Geophysical Fluid Dynamics

Lecture 16

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Balanced Flow

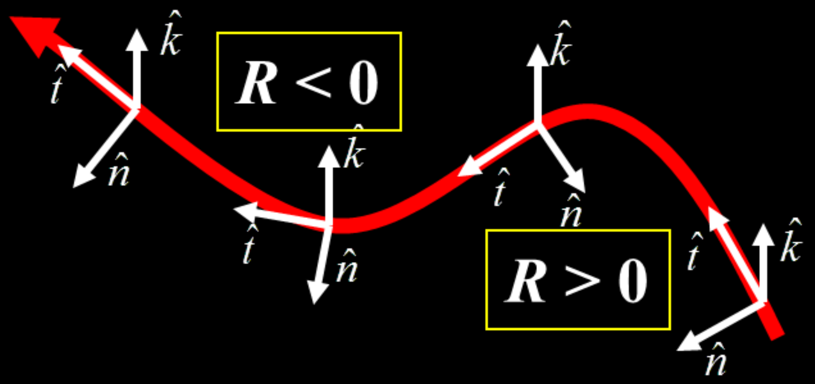
The pressure and velocity distributions in atmospheric systems are related by relatively simple, approximate force balances.

We can gain a qualitative understanding by considering steady-state conditions, in which the fluid flow does not vary with time, and by assuming there are no vertical motions.

To explore these balanced flow conditions, it is useful to define a new coordinate system, known as natural coordinates.

Natural Coordinates

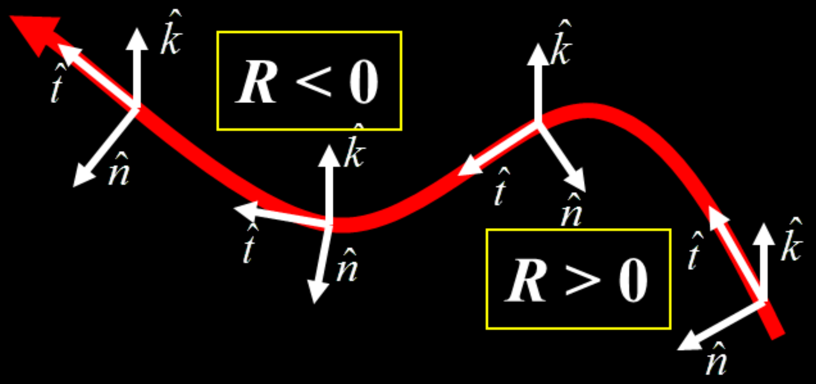
Natural coordinates are defined by a set of mutually orthogonal unit vectors whose orientation depends on the direction of the flow.



Unit vector $\hat{t}$ points along the direction of the flow.

Unit vector is perpendicular to the flow, with positive to the left.

Unit vector points upward.



Horizontal velocity: $\vec{V} = V\hat{t}$

V is the horizontal speed, defined by

$$V \equiv ds/dt,$$

where $s(x, y, t)$ is the curve followed by a fluid parcel moving in the horizontal plane.

To determine acceleration following the fluid motion,

$$\frac{d\vec{V}}{dt} = \frac{d}{dt}\left(V\hat{t}\right)$$

$$\frac{d\vec{V}}{dt} = \hat{t}\frac{dV}{dt} + V\frac{d\hat{t}}{dt}$$

$$\delta\psi = \frac{\delta s}{R} = \frac{|\delta\hat{t}|}{|\hat{t}|} = |\delta\hat{t}|$$

