Atmospheric Physics

Lecture 19

J. Sahraei

Physics Department,

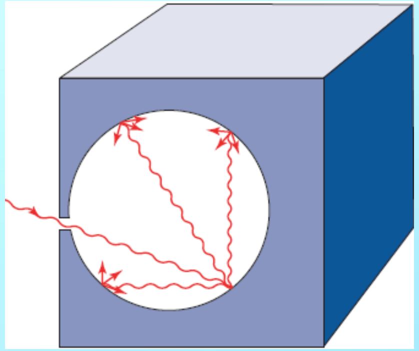
Razi University

https://sci.razi.ac.ir/~sahraei

Blackbody Radiation

A blackbody – the *maximum* amount of radiation that can be emitted by a surface at a given temperature.

The concept of a black body is an idealisation



A real body will emit less radiation than this

It follows that a black body emits the maximum possible amount of energy in each frequency interval, at a given temperature

Spectral Emittance

The spectral emittance ε_ν of a body is the ratio of the spectral radiance from that body to the spectral radiance from a black body;

therefore $\varepsilon_\nu \leq 1$

We can also define the spectral absorptance α_ν

as the fraction of energy per unit frequency interval falling on a body that is absorbed

Kirchhoff's law states that $\varepsilon_\nu = \alpha_\nu$

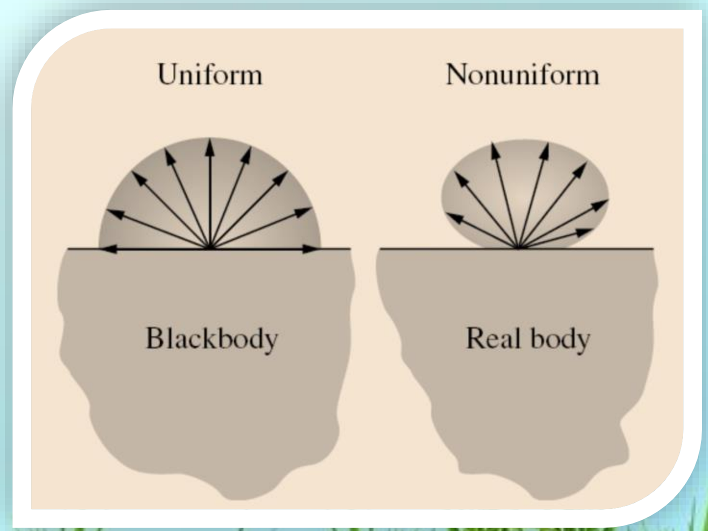
that is, at a given temperature and frequency the spectral emittance of a body equals its spectral absorptance

ATransparent bodies radiate energy in spherical space.

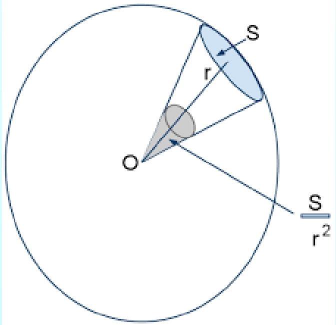
Non-transparent bodies radiate energy in hemi-spherical space.

The radiation energy emitted by a body is distributed in space at various wavelengths.

This complex phenomenon requires simplified laws for engineering use of radiation.



The Planck function



Black-body radiation

Planck's law states that the spectral energy density of black-body radiation at absolute temperature *T is given by*

$$u_\nu(T) = \frac{8\pi h\nu^3}{c^3(e^{h\nu/k_BT} - 1)}$$

where k_B is Boltzmann's constant

Since the photons carrying this energy are moving isotropically, the energy density associated with the group of photons moving within a small solid angle $\Delta\Omega$

$$u_\nu \Delta\Omega / (4\pi)$$

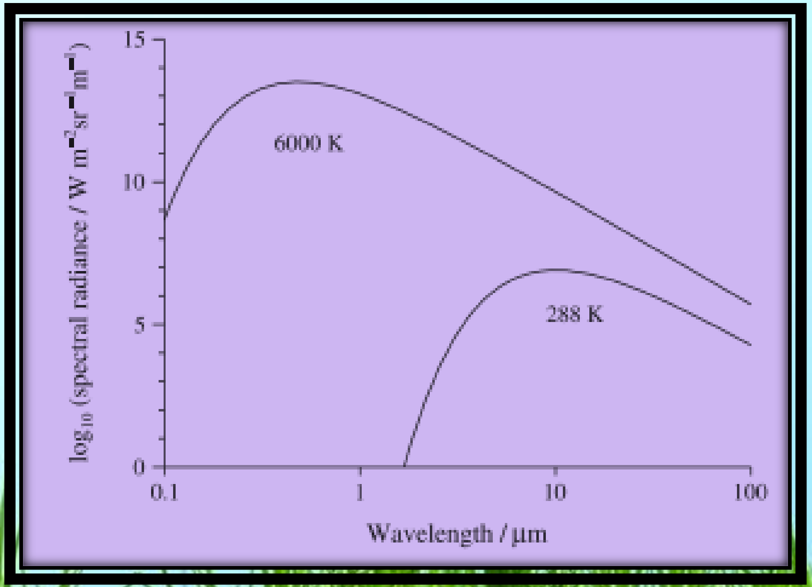
The power per unit area, per unit solid angle, per unit frequency interval (the spectral radiance) for black-body radiation at temperature *T is*

