



# *Atmospheric Physics*

## *Lecture 8*

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**Dew Point** = Temperature above freezing at which saturation occurs (i.e., dew forms)

**Frost Point** = Temperature where saturation occurs below the freezing point (i.e., frost forms)

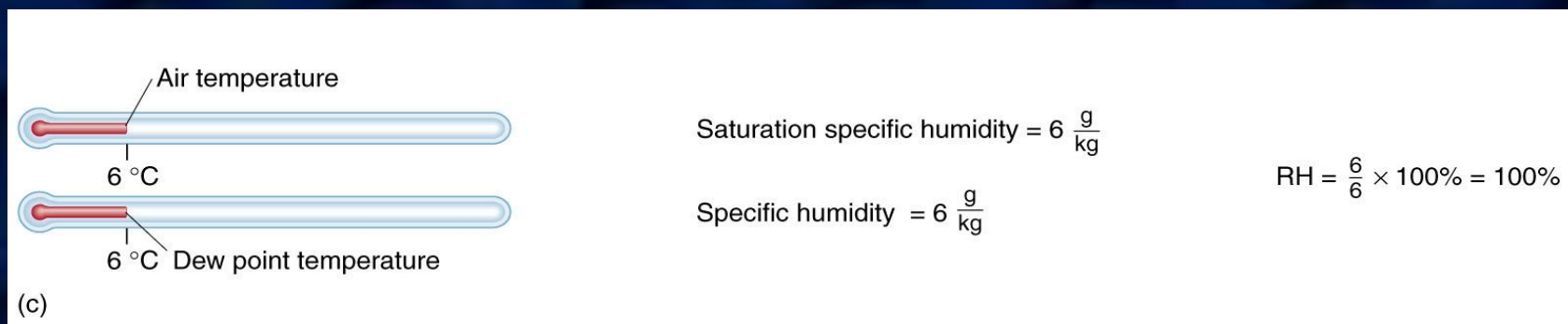
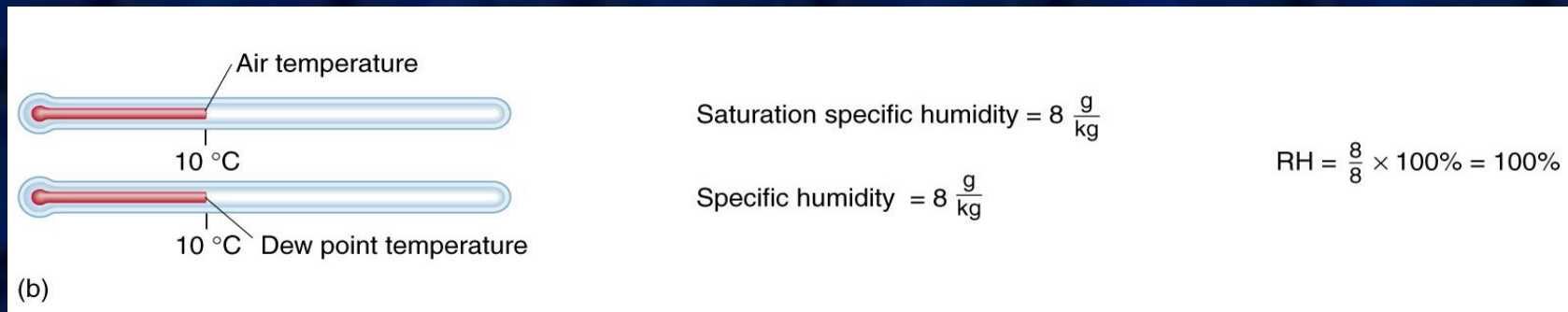
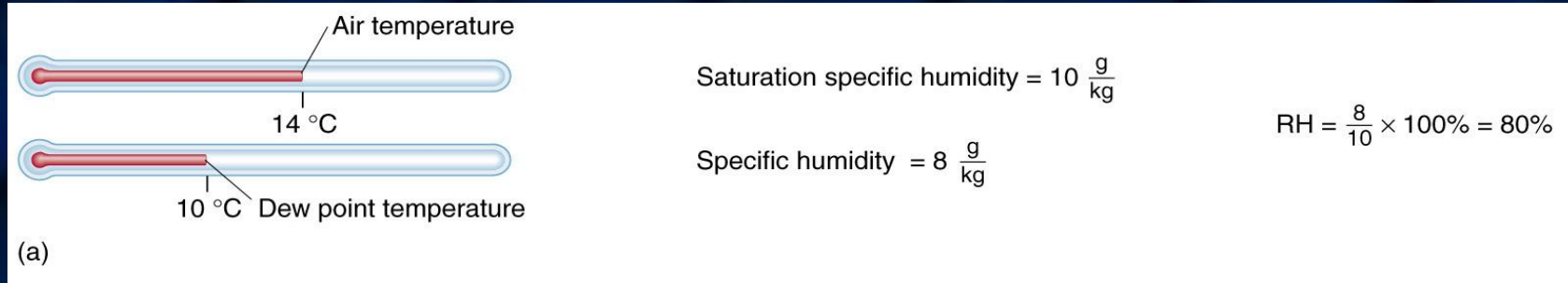