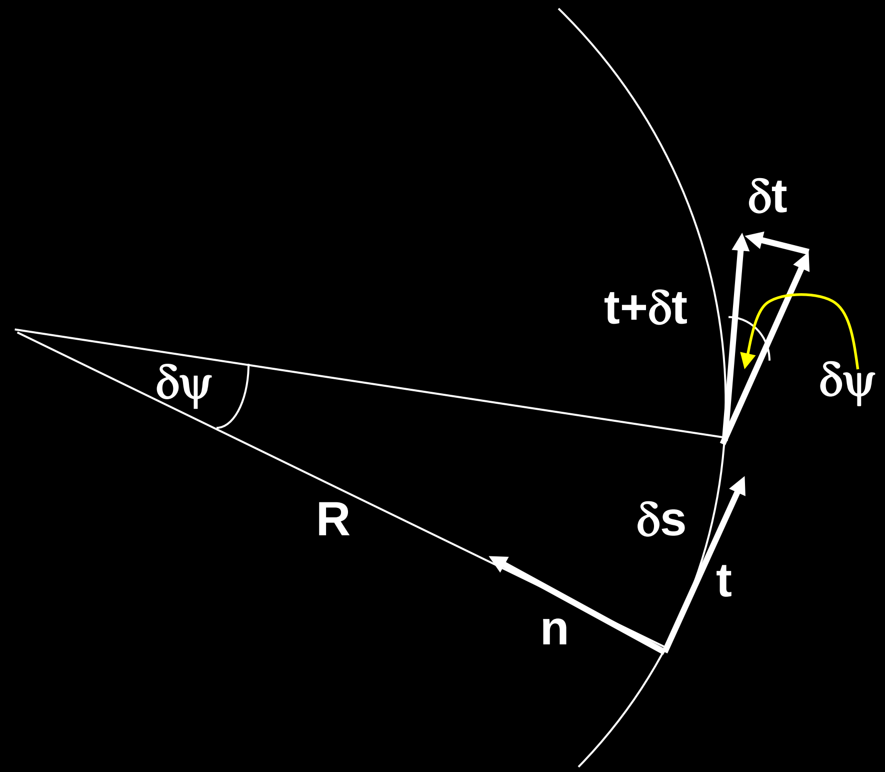
$R =$ radius of curvature (positive in positive n direction)

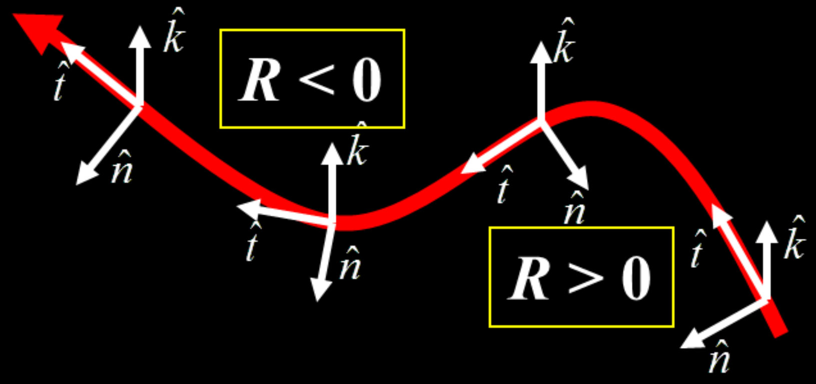
$R > 0$ if air parcels turn toward left

$R < 0$ if air parcels turn toward right

$$\frac{d\hat{t}}{dt} = \frac{d\hat{t}}{ds}\frac{ds}{dt} = \frac{\hat{n}}{R}V$$

$$\frac{d\hat{t}}{ds} = \frac{\hat{n}}{R}$$ (taking limit as $\delta s \to 0$)





$$\frac{d\vec{V}}{dt} = \hat{t}\frac{dV}{dt} + V\frac{d\hat{t}}{dt}$$

$$\frac{d\vec{V}}{dt} = \hat{t}\frac{dV}{dt} + \hat{n}\frac{V^2}{R}$$

vector form of acceleration following fluid motion in natural coordinates

$$\frac{d\vec{V}}{dt} = -\nabla_p\Phi - f\hat{k}\times\vec{V}$$

$$-f\hat{k}\times\vec{V} = -fV\hat{n}$$

Coriolis (always acts normal to flow)

$$-\nabla_p\Phi = -\left(\hat{t}\frac{\partial\Phi}{\partial s} + \hat{n}\frac{\partial\Phi}{\partial n} + \hat{k}\frac{\partial\Phi}{\partial z}\right)$$

pressure gradient

$$\hat{t}\frac{dV}{dt} + \frac{V^2}{R}\hat{n} = -\hat{t}\frac{\partial\Phi}{\partial s} - \left(\frac{\partial\Phi}{\partial n} + fV\right)\hat{n}$$

$$\frac{dV}{dt} = -\frac{\partial\Phi}{\partial s}$$

$$\frac{V^2}{R} + fV = -\frac{\partial\Phi}{\partial n}$$

component equations of horizontal momentum equation (isobaric) in natural coordinate system

$$\frac{dV}{dt} = -\frac{\partial \Phi}{\partial s}$$ Balance of forces parallel to flow