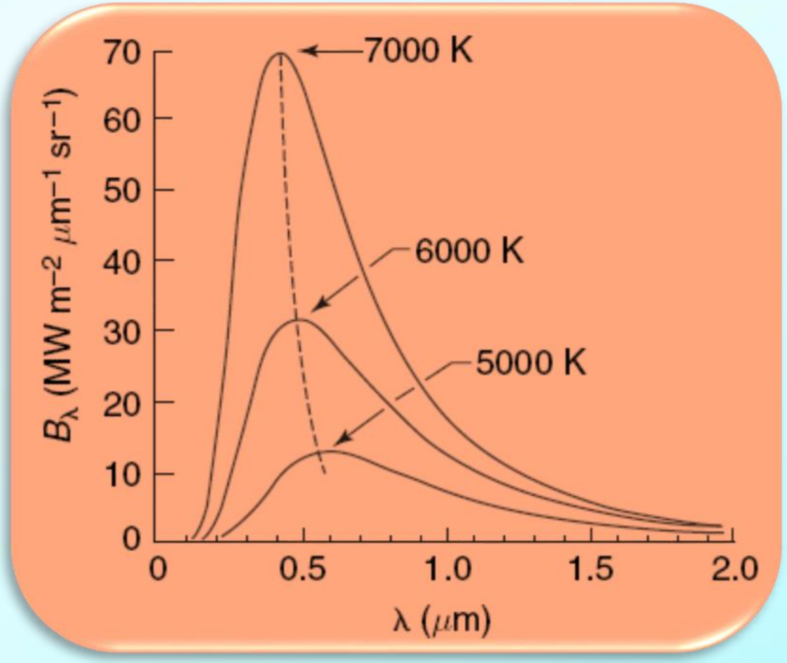
$$B_\nu(T) = \frac{2h\nu^3}{c^2(e^{h\nu/k_BT} - 1)}$$

The Planck function

The black-body spectral radiance can also be written in terms of the power per unit area, per unit solid angle, per unit wavelength interval,

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5(e^{hc/\lambda k_BT} - 1)}$$

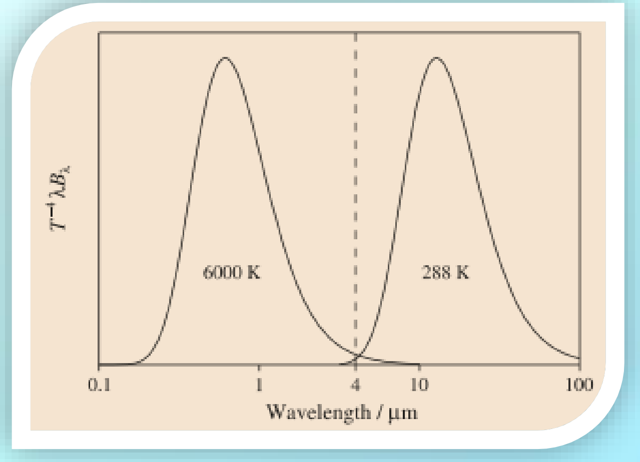




Blackbody radiation curves at three different temperatures (K; see legend in upper right corner).

