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Lecture 19

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VERTICAL MOTION

As mentioned previously, for synoptic-scale motions the vertical velocity component is typically of the order of a few centimeters per second.

Routine meteorological soundings, however, only give the wind speed to an accuracy of about a meter per second.

Thus, in general the vertical velocity is not measured directly but must be inferred from the fields that are measured directly.

Two methods for inferring the vertical motion field are the kinematic method, based on the equation of continuity, and the adiabatic method, based on the thermodynamic energy equation.

Both methods are usually applied using the isobaric coordinate system so that $\omega(p)$ is inferred rather than $w(z)$.

These two measures of vertical motion can be related to each other with the aid of the hydrostatic approximation.

Expanding dp/dt in the (x, y, z) coordinate system yields

$$\omega = \frac{dp}{dt} = \frac{\partial p}{\partial t} + \mathbf{V} \cdot \nabla p + w\left(\frac{\partial p}{\partial z}\right)$$

Now, for synoptic-scale motions, the horizontal velocity is geostrophic to a first approximation.

Therefore, we can write

$$V = V_g + V_a$$

where V_a is the ageostrophic wind and $|\mathbf{V}_a| \ll |\mathbf{V}_g|$

$$\mathbf{V}_g = (\rho f)^{-1}\mathbf{k} \times \nabla p$$ so that $$\mathbf{V}_g \cdot \nabla p = 0$$

Using this result plus the hydrostatic approximation

$$\omega = \frac{\partial p}{\partial t} + \mathbf{V}_a \cdot \nabla p - g\rho w$$

Comparing the magnitudes of the three terms on the right in equ., we find that for synoptic-scale motions

$$\partial p/\partial t \sim 10 \text{ hPa d}^{-1}$$

$$\mathbf{V}_a \cdot \nabla p \sim \left(1 \text{ m s}^{-1}\right)\left(1 \text{ Pa km}^{-1}\right) \sim 1 \text{ hPa d}^{-1}$$

$$g\rho w \sim 100 \text{ hPa d}^{-1}$$

Thus, it is quite a good approximation to let

$$\omega = -\rho g w$$

1) The Kinematic Method

One method of deducing the vertical velocity is based on integrating the continuity equation in the vertical.

Integration of

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p + \frac{\partial \omega}{\partial p} = 0$$

with respect to pressure from a reference level p_s to any level p yields

$$\omega(p) = \omega(p_s) - \int_{p_s}^{p} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p dp = \omega(p_s) + (p_s - p)\left(\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y}\right)_p$$

$$\omega(p) = \omega(p_s) + (p_s - p)\left(\frac{\partial\langle u\rangle}{\partial x} + \frac{\partial\langle v\rangle}{\partial y}\right)_p$$ *

Here the angle brackets denote a pressure-weighted vertical average:

$$\frac{w_1 \cdot x_1 + w_2 \cdot x_2 + \ldots + w_n \cdot x_n}{w_1 + w_2 + \ldots + w_n}$$

$$\langle\,\rangle \equiv (p - p_s)^{-1} \int_{p_s}^{p} ()\,dp$$

weighted average value of f on the interval $[a, b]$ with weight w

$$= \frac{\int_a^b w(x) \cdot f(x)\,dx}{\int_a^b w(x)\,dx}$$

With the aid of $\omega = -\rho g w$ the averaged form of equ. *

can be rewritten as

$$w(z) = \frac{\rho(z_s)\,w(z_s)}{\rho(z)} - \frac{p_s - p}{\rho(z) g}\left(\frac{\partial\langle u\rangle}{\partial x} + \frac{\partial\langle v\rangle}{\partial y}\right)$$

where z and z_s are the heights of pressure levels p and p_s, respectively.

Application of this Equ. to infer the vertical velocity field requires knowledge of the horizontal divergence.

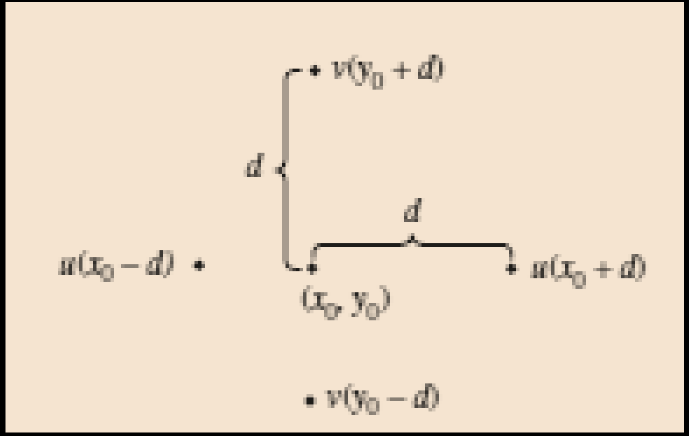
To determine the horizontal divergence, the partial derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$

are generally estimated from the fields of u and v by using finite difference approximations.

