دانشگاه رازی

Micrometeorology

Lecture 6

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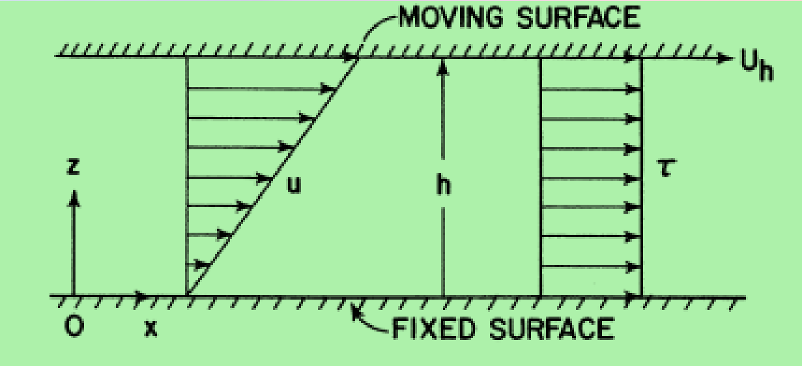
Momentum Fluxes

$$\tau = \mu \frac{\partial u}{\partial z} - \rho \overline{u'w'}$$



$$\tau = \mu(\partial u/\partial z)$$

$$Momentum\ Flux = \rho\overline{uw}$$

$$u = \bar{u} + u'$$

$$w = \bar{w} + w'$$

$$\overline{uw} = \bar{u}\bar{w} + \overline{u'w'}$$

$$\bar{w} = 0$$

$$\overline{uw} \approx \overline{u'w'}$$

$$\tau = -\rho\overline{u'w'}$$

How do we measure u'?

$$u = (\bar{u} + u')$$

A sonic anemometer measures at very high sampling rates. This is typically at 10 or 20 Hz. Data from a high temporal resolution time series is used to calculate the mean of the time series and subsequently a perturbation from the mean.

Once the perturbations are calculated, turbulent statistics can be calculated.

$$u' = u - \bar{u}$$

Variance, Standard Deviation and Turbulence Intensity

One statistical measure of the dispersion of data about the mean is the variance, σ_A^2, defined by:

$$\sigma^2 = \frac{1}{N}\sum_{i=0}^{N-1}(A_i - \overline{A})^2$$

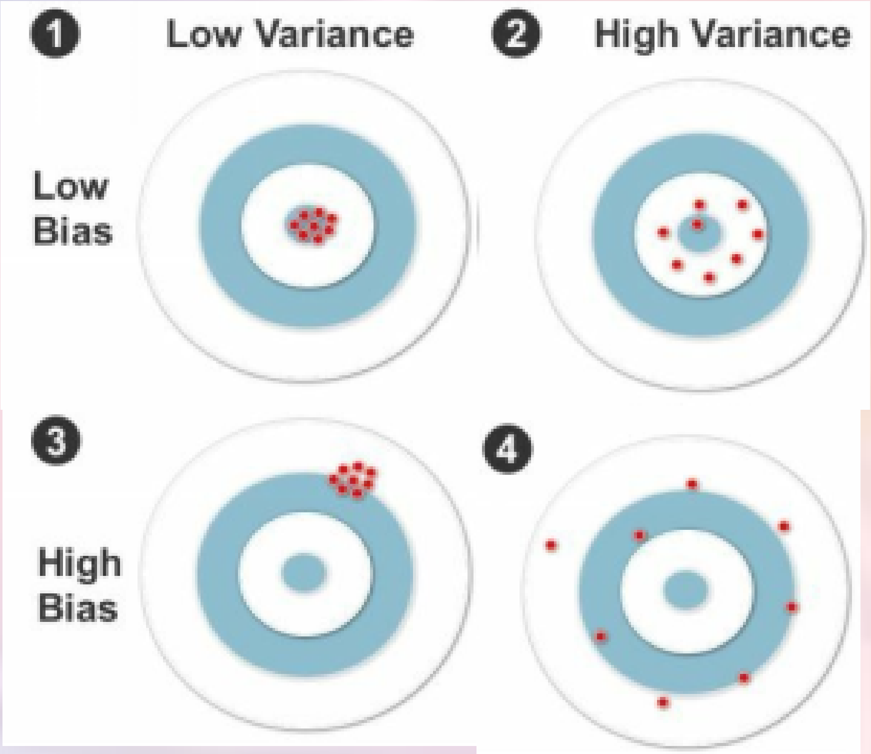
$$a' = A - \overline{A}$$

$$\sigma^2 = \frac{1}{N}\sum_{i=0}^{N-1}(a_i')^2 = \overline{a'^2}$$

This is known as the biased variance.