deposition

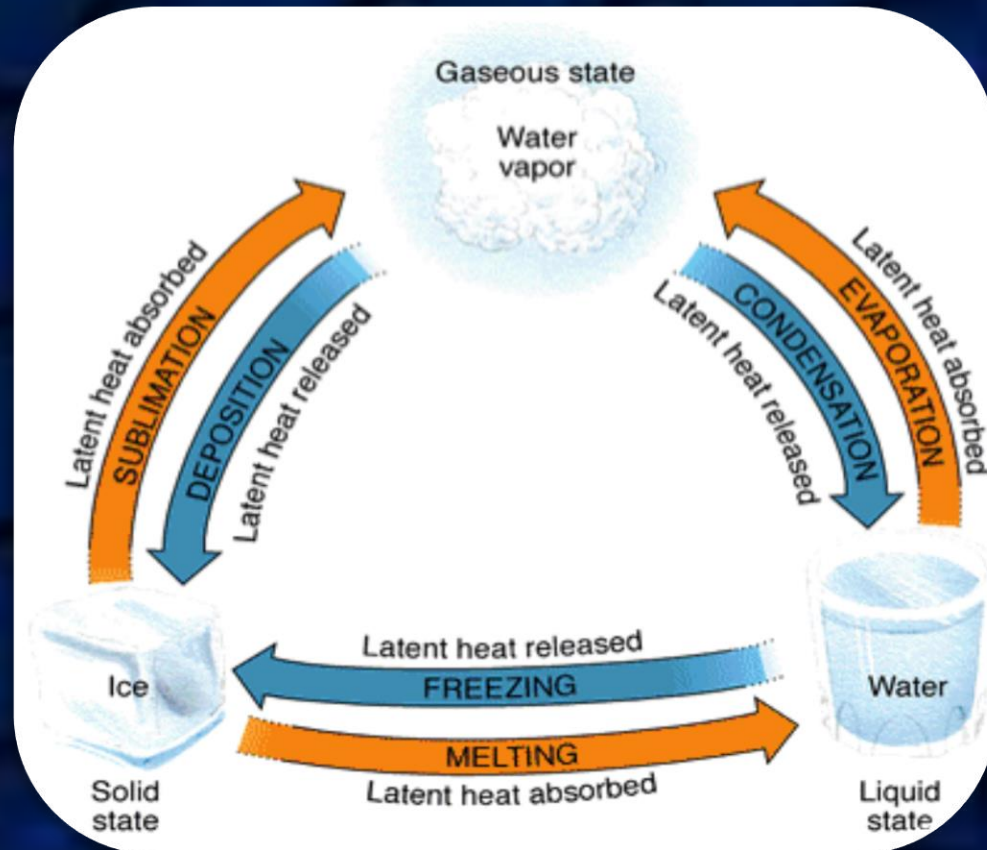


condensation

When the air temperature drops to the dewpoint,  
the relative humidity is 100%



## Phase changes of water (latent heat transfer)

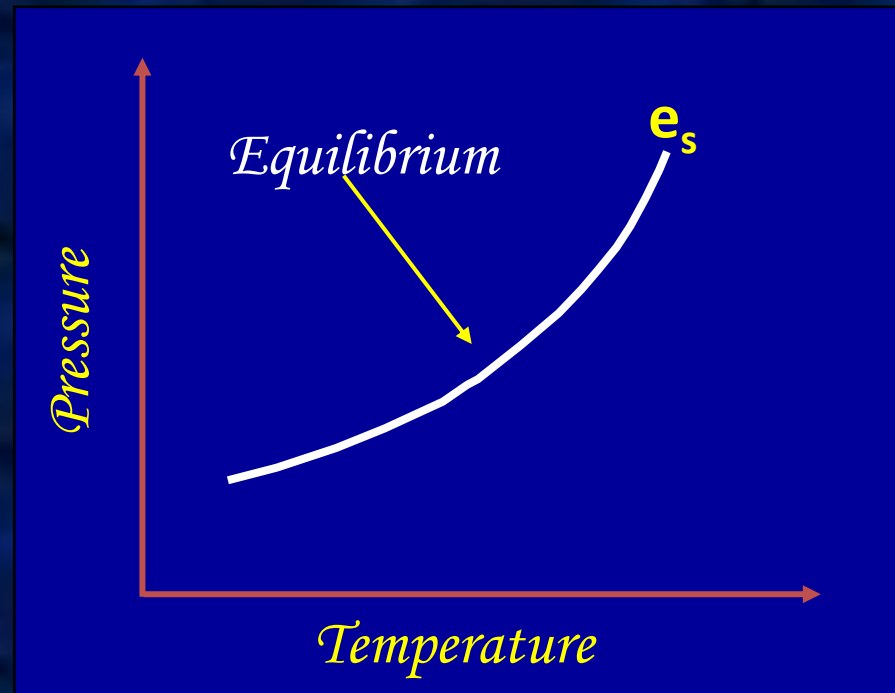


## Equilibrium Curve

Equilibrium:

Rate of condensation = Rate of evaporation

$e_s$ : water vapor pressure at equilibrium (saturation)



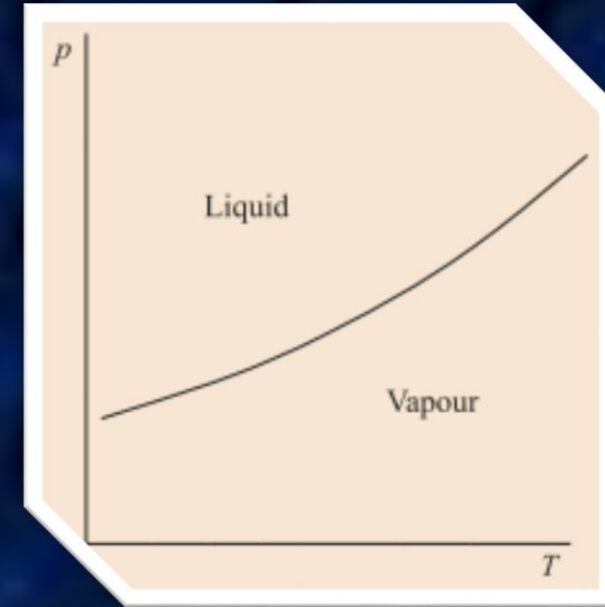
## Clausius-Clapeyron equation

Water vapour is a minor constituent of the atmosphere

It is responsible for the precipitation

Volume mixing ratio ( $\leq 0.03$ )

$$\frac{dp}{dT} = \frac{\delta S}{\delta V} = \frac{L}{T \delta V} \quad L = T \delta S$$



$$\delta V \approx V_v = 1/\rho_v = R_v T/p$$

Hence the Clausius-Clapeyron equation can be written in the more convenient form

$$\frac{dp}{dT} = \frac{Lp}{R_v T^2}$$

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2}$$

Note that if  $L$  is constant (a fairly good approximation at typical atmospheric temperatures), this can be integrated to give

$$e_s(T) = e_s(T_0) \exp \frac{L}{R_v} \left( \frac{1}{T_0} - \frac{1}{T} \right)$$

where  $T_0$  is a constant reference temperature.