For example, to determine the divergence of the horizontal velocity at the point x_0, y_0 in Figure, we write

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \approx \frac{u(x_0 + d) - u(x_0 - d)}{2d} + \frac{v(y_0 + d) - v(y_0 - d)}{2d}$$

However, for synoptic-scale motions in midlatitudes, the horizontal velocity is nearly in geostrophic equilibrium.



Grid for estimation of the horizontal divergence

Except for the small effect due to the variation of the Coriolis parameter, the geostrophic wind is nondivergent; that is, $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ are nearly equal in magnitude but opposite in sign.

Thus, the horizontal divergence is due primarily to the small departures of the wind from geostrophic balance (i.e., the ageostrophic wind).

A 10% error in evaluating one of the wind components in

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \approx \frac{u(x_0 + d) - u(x_0 - d)}{2d} + \frac{v(y_0 + d) - v(y_0 - d)}{2d}$$

can easily cause the estimated divergence to be in error by 100%.

For this reason, the continuity equation method is not recommended for estimating the vertical motion field from observed horizontal winds.

2) The Adiabatic Method

A second method for inferring vertical velocities, which is not so sensitive to errors in the measured horizontal velocities, is based on the thermodynamic energy equation.

If the diabatic heating J is small compared to the other terms in the heat balance, $\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) - S_p\omega = \frac{J}{c_p}$ where $S_p \equiv \frac{RT}{c_p p} - \frac{\partial T}{\partial p} = -\frac{T}{\theta}\frac{\partial \theta}{\partial p}$

where, with the aid of the equation of state and Poisson's equation $\theta = T(p_s/p)^{R/c_p}$

which is the static stability parameter for the isobaric system.

Using $\frac{T}{\theta}\frac{\partial \theta}{\partial z} = \Gamma_d - \Gamma$ and the hydrostatic equation, S_p may be rewritten as

$$S_p = (\Gamma_d - \Gamma)/\rho g$$

yields $$\omega = S_p^{-1}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

Because temperature advection can usually be estimated quite accurately in midlatitudes by using geostrophic winds, the adiabatic method can be applied when only geopotential and temperature data are available.

A disadvantage of the adiabatic method is that the local rate of change of temperature is required.

Unless observations are taken at close intervals in time, it may be difficult to accurately estimate $\partial T/\partial t$ over a wide area.

This method is also rather inaccurate in situations where strong diabatic heating is present, such as storms in which heavy rainfall occurs over a large area.

Chapter 6 presents an alternative method for estimating ω, based on the so-called *omega equation*, that does not suffer from these difficulties.

SURFACE PRESSURE TENDENCY

The development of a negative surface pressure tendency is a classic warning of an approaching cyclonic weather disturbance. A simple expression that relates the surface pressure tendency to the wind field, and thus in theory might be used as the basis for short-range forecasts, can be obtained by taking the limit p ⟶ 0 in

$$\omega(p) = \omega(p_s) - \int_{p_s}^{p} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p dp = \omega(p_s) + (p_s - p)\left(\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y}\right)_p$$

to get

$$\omega(p_s) = -\int_0^{p_s} (\nabla \cdot \mathbf{V})\, dp$$

followed by substituting from

$$\omega = \frac{\partial p}{\partial t} + \mathbf{V}_a \cdot \nabla p - g\rho w$$

to yield

$$\frac{\partial p_s}{\partial t} \approx -\int_0^{p_s} (\nabla \cdot \mathbf{V})\, dp$$

Here we have assumed that the surface is horizontal so that w_s = 0, and have neglected advection by the ageostrophic surface velocity in accord with the scaling arguments in Section 3.5.1.

According to $\frac{\partial p_s}{\partial t} \approx -\int_0^{p_s} (\nabla \cdot \mathbf{V})\, dp$ the surface pressure tendency at a given point is determined by the total convergence of mass into the vertical column of atmosphere above that point.

This result is a direct consequence of the hydrostatic assumption, which implies that the pressure at a point is determined solely by the weight of the column of air above that point. Temperature changes in the air column will affect the heights of upper-level pressure surfaces, but not the surface pressure.

This is difficult to compute accurately from observations because it depends on the ageostrophic wind field.

In addition, there is a strong tendency for vertical compensation. Thus, when there is convergence in the lower troposphere, there is divergence aloft, and vice versa.

Adjustment of surface pressure to a midtropospheric heat source. Dashed lines indicate isobars. (a) Initial height increase at upper level pressure surface. (b) Surface response to upper level divergence.