It is a good measure of the dispersion of a sample of BL observations, but not the best measure of the dispersion of the whole population of possible observations. A better estimate of the variance (an unbiased variance) of the population, given a sample of data, is

$$\sigma_A^2 = \frac{1}{(N-1)}\sum_{i=0}^{N-1}(A_i - \overline{A})^2$$


1
Low Variance
2
High Variance
Low
Bias
3
High
Bias
4

Recall that the turbulent part (or the perturbation or gust part) of a turbulent variable is given by $A = \overline{A} + a'$

Substituting this into the biased definition of variance gives:

$$\sigma_A^2 = \frac{1}{N}\sum_{i=0}^{N-1} a'^{\,2}_i = \overline{a'^2}$$

Thus, whenever we encounter the average of the square of a turbulent part of a variable, such as

$\overline{u'^2}$, $\overline{v'^2}$, $\overline{w'^2}$, $\overline{\theta'^2}$, $\overline{r'^2}$, or $\overline{q'^2}$,

we can interpret ese as variances.

The standard deviation is defined as the square root of the variance:

$$\sigma_A = \left(\overline{a'^2}\right)^{1/2}$$

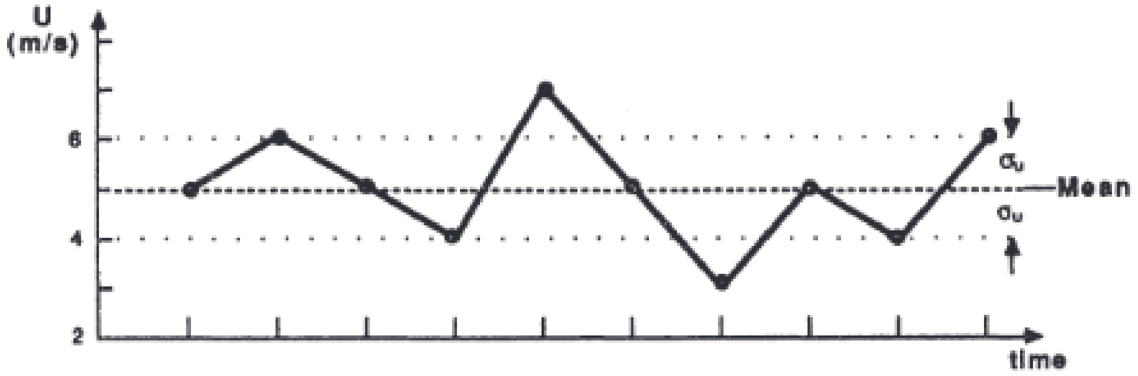


Fig. 2.6 Relationship between the standard deviation and turbulence variations. Solid line connects the data points; heavy dashed line is the average; dotted lines are drawn one standard deviation above and below the mean. Based on the example data in section 2.4.5.

$$I = \sigma_M / \overline{M}$$

Covariance and Correlation

In statistics, the covariance between two variables is defined as

$$\text{covar}(A,B) \equiv \frac{1}{N}\sum_{i=0}^{N-1}(A_i - \overline{A})\cdot(B_i - \overline{B})$$

Using our Reynolds averaging methods, we can show that:

$$\text{covar}(A,B) \equiv \frac{1}{N}\sum_{i=0}^{N-1} a_i'\, b_i' = \overline{a'\, b'}$$

The covariance indicates the degree of common relationship between the two variables, A and B.