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n}$$ Balance of forces normal to flow

For motion parallel to geopotential height contours, $\frac{\partial \Phi}{\partial s} = 0$, which means that the speed is constant following the motion.

$$\frac{dV}{dt} = 0 \rightarrow V = cte$$

If the geopotential gradient normal to the direction of motion is constant along a trajectory, the normal component equation implies that the radius of curvature *R* is also constant.

$$\frac{\partial \Phi}{\partial n} = 0 \qquad \frac{V^2}{R} + fV = 0 \qquad R = -V / f = cte$$

Geostrophic Flow

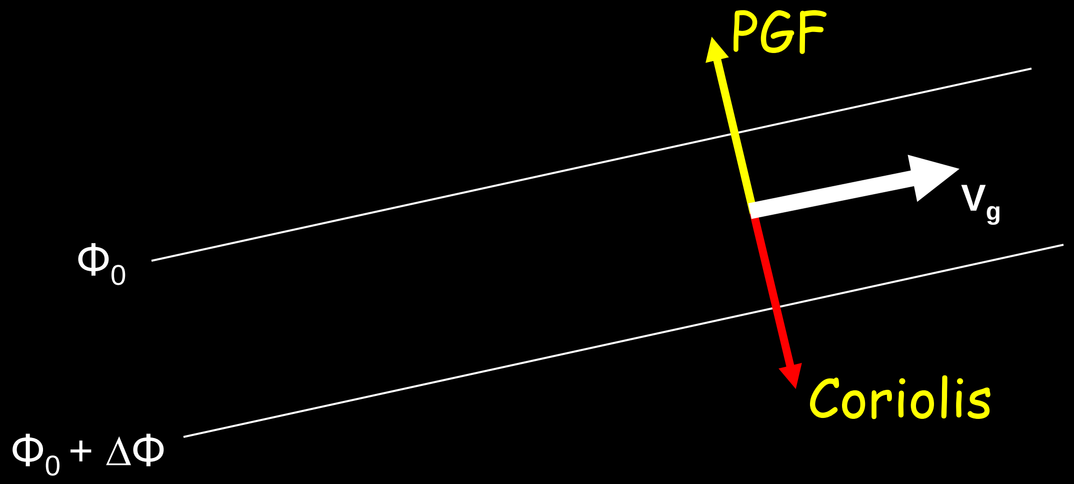
Flow in a straight-line $\left(R \rightarrow \pm\infty\right)$ flow parallel to the height contours is referred to as geostrophic motion.

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n}$$

(The term $\frac{V^2}{R}$ is crossed out with an arrow pointing to **0**.)

Horizontal components of Coriolis force and pressure gradient force are in exact balance.

$$fV_g = -\frac{\partial \Phi}{\partial n}$$



Inertial Flow

If the geopotential field is uniform on a constant pressure surface, so that the horizontal pressure gradient vanishes;

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n} \quad (\text{crossed out} \rightarrow 0)$$

$$R = -\frac{V}{f}$$

$$\frac{\partial \Phi}{\partial s} = 0 \rightarrow \frac{dV}{dt} = 0 \rightarrow V = const.$$

Because uniform geopotential implies constant speed, then the radius of curvature is constant if we assume f is approximately constant.

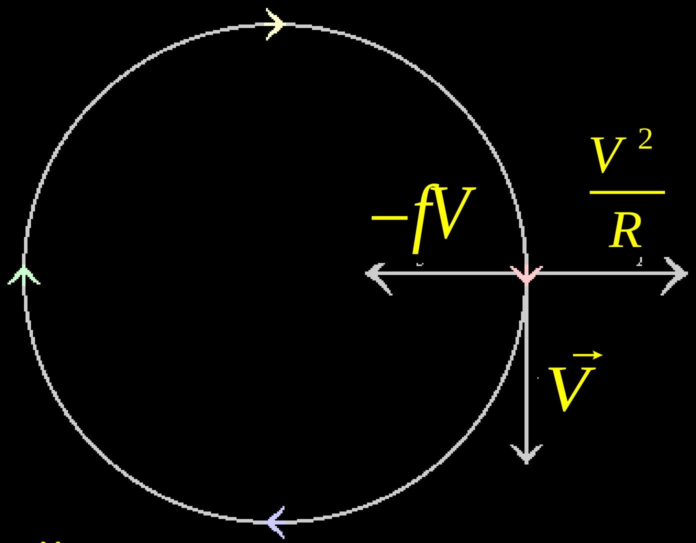
Inertial flow is not one of the more commonly seen flows in the atmosphere, yet it does exist.

