Atmospheric Physics

Lecture 4

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Hydrostatic balance

$$\frac{dp}{dz} = -g\rho.$$

equation for hydrostatic balance

$$p = R_a T \rho \rightarrow \rho$$

We can derive some basic properties of the atmosphere, given that it is an ideal gas and assuming that it is in hydrostatic balance.

$$\frac{dp}{dz} = -\frac{gp}{R_a T}.$$

If the temperature is a known function of height, T(z), we can in principle find the pressure and density as functions of height

$$\frac{d}{dz}(\ln p) = -\frac{g}{R_a T}$$

$$\ln p - \ln p_0 = -\frac{g}{R_a}\int_0^z \frac{dz'}{T(z')}$$

$$p = p_0 \exp\left(-\frac{g}{R_a}\int_0^z \frac{dz'}{T(z')}\right)$$

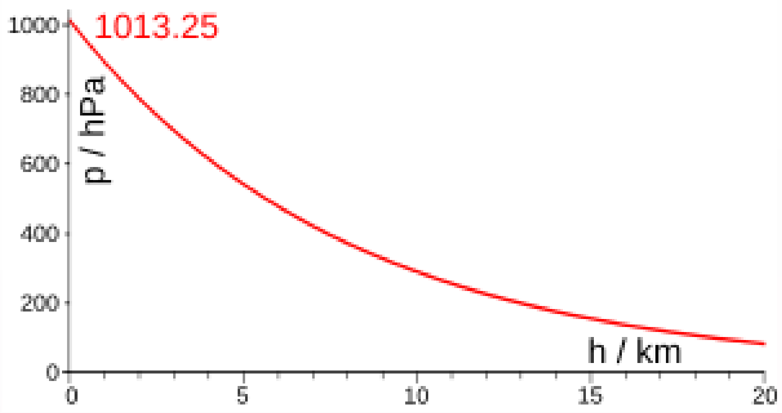
The simplest case is that of an isothermal temperature profile, i.e. $T = T_0$ = constant, when the pressure decays exponentially with height:

$$p = p_0 \exp\left(-\frac{gz}{R_a T_0}\right) = p_0 e^{-z/H},$$

$H = R_a T_0 / g$ the pressure scale height

The height over which the pressure falls by a factor of 1/e=0.368

Elevation	Density
0	ρ_0
H	$(1/e)\rho_0 = 0.368\rho_0$
2H	$(1/e^2)\rho_0 = 0.135\rho_0$
3H	$(1/e^3)\rho_0 = 0.050\rho_0$
4H	$(1/e^4)\rho_0 = 0.018\rho_0$



In this isothermal case the density also falls exponentially with height in the same way:

$$\rho = \rho_0 \exp(-z/H)$$

P_0 being the density at the ground.

For an isothermal atmosphere with $T_0 = 260$ K, *H is about 7.6 km*

For Earth, g = 9.81 meters/sec2 , T = 290 K.
The atmosphere consists of 22% O2 (m= $2 \times 2.67 \times 10^{-26}$ kg) and 78% N2 (m= $2 \times 2.3 \times 10^{-26}$ kg).
What is the scale height, H?

At what altitude on Earth would the density of the atmosphere P(z) be only 10% what it is at sea level, P0?

Calculate the total mass of the atmosphere in a column of air, below a height h with integral calculus. At what altitude, h, on Earth is half the atmosphere below you?

The lapse rate denotes the rate of decrease of temperature with height:

$$\Gamma(z) = -\frac{dT}{dz}$$

in general the temperature decreases with height ($\Gamma > 0$) in the troposphere and increases with height ($\Gamma < 0$) in the stratosphere;

A layer in which the temperature increases with height ($\Gamma < 0$) is called an inversion layer.

If Γ is constant in the region between the ground and some height z_1 ,say, then the temperature in that region decreases linearly with height and the integral in

$$p = p_0 \exp\left(-\frac{gz}{R_a T_0}\right) = p_0 e^{-z/H},$$

Another useful deduction from the hydrostatic equation in the form

$$\frac{dp}{dz} = -\frac{gp}{R_a T}.$$

is the 'thickness', or depth, of the layer between two given surfaces of constant pressure.

Suppose that the height of the pressure surface $p = p_1$ is z_1

and the height of the pressure surface $p = p_2$ is z_2

Then, if $p_1 > p_2$,we must have $z_1 < z_2$
since pressure decreases with height when hydrostatic balance applies

From $\frac{dp}{dz} = -\frac{gp}{R_a T}.$ $g\,dz = -R_a T\,d(\ln p);$ integration gives

$$z_2 - z_1 = -\frac{R_a}{g}\int_{p_1}^{p_2} T\, d(\ln p)$$

The integral can in principle be evaluated if the temperature T is known as a function of pressure p: this may be provided for example by a weather balloon or a satellite-borne instrument.

In particular, if T is constant,

$$z_2 - z_1 = \frac{R_a T}{g}\ln\left(\frac{p_1}{p_2}\right)$$

If T is not constant, we can still write

$$z_2 - z_1 = \frac{R_a \overline{T}}{g}\ln\left(\frac{p_1}{p_2}\right)$$

provided that we define T as a suitably weighted mean temperature within the layer:

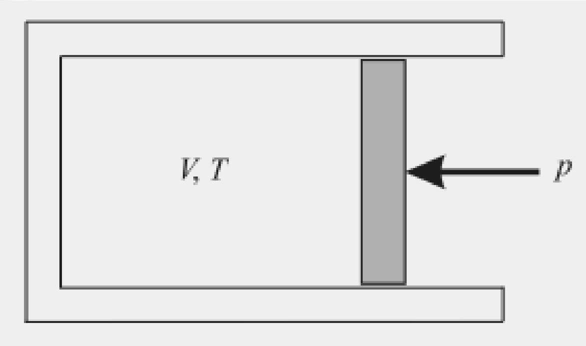
$$\overline{T} = \frac{\int_{p_2}^{p_1} T\, d(\ln p)}{\int_{p_2}^{p_1} d(\ln p)}.$$

Thus the thickness of the layer between two pressure surfaces is proportional to the mean temperature of that layer.