$$r_{AB} \equiv \frac{\overline{a'\, b'}}{\sigma_A\, \sigma_B}$$

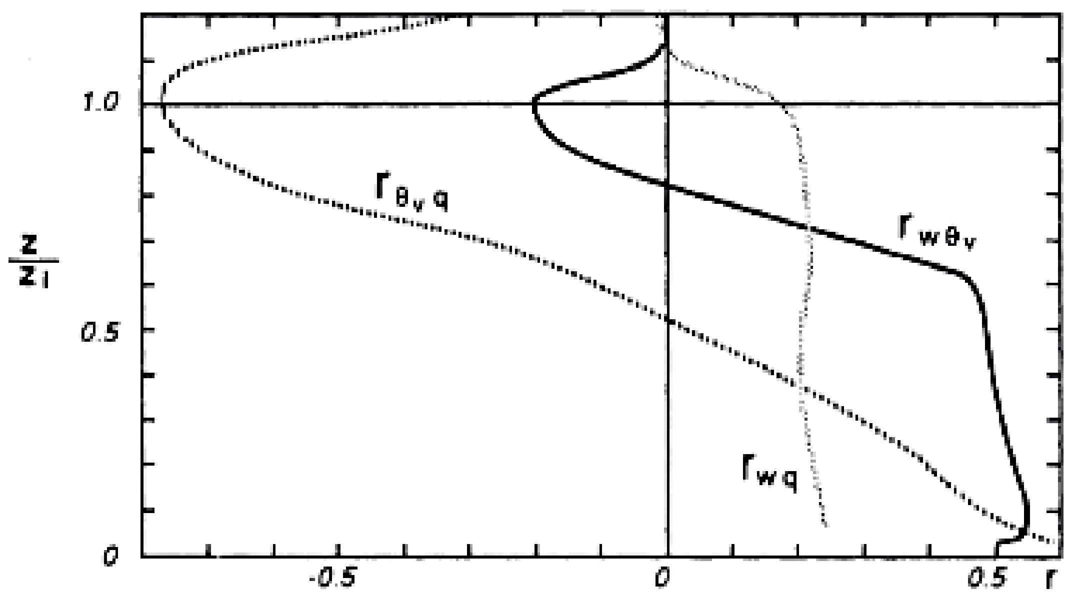


Fig. 2.7 Correlation coefficient profiles in the convective mixed layer.

Example

Suppose that we erect a short mast instrumented with anemometers to measure the U and W wind components. We record the instantaneous wind speeds every 6 s for a minute, resulting in the following 10 pairs of wind observations:

U (m/s):	5	6	5	4	7	5	3	5	4	6
W (m/s):	0	-1	1	0	-2	1	2	-1	1	-1

Find the mean, biased variance, and standard deviation for each wind component . Also, find the covariance and correlation coefficient between U and W.

$\overline{U} = 5\ m \cdot s^{-1}$

$\overline{W} = 0\ m \cdot s^{-1}$

$\overline{u'w'} = -1.10\ m^2 \cdot s^{-2}$

$\sigma_U^2 = 1.20\ m^2 \cdot s^{-2}$

$\sigma_W^2 = 1.40\ m^2 \cdot s^{-2}$

$\sigma_U = 1.10\ m \cdot s^{-1}$

$\sigma_W = 1.18\ m \cdot s^{-1}$

$r_{UW} = -0.85$ (dimensionless)

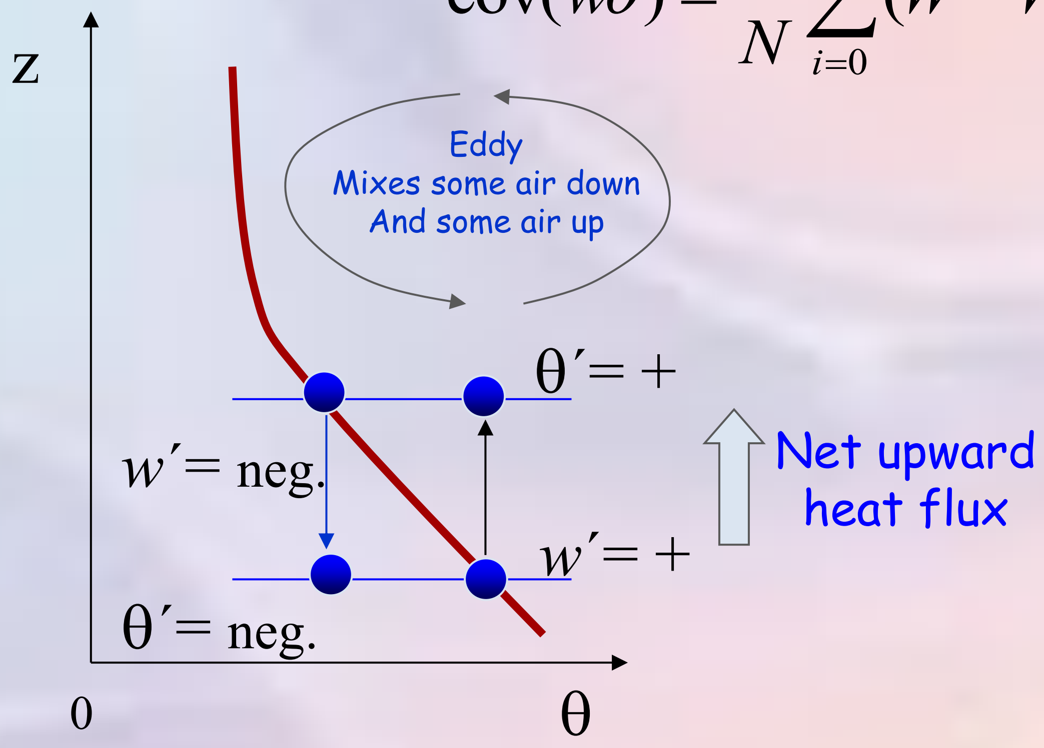
Turbulent Flux?