We can relate the partial pressure  $e$  of water vapour to its volume mixing ratio  $v$  and mass mixing ratio  $\mu$ , say:

$$\mu_i = \frac{nm_i p_i}{mp} = \frac{m_i}{\bar{m}} \frac{p_i}{p}$$

$$\bar{m} = \frac{m}{n}$$

is the mean molecular mass for the sample.

We also define the volume mixing ratio  $v_i$  (also known as the mole fraction) by

$$v = e/p$$

$$\mu = \varepsilon(e/p)$$

where  $p$  is the total air pressure, and

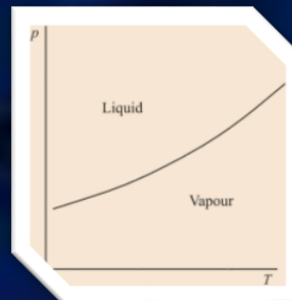
$$\varepsilon \approx \frac{m_v}{m_d} = \frac{18.02}{28.97} = 0.622 \quad \mu < 0.03$$

So long as no condensation or evaporation takes place, this mass  $\mu$  remains constant.

The vapour pressure of the parcel satisfies

$$e = \frac{\mu p}{\varepsilon}$$

and while this remains less than  $e_s(T)$  saturation does not occur, since the water vapour in the parcel remains below the vapour pressure curve in Figure.



Suppose that the parcel rises adiabatically from the surface at  $p_0$  and  $T_0$

then the potential temperature of the parcel remains constant at  $\theta = T_0$   
while the temperature  $T$  of the parcel falls according to

$$\theta = T \left( \frac{p_0}{p} \right)^k \quad T = T_0 \left( \frac{p}{p_0} \right)^k$$

we can eliminate the pressure  $p$  of the parcel to find how the vapour pressure varies as a function of temperature  $T$ , following the motion of the parcel:

$$e = \frac{\mu p}{\varepsilon} \quad e_{\text{parcel}}(T) = \frac{\mu p_0}{\varepsilon} \left( \frac{T}{T_0} \right)^{1/k}$$

As illustrated in Figure , eventually the temperature of the rising parcel falls enough for  $e_{\text{parcel}} = e_s$  and saturation occurs.

The pressure level at which this happens can be calculated from equation:

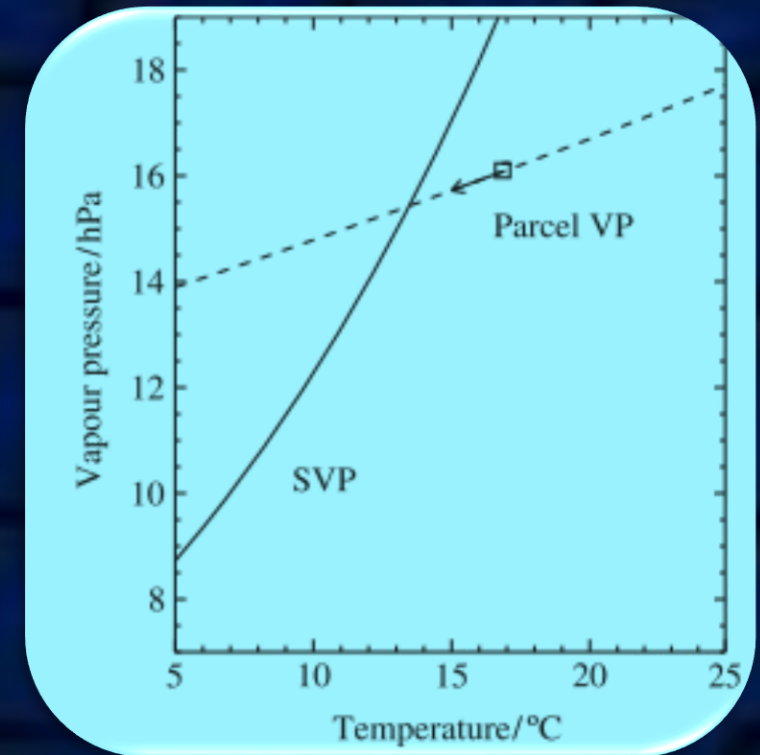
$$T = T_0 \left( \frac{p}{p_0} \right)^k$$

It is called the lifting condensation level. (However, it should be noted that in practice liquid need not form at saturation; the vapour may become supersaturated.