Entropy and potential temperature

The First Law of Thermodynamics, applied to a small change to a closed system, such as a mass of air contained in a cylinder with a movable piston at one end can be written

$$\delta U = \delta Q + \delta W, \qquad \delta U = T\,\delta S - p\delta V,$$



where S is the entropy of the system

An alternative form

$$\delta H = T\,\delta S + V\,\delta p$$

$H = U + pV$ the enthalpy

These equations apply both for reversible and for irreversible changes. However, we shall mostly restrict our attention to reversible changes, for which the equations

$$\delta Q = T\,\delta S, \qquad \delta W = -p\,\delta V$$

For unit mass of ideal gas, for which V = 1/ρ, it can be shown that

$$U = c_v T,$$

where c_v is the specific heat capacity at constant volume and is independent of T.

Therefore the ideal gas law, equation implies that, for unit mass of air

$$p = R_a T \rho$$

$$H = c_v T + R_a T = c_p T \qquad c_p = c_v + R_a$$

the specific heat capacity of air at constant pressure

On substituting the expression $H = c_v T + R_a T = c_p T$,

and $V = 1/\rho = R_a T/p$ into equation $\delta H = T\,\delta S + V\,\delta p$,

we get

$$T\,\delta S = c_p\,\delta T - \frac{R_a T}{p}\,\delta p.$$

Division by T gives

$$\delta S = c_p \frac{\delta T}{T} - R_a \frac{\delta p}{p} = c_p\,\delta(\ln T) - R_a\,\delta(\ln p).$$

and integration gives the entropy per unit mass

$$S = c_p \ln T - R_a \ln p + \text{constant} = c_p \ln(T p^{-\kappa}) + S_0,$$

$\kappa = R_a/c_p$, which is approximately 2/7 for a diatomic gas, and S_0 is a constan

An adiathermal process is one in which heat is neither gained nor lost, so that $\delta Q = 0$

An adiabatic process is one that is both adiathermal and reversible;

from equation $\delta Q = T\,\delta S,$ $\delta S = 0$

Imagine a cylinder of air, originally at temperature T and pressure p, that is compressed adiabatically until its pressure equals p_0.

We can find its resulting temperature, θ say, using equation

$$\delta S = c_p\,\delta(\ln T) - R_a\,\delta(\ln p).$$

with the fact that $\delta S = 0$,
For an adiabatic process, so that $c_p\,\delta(\ln T) = R_a\,\delta(\ln p)$

Integrating and using the end conditions T = θ and p = p_0 then gives

$$c_p \ln\left(\frac{\theta}{T}\right) = R_a \ln\left(\frac{p_0}{p}\right)$$

$$\kappa = R_a / c_p,$$

$$\theta = T\left(\frac{p_0}{p}\right)^{\kappa}$$

The quantity θ is called the potential temperature of a mass of air at temperature T and pressure p.

The value of p_0 is usually taken to be 1000 hPa.

Using equation

$$S = c_p \ln(Tp^{-\kappa}) + S_0.$$

it follows that the potential temperature is related to the specific entropy S by

$$S = c_p \ln\theta + S_1,$$

where S_1 is another constant

By definition, the potential temperature of a mass of air is constant when the mass is subject to an adiabatic change;

conversely, the potential temperature will change when the mass is subject to a non-adiabatic (or diabatic) change.

As we shall see, the potential temperature is often a very useful concept in atmospheric thermodynamics and dynamics.

Parcel concepts

For an adiabatically rising parcel, the potential temperature and entropy are constant as its height changes, so we can write

$$\left(\frac{d\theta}{dz}\right)_{\text{parcel}} = 0, \qquad \left(\frac{dS}{dz}\right)_{\text{parcel}} = 0$$

From equation

$$\delta S = c_p \frac{\delta T}{T} - R_a \frac{\delta p}{p} = c_p\, \delta(\ln T) - R_a\, \delta(\ln p),$$

we therefore have the following relation between the vertical derivatives of temperature and pressure, following the parcel:

$$0 = \frac{c_p}{T}\left(\frac{dT}{dz}\right)_{\text{parcel}} - \frac{R_a}{p}\left(\frac{dp}{dz}\right)_{\text{parcel}},$$

$$0 = \frac{c_p}{T}\left(\frac{dT}{dz}\right)_{\text{parcel}} - \frac{R_a}{p}\left(\frac{dp}{dz}\right)_{\text{parcel}},$$

$$-\left(\frac{dT}{dz}\right)_{\text{parcel}} = -\frac{R_a T}{c_p p}\left(\frac{dp}{dz}\right)_{\text{parcel}}$$

$$\frac{dp}{dz} = -\rho g \qquad pV_m = RT,$$

$$-\left(\frac{dT}{dz}\right)_{\text{parcel}} = \frac{g}{c_p} \equiv \Gamma_a,$$

The quantity Γ_a is the rate of decrease of temperature with height, following the adiabatic parcel as it rises.

It is called the adiabatic lapse rate;