As a conceptual tool, suppose we examine a small idealized eddy near the ground on a hot summer day (see Fig.).

Transport of a quantity by eddies or swirls.

The covariance of a velocity component and any quantity.

$$\mathrm{cov}(w\theta) = \frac{1}{N}\sum_{i=0}^{N-1}(W-\overline{W})\cdot(\theta-\overline{\theta}) = \frac{1}{N}\sum_{i=1}^{N} w_i'\theta_i' = \overline{w'\theta'}$$



Turbulent Sensible Heat Flux

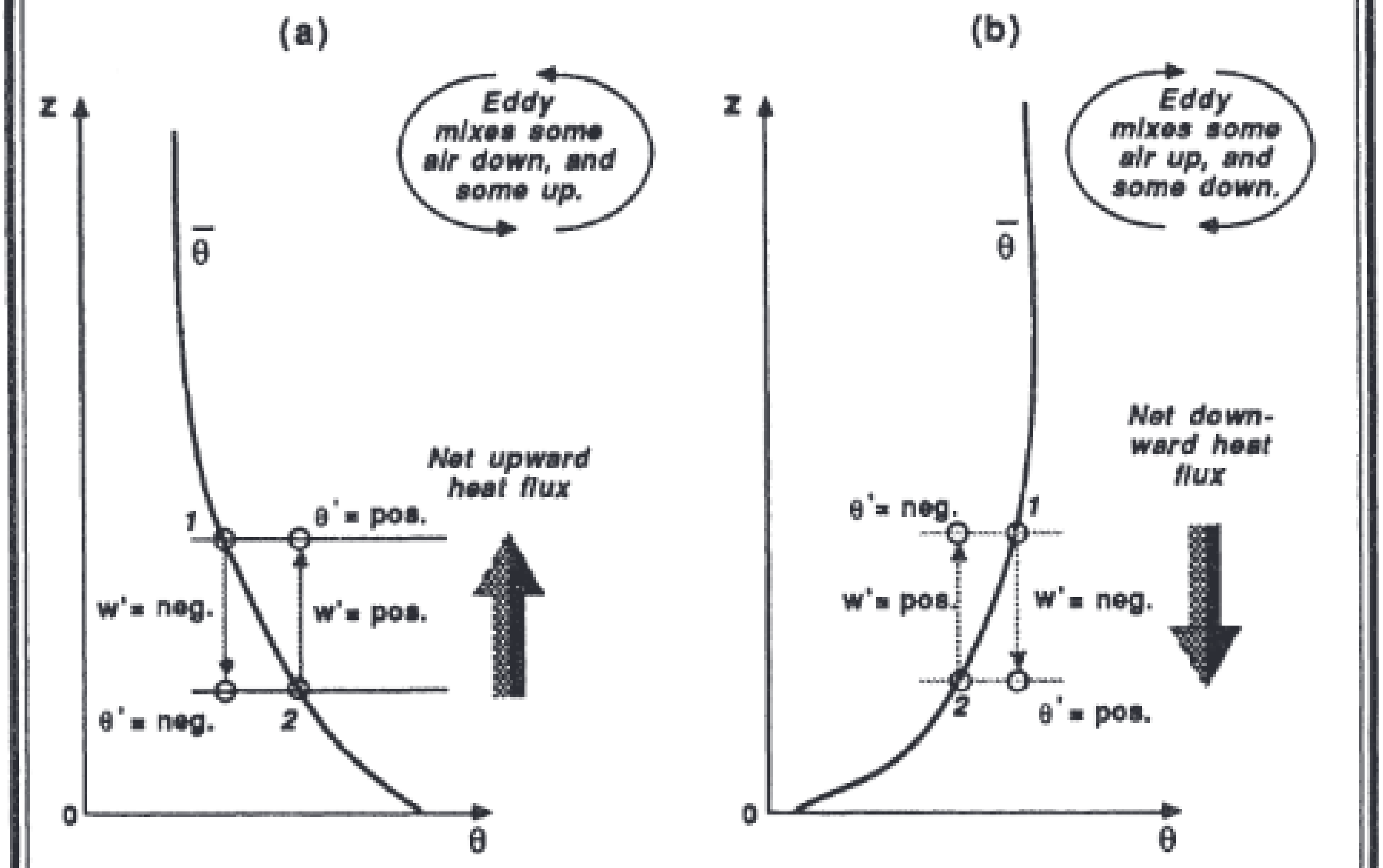


Fig. 2.12 Idealization of the small eddy mixing process, showing (a) net upward turbulent heat flux in a statically unstable environment, and (b) net downward turbulent heat flux in a stable environment.

As before, we can extend our arguments to write various kinds of eddy flux :

Vertical kinematic eddy heat flux $= \overline{w'\theta'}$

Vertical kinematic eddy moisture flux = $\overline{w'q'}$

x-direction kinematic eddy heat flux = $\overline{u'\theta'}$

Vertical kinematic eddy flux of u-momentum = $\overline{u'w'}$

The last flux is also the x-direction kinematic eddy flux of W-momentum.

Comparing the advective fluxes to the eddy fluxes, it is important to recognize that $\overline{W} \cong 0$ throughout most of the boundary layer.

Turbulence Kinetic Energy

The usual definition of kinetic energy (KE) is KE = 0.5 m M^2, where m is mass.