It is usually necessary for small 'condensation nuclei' to be present before liquid drops appear; see next Section.)

A useful related concept is the saturation mixing ratio, defined as:

$$\mu_s(T, p) = \frac{e_s(T) \varepsilon}{p} \quad (\text{g kg}^{-1})$$



If, at temperature  $T$  and pressure  $p$ , the mixing ratio

$$\mu < \mu_s(T, p) \implies e < e_s \quad \text{Air is unsaturated}$$

$$\mu = \mu_s(T, p), \implies e = e_s \quad \text{Air is saturated}$$

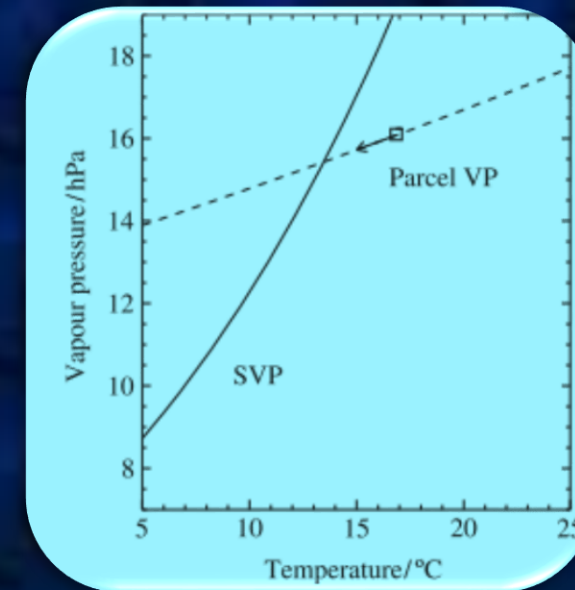
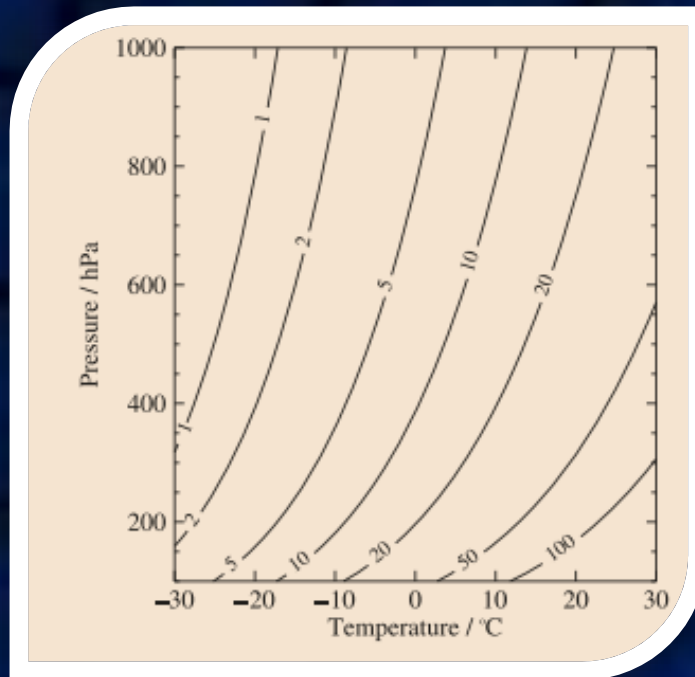
$$\mu > \mu_s(T, p), \implies e > e_s \quad \text{Air is supersaturated}$$

$$\text{from } \begin{cases} e = \frac{\mu p}{\varepsilon} \\ \mu_s(T, p) = \frac{e_s(T) \varepsilon}{p} \end{cases}$$

Fig. gives a plot of  $\mu_s$  as a function of temperature and pressure.

The overall behaviour of  $\mu_s$  is clearly consistent with the facts that it is inversely proportional to pressure and proportional to  $e_s$ , which increases with temperature (see previous Fig.).

$$\mu_s(T, p) = \frac{e_s(T) \varepsilon}{p}$$



Contours of the saturation mixing ratio  $\mu_s(T, p)$ , in units of  $\text{g kg}^{-1}$

The **dew point**  $T_d$  of a sample of air is the temperature to which the air must be cooled at constant pressure (i.e. not following a rising parcel), retaining its water vapour content, for it to become saturated.

