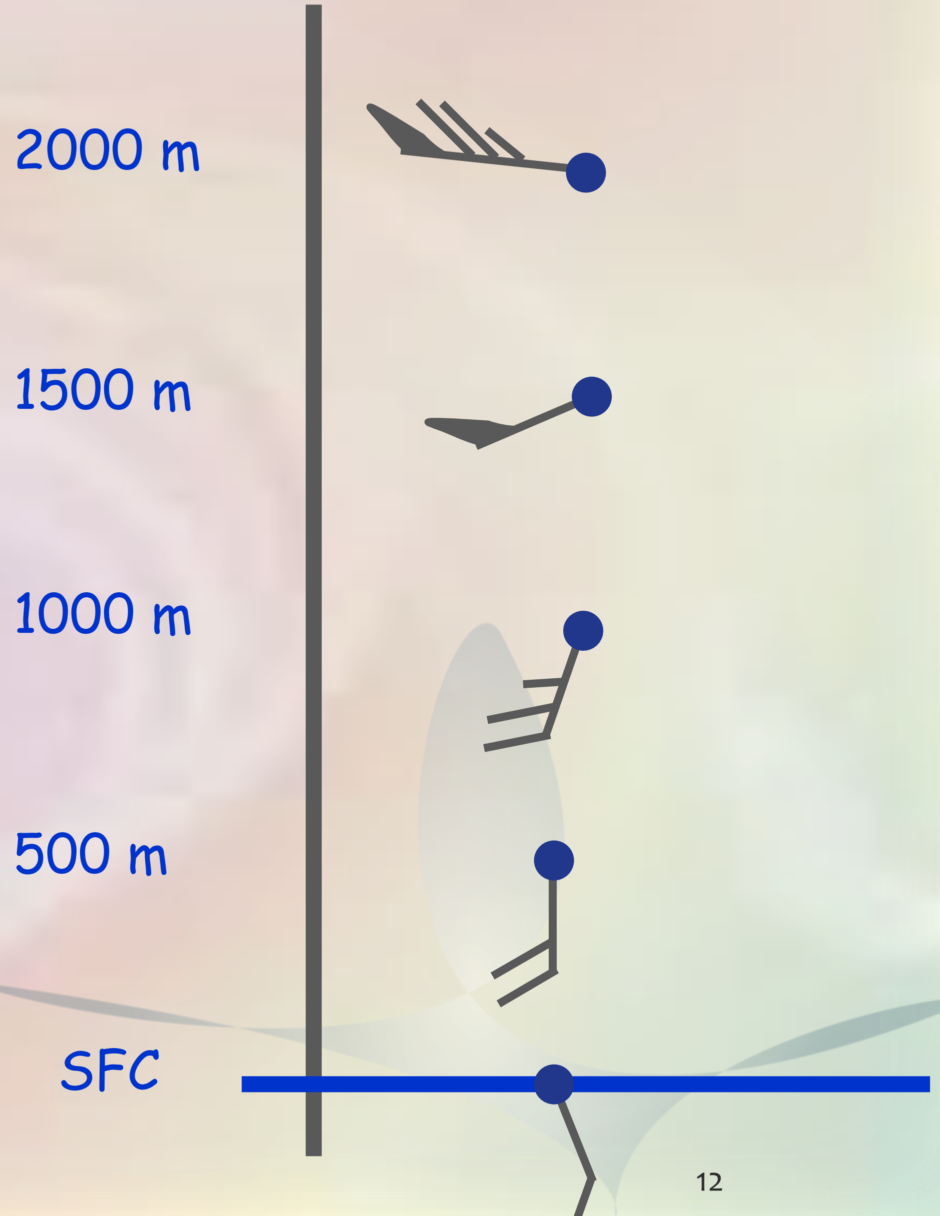
Normally only a dot is placed at the head of the arrow and the arrow itself is not drawn.
The hodograph is completed by connecting the dots!

Hodograph - Example

Height (MSL)	Direction	Speed (kt)
250 m (SFC)	160	10
500 m	180	20
1000 m	200	25
1500 m	260	50
2000 m	280	75

Just by looking at this table, it is hard (without much experience) to see what the winds are doing and what the wind shear is.