If B_λ is integrated over all wavelengths, we obtain the black-body radiance

$$\int_0^\infty B_\lambda(T)\,d\lambda = \frac{\sigma}{\pi}T^4$$



where σ is the Stefan–Boltzmann constant

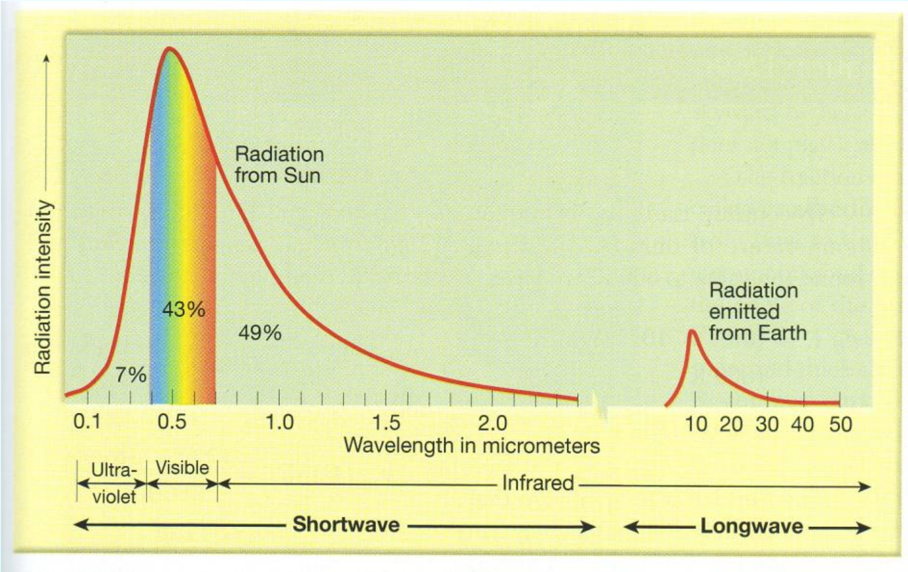
In terms of an integral over $\ln(\lambda)$, this gives

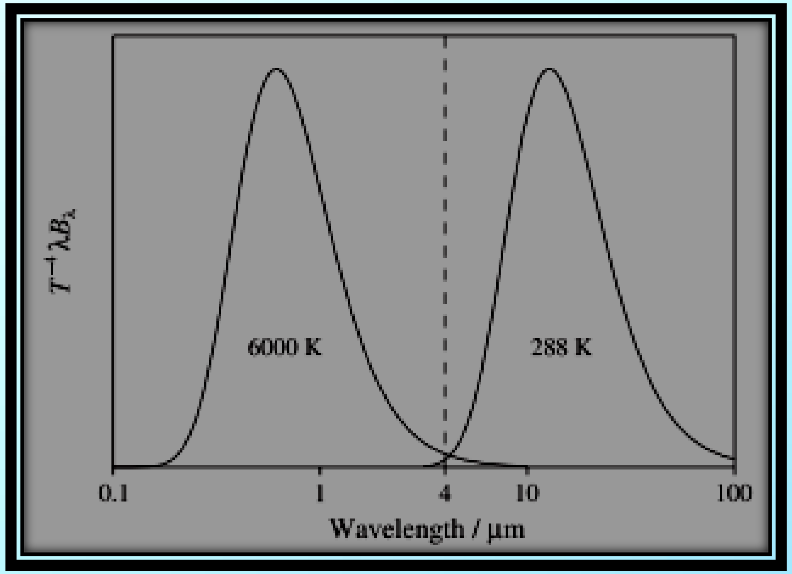
$$T^{-4}\int_{-\infty}^{\infty} \lambda B_\lambda(T)\, d(\ln\lambda) = \frac{\sigma}{\pi}$$

This suggests plotting $T^{-4}\lambda B_\lambda$ against $\ln\lambda$:

the area under the resulting curve is then independent of *T*.

Note that, with this normalisation, there is little overlap between the black-body spectral radiances at 6000





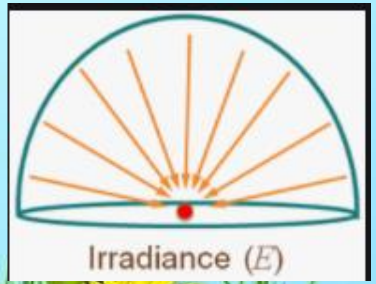
What is the total irradiance of any object?

If the Planck distribution function spectral irradiance is integrated over all wavelengths, then the total irradiance emitted into a hemisphere is given by the Stefan–Boltzmann Law:

$$R = \sigma T^4$$

R has SI units of W m^{-2}, where the m^2 refers to the surface area of the object that is radiating.

The Stefan–Boltzmann law (total) irradiance applies to an object that radiates according to the Planck distribution function spectral irradiance.



Stefan-Boltzmann Law

This law expresses the rate of radiation emission per unit area

$$R = \sigma T^4 \qquad \sigma = 5.67 \times 10^{-8}\, W / m^2 K^4$$

Compare the difference between the radiation emission from the sun and the Earth.