Therefore, if the water vapour mixing ratio is  $\mu$ , the dew point  $T_d$  satisfies the implicit equation

$$\mu_s(T_d, p) = \mu$$

Equivalently, if the air sample initially has vapour pressure  $e$ , then

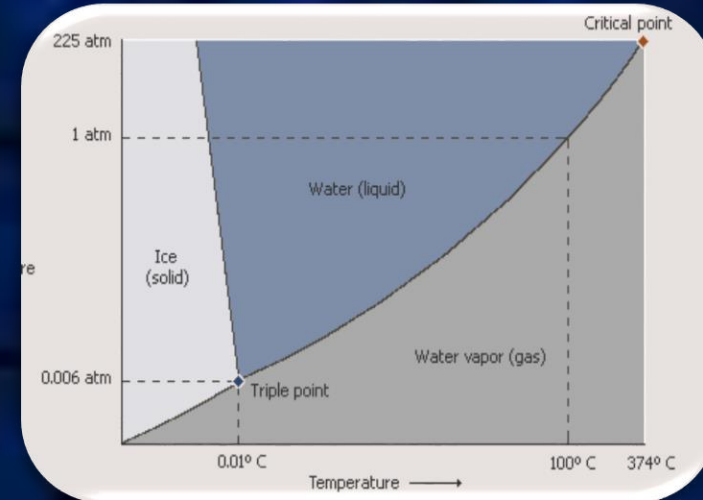
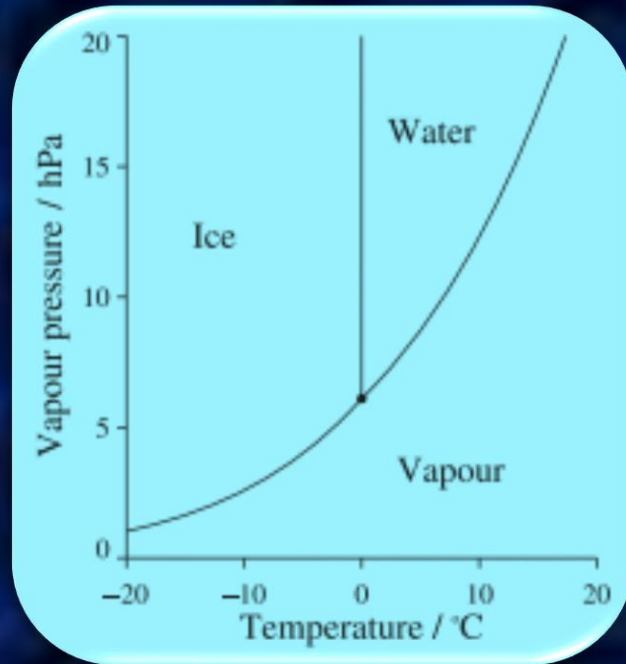
$$e_s(T_d) = e$$

We now briefly mention the ice phase.

In addition to the vapour-water phase transition just considered, there are also ice-water and ice-vapour transitions, as shown in Fig.

The three transition lines meet at the triple point,

$$T_t = 273 \text{ K} \quad \text{and} \quad p_t = 6.1 \text{ hPa.}$$

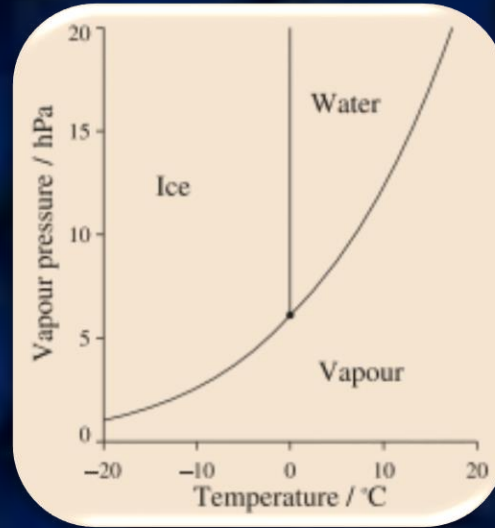


The phase transitions between ice, liquid water and water vapour.