Inertial flows are also known as inertial oscillations, since air parcels under the influence of inertial balance follow circular paths in an anti-cyclonic manner.

Air parcels will follow circular paths in anticyclonic rotation with period

Period of Inertial Oscillation

$$P = \left|\frac{2\pi R}{V}\right| = \left|\frac{2\pi R}{fR}\right| = \frac{2\pi}{f} = \frac{2\pi}{2\Omega|\sin\varphi|} = \frac{\frac{1}{2}\,day}{|\sin\varphi|}$$



Where ϕ is the latitude and ***P*** is the period of the oscillation

As stated above, inertial flow is not one of the more commonly seen forces in the atmosphere. The reason is that the pressure gradient force drives most flows in the atmosphere.

Since the pressure gradient force is assumed to be zero in inertial flow, this line of reasoning is justified. The only situations where inertial flow may be observed are in the center of large anti-cyclones or cyclones, where pressure gradients are very weak.

Although atmospheric conditions may not be favorable at most times for inertial flow, it is more prevalent in oceanic currents, where transient winds blowing across the surface are more likely to drive the current, as opposed to internal pressure gradients.

Cyclostrophic Flow

If the horizontal scale of a disturbance is small enough, the coriolis force may be neglected;

$$\frac{V^2}{R} + \cancel{fV} = -\frac{\partial \Phi}{\partial n}$$

This balance is between to the pressure gradient force and the centrifugal force.

If this equation is solved for V, we obtain the speed of the cyclostrophic wind:

$$V = \left(-R \frac{\partial \Phi}{\partial n} \right)^{1/2}$$

With the balance between the pressure gradient force in the n equation of motion and the centrifugal force, this constricts the possible types of flow to two types.

The flow can be either cyclonic or anti-cyclonic with a circular motion as a result of the centrifugal force.

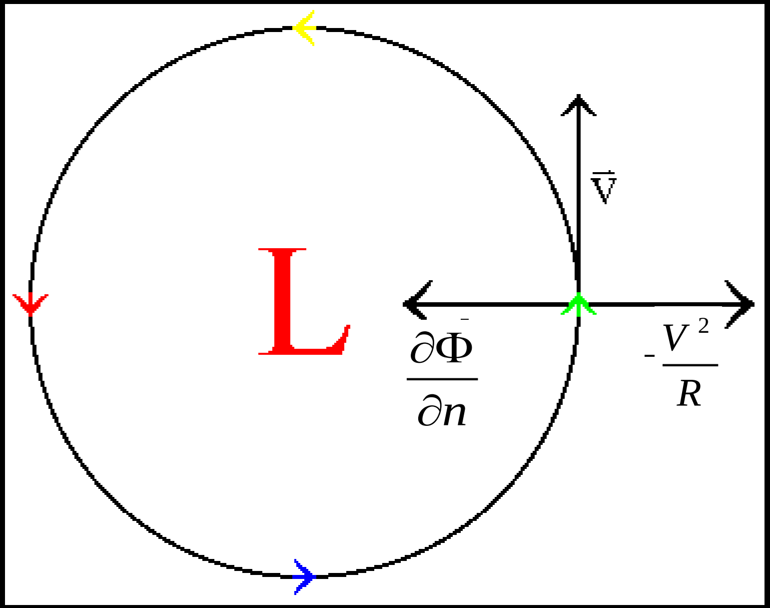
However, the pressure gradient force always points inward, making the center of circulation an area of low pressure.
Since only two forces are considered, there are certain assumptions the also have to be made.