The sun with an average temperature of 6000 K emits 73,483,200 W/m^2

By contrast, Earth with an average temperature of 300 K emits 459 W/m^2

The sun has a temperature 20 times higher than Earth and thus emits about 160,000 times more radiation

This makes sense, $20^4 = 160{,}000$

Local thermodynamic equilibrium

$$B_\nu(T) = \frac{2h\nu^3}{c^2(e^{h\nu/k_BT} - 1)}$$

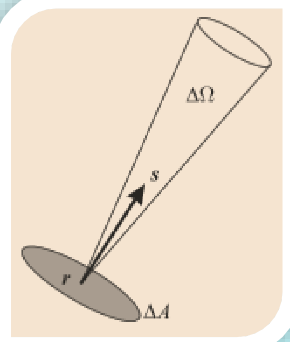
The Planck function

h = 6.6262x10^{-34} joule sec
k =1.3806x10^{-23} joule deg^{-1}
c =2.99793x10^{+8} m/s
T = object temperature in Kelvins

$$n_1 = g_1 e^{-E_1/k_BT}$$

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} e^{-(E_1 - E_2)/(k_BT)}$$

The radiative-transfer equation



Radiometric quantities

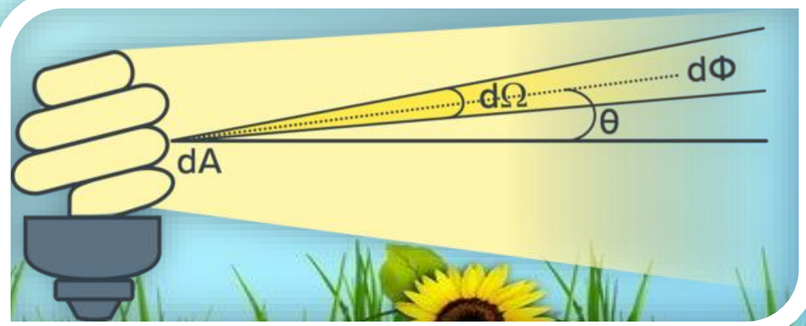
Several different, but related, quantities are used in the description and measurement of radiation. The most important are as follows.

The spectral radiance (or monochromatic radiance) $L_\nu(r, s)$

is the power per unit area, per unit solid angle, per unit frequency interval in the neighbourhood of the frequency ν, at a point r, in the direction of the unit vector s.

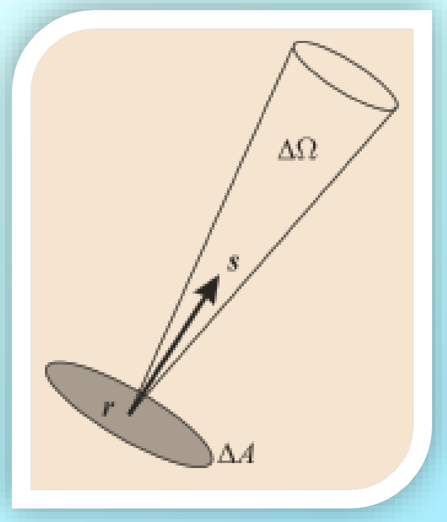
It is measured in W m^{-2} stradian^{-1} Hz^{-1}

$$L_\nu(r,s) = \frac{d\Phi(\nu)}{dA \, \cos\theta \, d\Omega \, d\nu}$$



The spectral radiance can be visualised in terms of the photons emerging from a small area ΔA with unit normal s, centred at a point r

$$L_\nu \, \Delta A \, \Delta\Omega \, \Delta\nu$$



Is energy transferred by these photons, per unit time, from 'below ' the area A to 'above'. (Here 'below' means in the direction -s and 'above' means in the direction s.)

We have already encountered a special case of the spectral radiance, for isotropic black body radiation in an isothermal cavity, when

$$L_\nu = B_\nu(T),$$

The radiance $L(r,s)$ is the power per unit area, per unit solid angle at a point r in the direction of the unit vector s; in other words it is the integral of L_ν over frequency

$$L(r,s) = \int_0^\infty L_\nu(r,s)\,d\nu$$

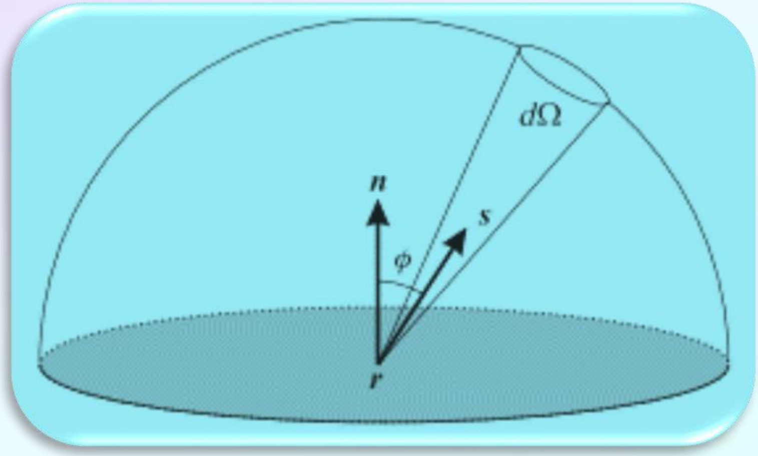
Its units are in W m^{-2} $stradian^{-1}$