The triple point is indicated by the small solid circle.

Note that the ice-water curve is not quite vertical, but has a large negative slope of about  $-1.4 \times 10^5 \text{ hPa K}^{-1}$  near the triple point.

The densities and, therefore, the specific volumes  $V$  of the three phases are different at the triple point:



the specific volume of liquid water is  $V_l = 1.00 \times 10^{-3} \text{m}^3 \text{kg}^{-1}$

the specific volume of ice is  $V_i = 1.09 V_l$

(unlike most substances, water expands on freezing)

the specific volume of water vapour is  $V_v \approx 2 \times 10^5 V_l$

The basic form

$$\frac{dp}{dT} = \frac{\delta S}{\delta V} = \frac{L}{T \delta V}$$

of the Clausius-Clapeyron equation applies to each of these transitions

$$\text{Since } V_v \gg V_i$$

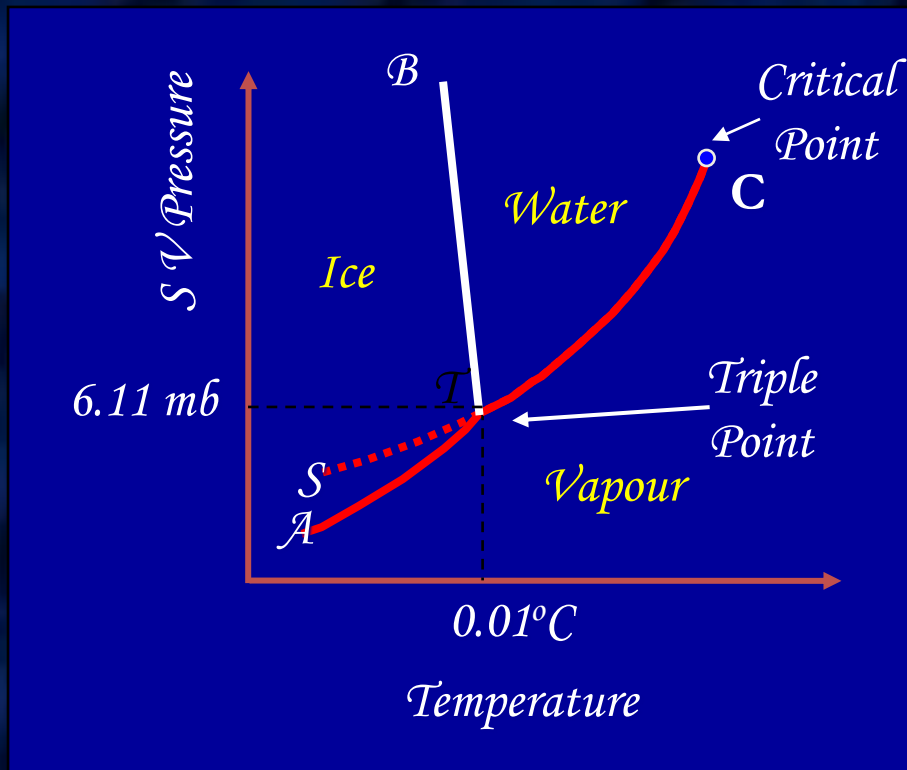
and the latent heat of sublimation (ice-vapour) is approximately constant, an approximate form of the sublimation curve can be found as in equation

$$e_s(T) = e_s(T_0) \exp \frac{L}{R_v} \left( \frac{1}{T_0} - \frac{1}{T} \right)$$

indeed, since the two latent heats are fairly similar in size, the two curves are quite close.

However, the form of the ice-water transition curve is very different: here  $\delta V$  is small and negative, since  $V_l$  is slightly less than  $V_i$ , but the latent heat of fusion is positive and non-negligible.

Hence the ice-water transition has a large negative slope, as indicated in Fig.



$$\frac{dp}{dT} = \frac{L_{tv}}{T(V_v - V_l)}$$

$$\frac{dp}{dT} = \frac{L_{tv}}{T(V_v - V_i)}$$

$$\frac{dp}{dT} = \frac{L_{tl}}{T(V_l - V_i)}$$