The flow must be frictionless, always parallel to the height contours, and the scale of the flow is either small in scale or near the equator, where the coriolis force is essentially zero. The following picture illustrates cyclostrophic flow:

Cyclonic Flow

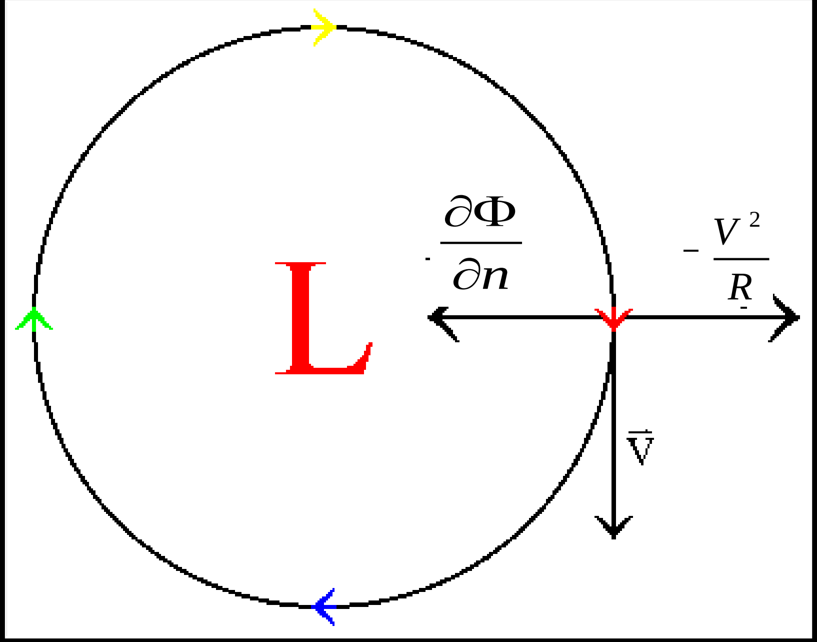
$$R > 0 \quad \frac{\partial \Phi}{\partial n} < 0 \rightarrow$$

$$\rightarrow R\frac{\partial \Phi}{\partial n} < 0 \Rightarrow V > 0$$



Anticyclonic Flow

$$R < 0 \quad \frac{\partial \Phi}{\partial n} > 0$$



When we assume that a flow is cyclostrophic in nature, the coriolis force is defined as being zero.

Therefore, a method must exist to determine if the Coriolis force can be neglected.

If the ratio of the centrifugal force to the coriolis force is large, then the cyclostrophic assumption can be made.

This ratio, called the *Rossby* number is defined below:

$$R_0 = \frac{V^2 / R}{fV} = \frac{V}{fR} \sim 10^3 \gg 1$$

There are some real world applications to cyclostrophic flow.

Small scale circulations, such as tornados, waterspouts, and dust devils are small enough so that the coriolis force can be neglected.

Several Types of Balanced Flow

Name	Approximation	Balance
Geostrophic Flow	$R \to \infty;\ \frac{\partial \Phi}{\partial s} = 0$	$\mathbf{F}_{Co} = -\mathbf{F}_{PG}$
Inertial Flow	$\nabla_p \Phi = 0$	$\mathbf{F}_{Co} = -\mathbf{F}_{Centr}$
Cyclostrophic Flow	$Ro = \frac{V}{fR} \gg 1$	$\mathbf{F}_{PG} = -\mathbf{F}_{Centr}$
Gradient Wind	$\text{finite R};\ \frac{\partial \Phi}{\partial s} = 0$	$\mathbf{F}_{Co} + \mathbf{F}_{Centr} = -\mathbf{F}_{PG}$

Several Types of Balanced Flow

balance	friction	Coriolis force	pressure gradient force	centrifugal force
geostrophic		√	√	
cyclostrophic			√	√
inertial		√		√
antitriptic	√		√	

Antitriptic balance

A coastal sea breeze reaches its typical strength around noon and continues to blow at about the same speed for several hours.

Under a steady onshore pressure gradient force which is due to the temperature gradient, the sea breeze should continue to gain strength.

The main force opposing this pressure gradient force is surface friction.

This balance is known as 'antitriptic'. Density currents, such as shallow outflows of cooler air from a mature thunderstorm, are largely antitriptically balanced.