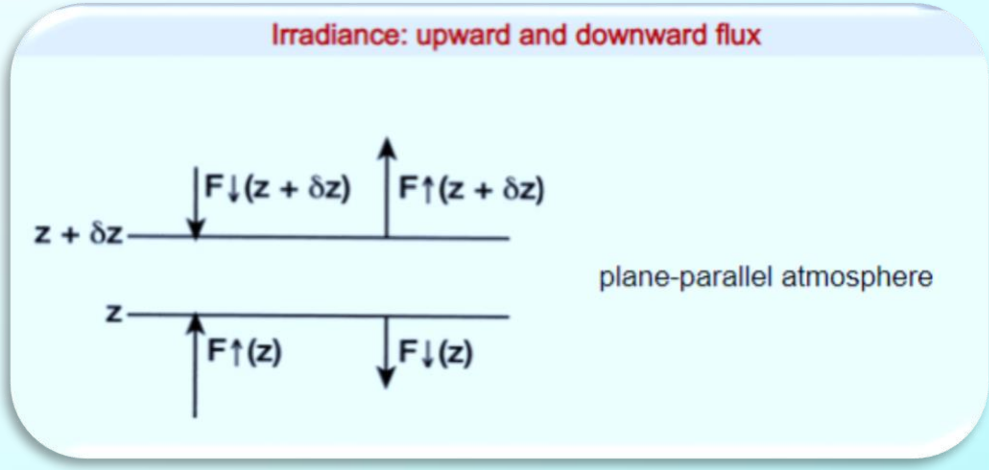
The spectral irradiance (or monochromatic irradiance) $F_\nu(r, n)$:

is the power per unit area, per unit frequency interval in the neighbourhood of the frequency ν, at a point r through a surface of normal n;

$$F_\nu(\hat{r},\hat{n}) = \int_{2\pi} L_\nu(\hat{r},\hat{s})\,\hat{n}.\hat{s}\;d\Omega(s)$$

its units are W m^{-2} Hz^{-1}





The irradiance (or flux density) F(r, n) is the power per unit area at a point r through a surface of normal n, i.e. the integral of F_ν over frequency, and also the integral of the radiance L over a hemisphere:

$$F(\hat{r},\hat{n})=\int_0^\infty F_\nu(\hat{r},\hat{n})d\nu = \int_{2\pi} L(\hat{r},\hat{s})\,\hat{n}.\hat{s}\,d\Omega(s)$$

its units are W m^{-2}

$$F^{\downarrow} = F(\hat{r},-\hat{n}) \qquad F_z = F^{\uparrow} - F^{\downarrow} \qquad L_\nu = B_\nu(T)$$

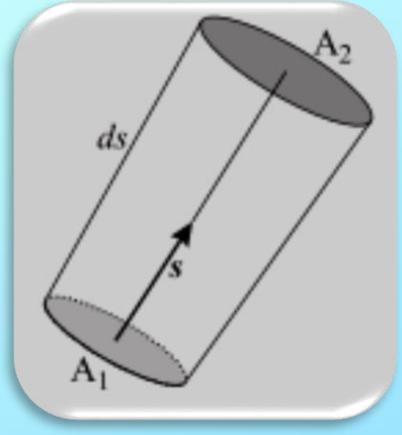
Equation

$$F_\nu(\hat{r},\hat{n})=\int_{2\pi} L_\nu(\hat{r},\hat{s})\,\hat{n}.\hat{s}\; d\Omega(s)$$

for the spectral irradiance becomes

$$F_\nu(\hat{r},\hat{n})=\int_{2\pi} L_\nu \hat{n}.\hat{s}\; d\Omega(s)=2\pi B_\nu \int_0^{\pi/2} \cos\phi \sin\phi d\phi=\pi B_\nu(T)$$

$$d\Omega=2\pi \sin\phi d\phi$$



since there is axisymmetry around the normal. Integrating over all v we obtain the Stefan–Boltzmann law for the irradiance

$$F(\hat{r},\hat{n})=\pi\int_0^{\infty} B_\nu(T)d\nu=\sigma T^4$$