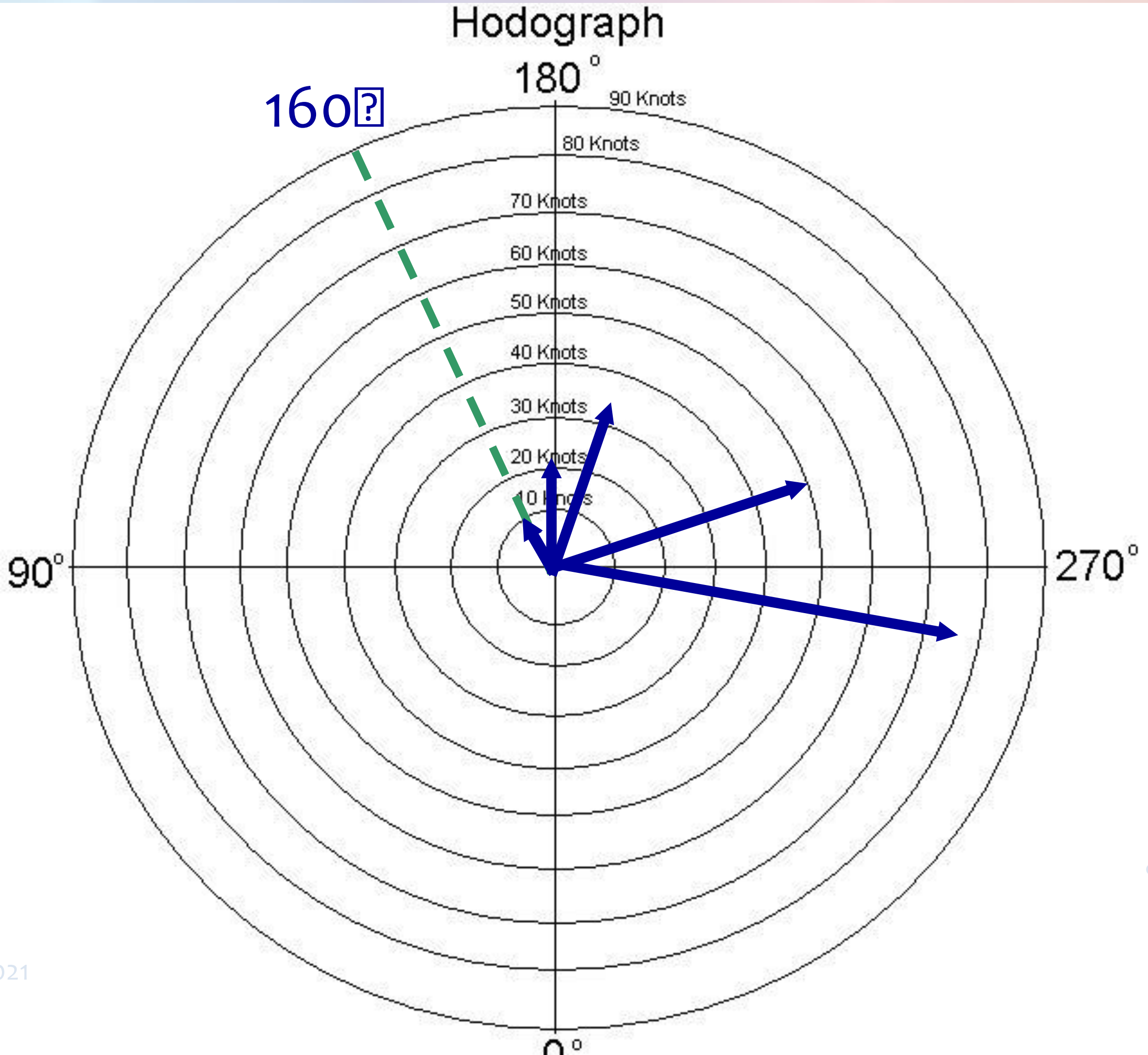
Let us plot the winds using a station model diagram.