When dealing with a fluid such as air it is more convenient to talk about kinetic energy per unit mass, which is just 0.5 M^2

$$MKE/m = \frac{1}{2}\left(\overline{U}^2 + \overline{V}^2 + \overline{W}^2\right)$$

$$e = \frac{1}{2}\left(u'^2 + v'^2 + w'^2\right)$$

where e represents an instantaneous turbulence kinetic energy per unit mass.

There is an additional portion of the total KE consisting of mean-turbulence products, but this disappears upon averaging.

By averaging over these instantaneous values, we can define a mean turbulence kinetic energy (TKE) that is more representative of the overall flow:

$$\frac{TKE}{m} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) = \bar{e}$$

We can immediately see the relationship between TKE/m and the definition of variance defined in the last section. It is apparent that statistics will play an important role in our quantification of turbulence.

The turbulence kinetic energy is one of the most important quantities used to study the turbulent BL.

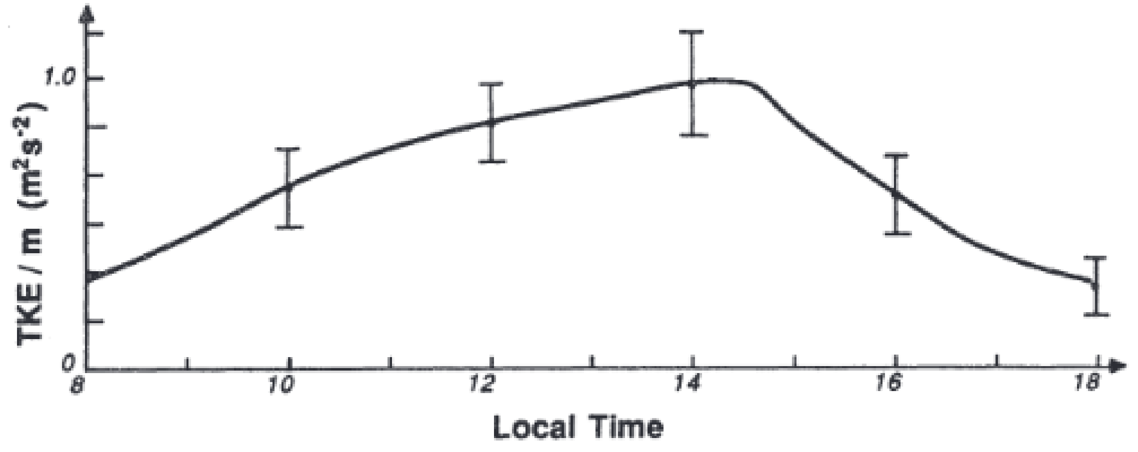


Fig. 2.8 Diurnal cycle of TKE/m = 1/2 ($\overline{u'^2}$+ $\overline{v'^2}$ +$\overline{w'^2}$) measured by aircraft below 300 m agl over Tennessee, August 1978. Adapted from Bean and Repoff, 1983.