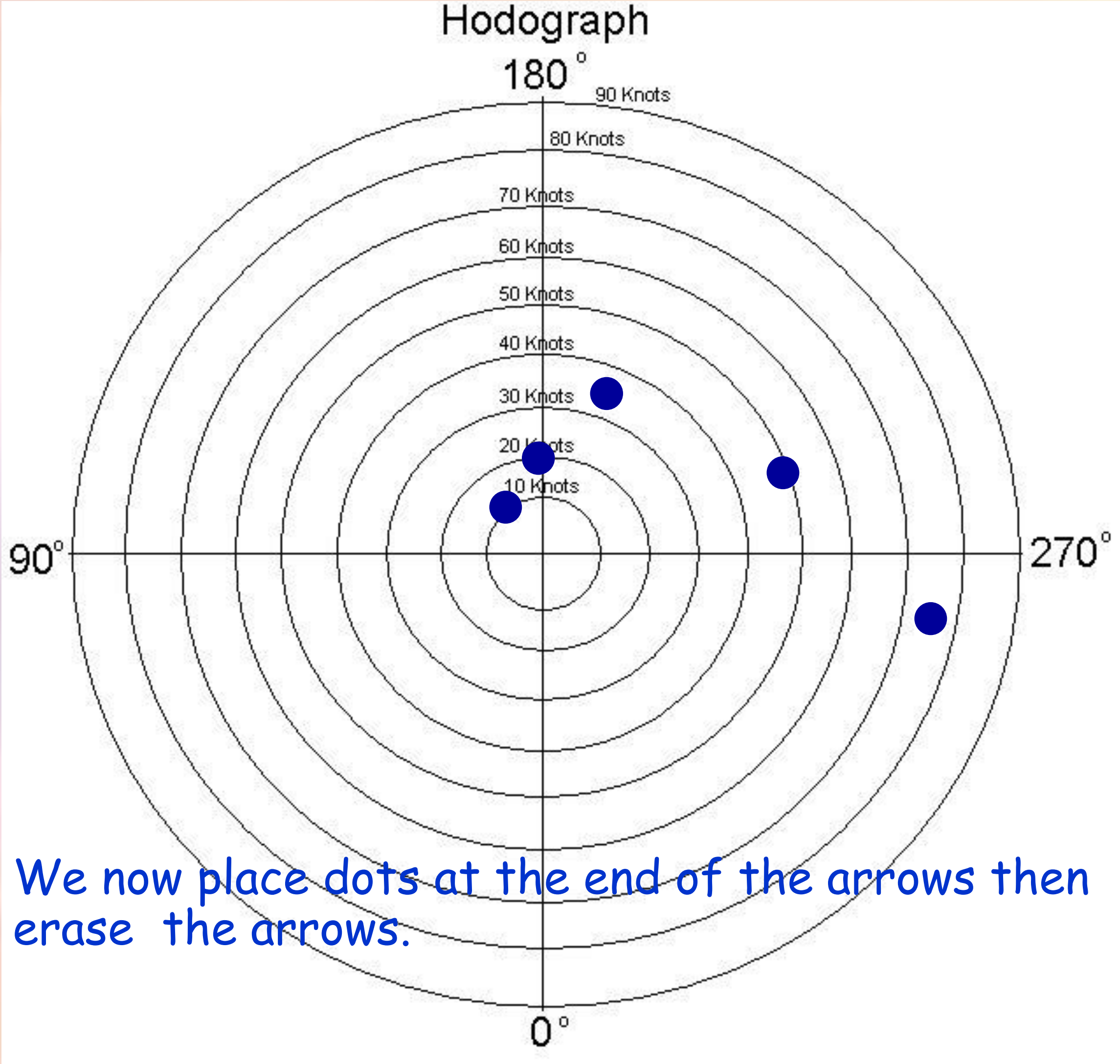


Let us now draw the hodograph!

Let us draw the surface observation. 160º at 10 kts

Let us draw the 500 m observation: 180º at 20 kts





We now place dots at the end of the arrows then erase the arrows.

We then connect the dots with a smooth curve and label the points.