A typical daytime variation of TKE in convective conditions is shown in Fig

Examples of the vertical profile of TKE for various boundary layers are shown in Fig.

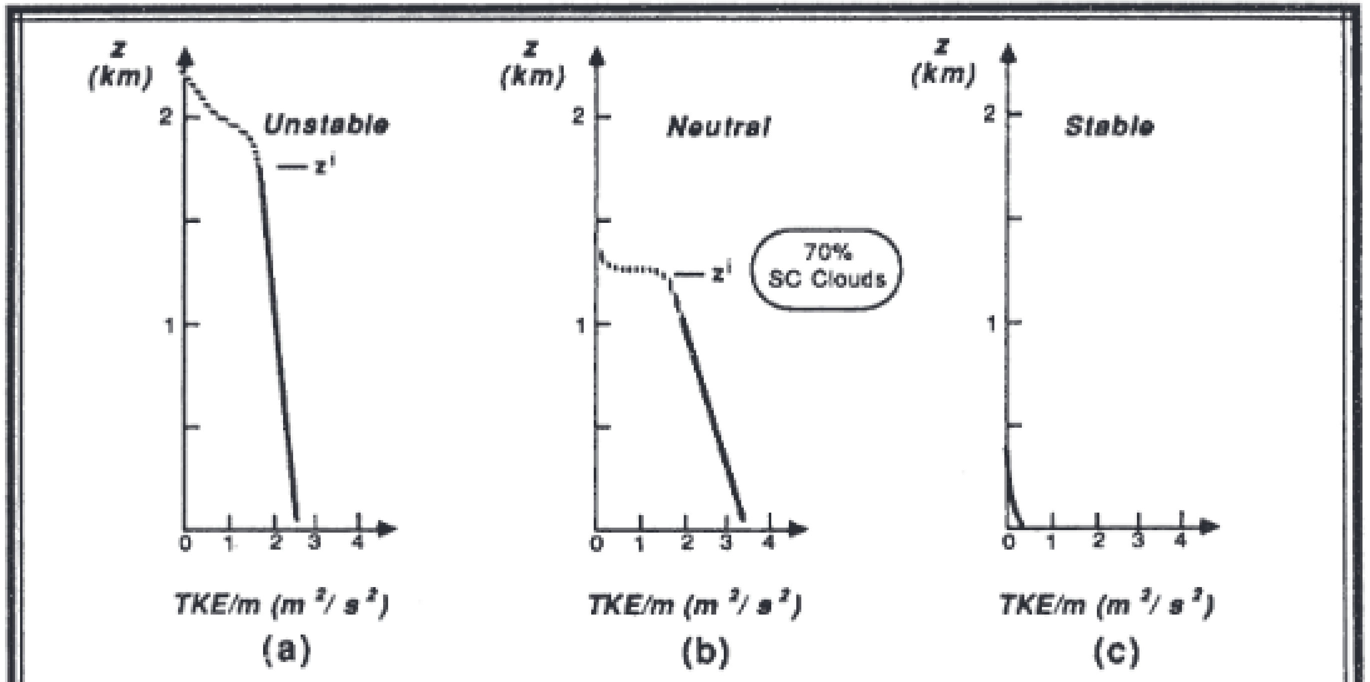


Fig. 2.9 Examples of turbulence kinetic energy per unit mass (TKE/m = $1/2\,(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$) for various boundary layers: (a) Daytime convective mixed layer with mostly clear skies and light winds 1600 local time, 14 June 1983, Chickasha, OK; (b) Near neutral day with strong winds (10-15 m/s at surface) and broken clouds at 1100 local time, 12 June 1983, Chickasha, OK; (c) Nocturnal stable boundary layer at 1000 local time, 11 September 1973 in Minnesota (from Caughey, et al, 1979,) Part (c) data excludes $\overline{v'^2}$ contributions because of measurement errors.