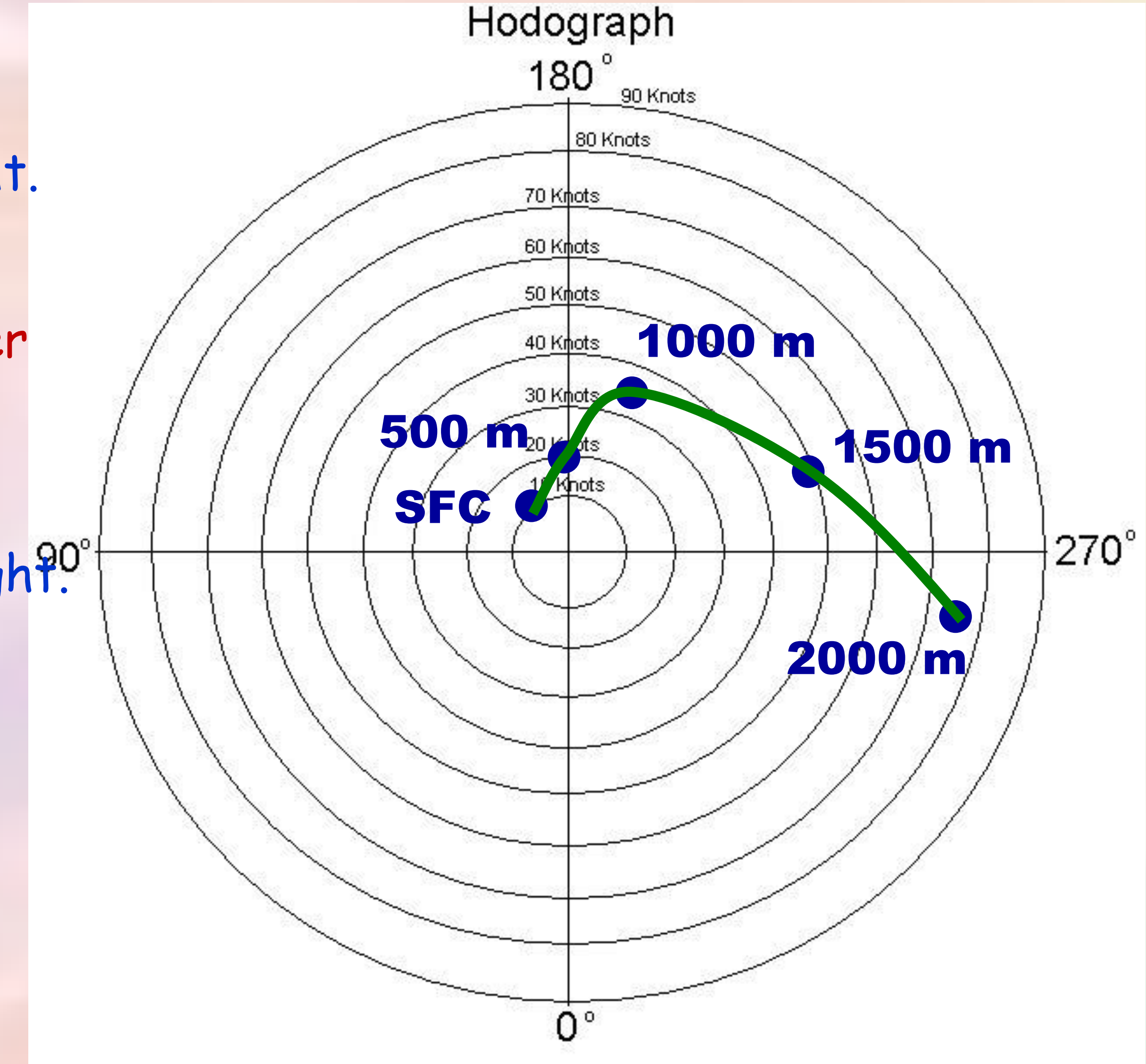
This is the final hodograph

We see that the wind speeds increase with height.

We know this since the plotted points get farther from the origin as we go up.

We see that the winds change direction with height. it is curved ***clockwise***.

If we start at the surface (SFC) and follow the hodograph curve, we go in a clockwise direction!



$$\frac{\partial u}{\partial t} = -\vec{V}.\nabla u - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv - ew + F_x$$

$$\frac{\partial v}{\partial t} = -\vec{V}.\nabla v - \frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_y$$

$$\frac{\partial w}{\partial t} = -\vec{V}.\nabla w - \frac{1}{\rho}\frac{\partial p}{\partial z} + eu - g + F_z \qquad \rightarrow -\frac{1}{\rho}\frac{\partial p}{\partial z} = g$$

$$\frac{\partial p}{\partial t} = -\vec{V}.\nabla p - \gamma p \nabla.\vec{V} + \frac{R}{c_v}\rho\frac{dH}{dt}$$

$$\frac{\partial \rho}{\partial t} = -\vec{V}.\nabla \rho - \rho \nabla.\vec{V}$$

$$\frac{\partial q_v}{\partial t} = -\vec{V}.\nabla q_v + Q_v$$

$$p = \rho R T$$